Application of Pattern-Mixture Models for Evaluation of Estimation Methods Under Responsive Designs October 2012

Randall Powers and John L. Eltinge
Office of Survey Methods Research, U.S. Bureau of Labor Statistics
powers.randall@bls.gov

Abstract
In work with data collected under a responsive design, most analytic approaches may be viewed as extensions of methods developed previously under, respectively, *selection models* or *pattern-mixture models* for nonresponse. Under selection models, one approximates the probability of specified responses (or, more generally, the probability of observing certain profiles of paradata) as a function of observable information from frame data, survey data and paradata. Under pattern-mixture models, one views the moment structure of observed survey data as functions of specified response patterns (or, more generally, specified patterns of observed paradata). For the pattern-mixture approach, an especially important issue is the use of constraints on subpopulation moments to ensure that the resulting models are estimable from available data.

Following a brief review of these concepts, this paper presents some simulation-based evaluations of the properties of the estimators based on the pattern-mixture approach. Special attention is directed toward evaluation of these properties under moderate deviations from assumed conditions.

Key Words: Cost-quality trade-offs; Model identification informaton; Nonresponse; Paradata; Pattern-mixture model; Selection model.

1. Introduction

This paper consider the use of pattern-mixture model methods for analysis of survey data collected under a responsive design. Section 2 provides an overview of selection-model and pattern-mixture model approaches to survey nonresponse. Section 3 gives a brief review some of the recent literature on responsive designs. Section 4 outlines some ways in which one may extend previous pattern-mixture approaches to data collected under a responsive design. Section 5 presents some results from a simulation study and related discussion.

2. Approaches to Nonresponse

Our work will center on standard survey environments, in which an analyst wants to carry out estimation and inference for population means, totals, quantiles or model coefficients associated with a finite population U or the with the associated superpopulation. As usual, the survey organization selects a sample through a complex design. However, in most practical applications, we do not observe all of the sample data of interest. In some cases, we have *unit* nonresponse, in which we do not observe any survey data for some sample units i. In other cases, we have *wave* or *item* nonresponse, in which we are unable to observe some parts of our data X for some sample units i.

To develop the ideas for this paper, we will use the following notation. Let R_{ij} be the response indicator for item j from sample unit i. In addition, Z_i will be a vector of *paradata* related to the survey process and other factors that may affect survey response.

With this notation, the survey literature has considered two general approaches to nonresponse. The first centers on *selection models*, in which we develop an explicit model for the probability of response as a function of the observed portions of our data X and Z. The second approach centers on *pattern-mixture models*, in which we focus attention on the moments (usually the mean and covariance matrix) of our survey variables X, *conditional on certain observed "patterns" of nonresponse*.

This pattern-mixture approach has been primarily of interest for cases with multivariate X, subject to differing patterns of nonresponse across items or waves. For example, in a panel survey, one pattern would arise if a unit i responds on waves 1 and 3, but not on wave 2. For the current discussion, it is useful to note that indicators for these observed "response patterns" can be viewed as a form of paradata.

Work with pattern-mixture models can involve a number of technical issues. One issue of special importance centers on *model identification*. To have an identified model in a pattern-mixture setting, one generally imposes identifying restrictions on some of the mean and covariance parameters, or on related regression coefficients. Pattern-mixture models have both Bayesian and frequentist interpretations. Please refer to Glynn, Laird and Rubin (1986), Little (1993, 1994), Little and Wang (1996), and Andridge and Little (2011) for general background on these models and on interpretation of the resulting analyses.

3. Responsive Designs

Within the general area of nonresponse, most of the formal mathematical literature has considered survey procedures – including fieldwork and adjustment methods – as fully specified before one begins sample selection and data collection. However, practical applications often require some degree of field intervention to address nonresponse and other problems in data quality and cost. In recent years, the statistical literature has explored these *field-intervention* ideas through development of methods known as "responsive design." For more details, please refer to Groves and Heeringa (2006); Laflamme and Karaganis (2010); Peytchev et al. (2010); Gonzalez (2012); Beaumont, Haziza and Bocci (2012); and references cited therein.

In exploration of responsive-design and related approaches to field intervention, one can consider two fairly distinct cases. For the first case, we have field intervention with sample units for which we already have some paradata Z, and may have some of our survey data X. This setting is conceptually similar to some aspects of on-line industrial quality control. For the second case, we have field intervention with "new" sample units. In this second case, we do not have any survey data or paradata for these "new" sample units, but our previous sample data may have provided us with improved information on the parameters of a given selection model or pattern-mixture model. This second case is conceptually similar to some aspects of off-line industrial quality control. For the remainder of this paper, we will focus attention on the first case.

4. Application of Pattern-Mixture Models to Data from Responsive Designs

In contrast with the previous responsive-design literature – which has tended to emphasize a selection-modeling approach – this paper will consider some ways in which to extend previous pattern-mixture work to the context of a responsive design. Here, we will place special emphasis on the moments of our survey variables X conditional on our observed paradata Z .Some examples of Z are simple indicators of item- or wave-level nonresponse, as considered previously for pattern-mixture models. Other examples of Z are process variables (for example, soft-refusal indicators), or observable demographic or economic characteristics that are considered important predictors of the response process and the survey variables.

To explore these ideas in some detail, we will consider a relatively simple example. Our paradata vector Z will have three components – Z_1, Z_2 and Z_3, where each of these components is an independent Bernoulli random variable. Our survey data X will follow a four-dimensional normal distribution. For this work, both the mean vector and the covariance matrix for X_i are allowed to vary across i. In particular, these moments will depend on the paradata Z_i, through the functions

$$\mu_{Xi} = \beta_0 \mu^*_{X0} + \beta_1 \mu^*_{X1} Z_{i1} + \beta_2 \mu^*_{X2} Z_{i2} + \beta_3 \mu^*_{X3} Z_{i3}$$

and

$$\sigma_{Xi} = \sigma^*_{X1} Z_{i1} + \sigma^*_{X2} Z_{i2} + \sigma^*_{X3} Z_{i3}$$

The fixed four-dimensional vectors μ^*_{X0} through μ^*_{X3}, the scalar coefficients β_0 through β_3, and the covariance matrices σ^*_{X1} through σ^*_{X3} will be discussed in greater detail in the next section.

Although we are emphasizing a pattern-mixture approach, it is still useful to consider response probabilities. For our example, we will consider final response indicators R_i that are conditionally independent Bernoulli random variables, with response probabilities p_i that depend on the paradata Z_i through a standard logistic regression model. For this model, note especially that the logistic regression coefficient vector γ^* depends on the specific field-intervention rules established by our responsive design. Thus, our model absorbs the responsive-design process into the model for our final response indicators R_i.

For the analysis, we will need some additional notation. Let S be the full set of sample units, and partition S into two subsets. The first subset S_{AB}, is the group of sample units for which we observe the full vector X. In addition, we will partition X into two subvectors - X_A (which contains the first two elements of X), and X_B (which contains the final two elements of X). The second subset of our sample is S_B. For units in S_B, we are only able to observe the subvector X_B. For this example, the response indicator R_i equals 1 if the sample unit i is in S_{AB}, and R_i equals zero otherwise.

With this notation, we can extend previous pattern-mixture approaches to patterns, or groups, defined by our paradata Z .Specifically, we partition S_B into a total of G groups, S_{Bg}, and we consider the standard multivariate-analysis coefficients for the expected value of X_A given X_B.

In the initial discussion of pattern-mixture models, we emphasized the importance of restrictions on certain parameters, to ensure that we had an identified model. For our paradata setting, we will obtain the corresponding model-identification information through the assumption that the coefficients in the expression for the expected value of X_A given X_B are equal for all units i within a given group g, regardless of response status. Finally, we define the weights w_{AB} and w_{Bg} for each group g, based on the observed prevalence of each group within our survey.

Pulling together these components, we have an estimator of the mean of X_A that combines data from our two sample components S_{AB} (for which we have direct observations on X_A), and S_B (for which we have only data on X_B).

$$\bar{X}_A^* = w_{AB}\tilde{\bar{X}}_{A(S_{AB})} + \sum_{g=1}^{G} w_{Bg}\hat{E}(\bar{X}_{Ag} \mid \hat{\bar{X}}_{Bg})$$

This estimator is a weighted average of the sample mean obtained directly from S_A, and of the estimated conditional expectations of X_A given X_B for each of the groups g.

Up to this point, the responsive design literature appears to have developed largely in the context of selection models and related analytic approaches. In particular, these approaches have viewed responsive designs as variants on traditional two-phase designs, as considered by Särndal and Swensson (1987) and Särndal, Swensson and Wretman (1992). Within that framework, the "selection probabilities" in the second phase of sampling will depend on the assumed response-probability model, and on the specific field intervention rules that are used in the responsive design.

5. Simulation Study

To explore some features of a pattern-mixture approach to responsive design, we carried out a simulation study. Numerical values for our mean vectors were equal to:

$$\mu_{X1}^* = (1,0,0,0), \mu_{X2}^* = (0,0,1,1), \mu_{X3}^* = (1,1,1,1)$$

In addition, $\sigma_{X1}^* = I_4$;
σ_{X2}^* is a 4×4 -dimensional banded covariance matrix with diagonal elements equal to 2, elements on the first upper and lower off-diagonals equal to 1, and 0 elsewhere; and σ_{X3}^* is an equicovariance matrix with all diagonal elements equal to 10 and all off-diagonal elements equal to 1.

We considered six distinct cases, as summarized in Table 1. All six cases had the same probability distribution for our paradata, with 90% probability that Z_1 equals 1, and 10% probability that Z_2 equals 1.

The six cases differ in the ways in which the paradata Z_1 affects the mean of X, and in the ways in which the paradata affect the response probabilities. For cases 1, 2 and 3, the vector μ^*_{X1} equals zero, so Z_1 does not have a direct effect on the overall mean of X. In case 1, the vector γ^* has a nonzero value only in its first element, so the response probabilities depend only on Z_1. In case 2, the vector γ^* has entries equal to 1, 0.5 and 0, respectively. In case 3, the vector γ^* has all of its entries equal to one. For this reason, cases 2 and 3 have response probabilities that depend on Z_2 as well as on Z_1, and thus can be viewed as moderate deviations from case 1. For cases 4, 5 and 6, the vector μ^*_{X1} is not equal to zero, so Z_1 can have a substantial direct effect on the mean of X. Cases 4, 5 and 6 differ in their values of the vector γ^*, just as we had for cases 1 through 3. So again, cases 5 and 6 can be viewed as moderate deviations from case 4.

Graphs (Figures 1 through 3) of the results are presented following Table 1. They present plots of the probability of response against the sum of the X values, with the plotting symbol equal to the value of Z_1. Each plot provides points for 100 observations, generated independently.

Figure 1 gives results for Case 1. Note that we do not see much relationship between X and the response probabilities here. In contrast with this, Figure 2 gives results for Case 4, in which the units with Z_1 equal to 1 have higher response probabilities, and tend to have higher values for the sum of X. Figure 3 gives results for Case 6. Note that cases 4 and 6 are similar, except that Case 6 has a few units with larger response probabilities, due to the change in the value of γ^*.

Following that exploratory work, we carried out a simulation evaluation of our final weighted estimators. For each of Cases 1 through 6, we considered samples of size 1000, and produced 1000 independent replications of each of these samples. Table 2 summarizes the resulting bias and standard deviation of our weighted estimators for the means of X_1 and X_2. Cases 1 through 6 are covered separately in the final six rows of the table. The bias and standard deviation for the X_1 estimators are presented in the middle column, with the standard deviations in parentheses. The corresponding results for the X_2 mean estimators are in the final column. For Cases 1 and 4 (corresponding to simulation conditions that match the assumed conditions for model identification constraints), the absolute values of the bias estimates are less than the corresponding standard errors. For Cases 2, 3, 5 and 6 (representing moderate deviations from the ideal conditions), the simulation-based estimates of bias are substantially larger than the corresponding standard errors. In addition, note that the estimated biases are larger for Cases 3 and 6 than for Cases 2 and 5, respectively. This is consistent with the general idea that Cases 3 and 6 represent more pronounced deviations from the idealized conditions considered in Cases 1 and 4, respectively.

Tables 3 through 6 present related simulation results for the sample variance-covariance matrix $\sum_{12.34}$ of (X_1, X_2) conditional on (X_3, X_4). Table 3 presents results for the full sample. Tables 4 and 5 present the corresponding results for sample units with the paradata element Z_1 equal to 0 and 1, respectively. Note especially that the conditional variances are substantially larger for $Z_1 = 1$. Tables 6 and 7 present parallel results for the covariances of (X_1, X_2) conditional on (X_3, X_4), now restricted to units with the response indicator equal to 1. In addition, Tables 8 through 11 present the moments of the sample coefficients for the multivariate regression of (X_1, X_2) on (X_3, X_4) for specified cases defined by the paradata Z_1 and the response indicators *R*.

To explore our estimators in additional detail, we also have four graphs (Figures 4-7) that present side-by-side boxplots of the final weighted estimators of X_1, separately for ten cells. The ten cells are defined by the deciles of the full-sample means of X_1. Due to nonresponse, we are not able to observe these full-sample means in practice, but we are able to compute the final weighted estimators. Consequently, it is useful to explore the relationship between these full-sample means and final weighted estimators. Each set of results is based on 1000 independent replications of the process described earlier, with 1000 observations in each replication.

Figure 4 covers Case 1, with the final weighted estimators along the vertical axis and the ten groups of true full-sample means along the horizontal axis. Note that as the true sample means increase across the ten groups, we have a general increasing pattern in the distribution of the final weighted estimators. Figure 5 covers Case 3. As previously stated, the response probability structure for Case 3 was a moderate deviation from the probability structure for Case 1. Consistent with that idea, the boxplot pattern for Case 3 is similar to the pattern for Case 1. Figure 6 covers case 4. As previously stated, Cases 1 through 6 include cases in which the paradata Z_1 can have a relatively weak effect (as in Cases 1 and 3) or a relatively strong effect (as in Cases 4 and 6) for estimation of the mean of X_1. Again here for Case 4, as the true full-sample means increase across groups 1 through 10, the distributions of the true weighted estimators display a moderate increasing pattern. Finally, Figure 7 presents the boxplots for Case 6; these are similar to the boxplots for Case 4, again reflecting the idea that the response probability structure for Case 6 is a moderate deviation from that presented in Case 4. In addition, for all four cases presented in Figures 4 through 7, the variability of the estimators is somewhat greater in the tail groups 1 and 10, compared with the middle groups. This dispersion pattern is somewhat more pronounced for Cases 4 and 6, relative to the first two cases.

6. Discussion

In closing, this paper has considered extension of pattern-mixture models to account for responsive-design features. There are several areas of potential future work, including modeling to account for more complex interventions based on observed paradata; diagnostics to evaluate the homogeneity of coefficients within the S_{Bg} groups; and extensions to unequal-probability sampling.

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Table 1: Six cases based on $p_{Z1} = 0.9, p_{Z2} = 0.1, p_{Z3} = 0$

Case	μ^*_{X1}	γ^*
1	(0,0,0,0)	(1,0,0)
2	(0,0,0,0)	(1,0.5,0)
3	(0,0,0,0)	(1,1,1)
4	(10,10,1,1)	(1,0,0)
5	(10,10,1,1)	(1,0.5,0)
6	(10,10,1,1)	(1,1,1)

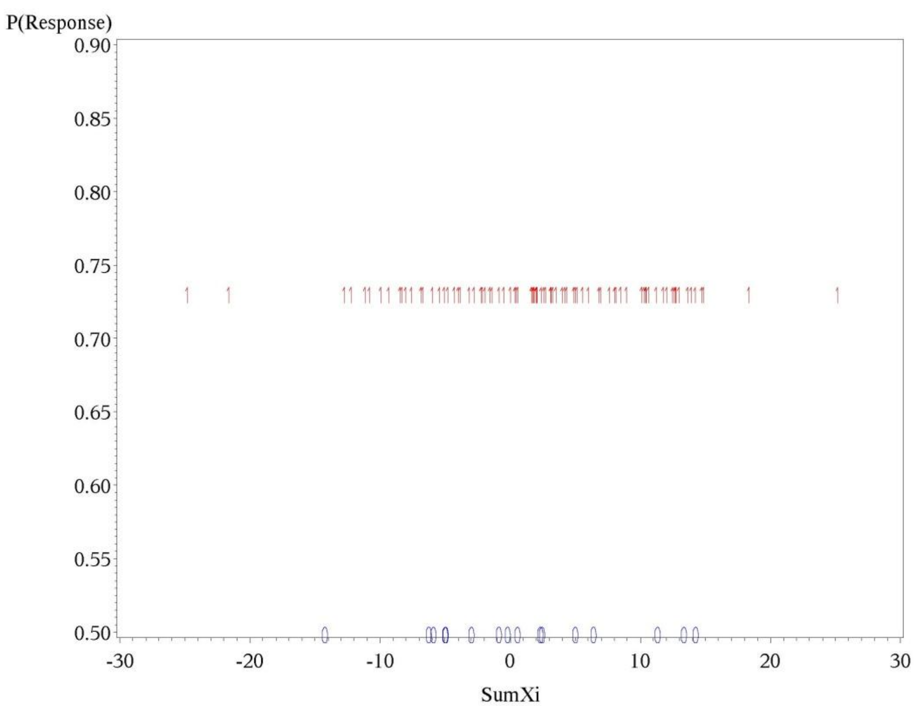

Figure 1: P(Response) vs. $X_1 + X_2 + X_3 + X_4$ for Case 1; Plotting Symbol=Value for Z_1

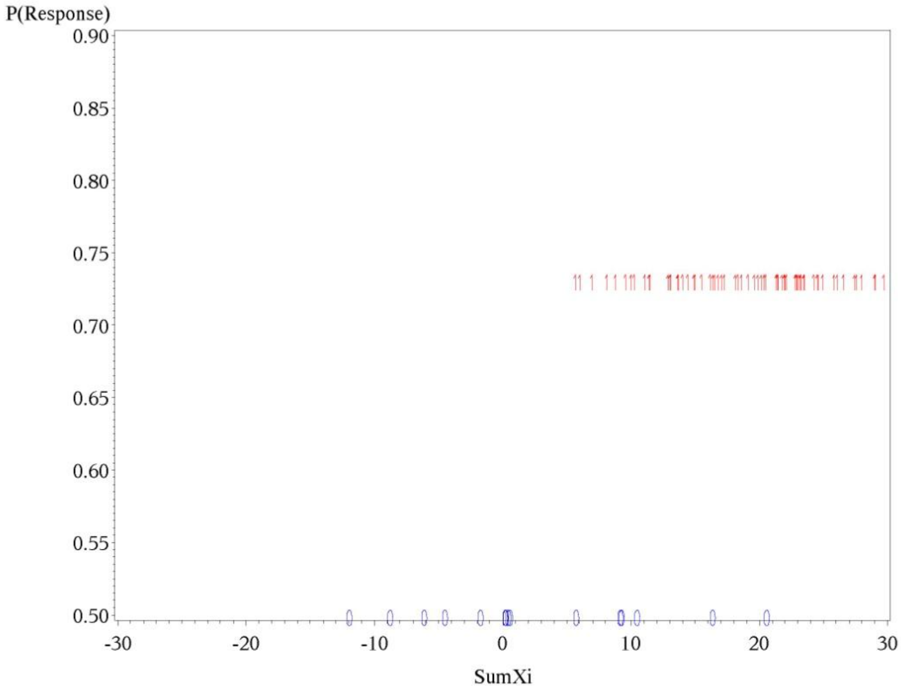

Figure 2: P(Response) vs. $X_1 + X_2 + X_3 + X_4$ for Case 4; Plotting Symbol=Value for Z_1

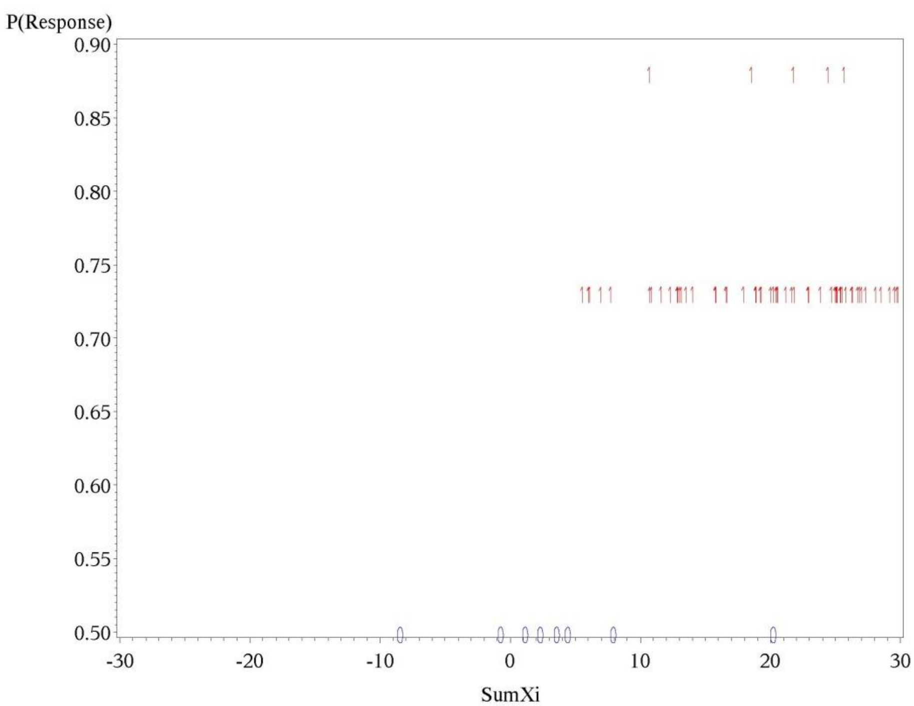

Figure 3: P(Response) vs. $X_1 + X_2 + X_3 + X_4$ for Case 6; Plotting Symbol=Value for Z_1

Table 2: Simulation Results: Bias (Standard Error) of $\overline{X}_1^*, \overline{X}_2^*$ for Cases 1-6

	$\overline{X}_1^* - \mu_1$	$\overline{X}_2^* - \mu_2$
Case	Mean (Std Error)	Mean (Std Error)
1	0.00053 (0.00077)	0.00050 (0.00077)
2	-0.00805 (0.00076)	-0.00915 (0.00075)
3	-0.01641 (0.00072)	0.01764 (0.00070)
4	-0.00032 (0.00081)	-0.00094 (0.00075)
5	-0.00931 (0.00076)	-0.00955 (0.00077)
6	-0.01800 (0.00078)	-0.01716 (0.00072)

Table 3: Simulation Results: Mean (Standard Deviation) of the Four Elements of $\Sigma_{12.34}$ for Cases 1-6 (Full Sample Results)

Case	$\sigma_{11.34}$	$\sigma_{12.34} = \sigma_{21.34}$	$\sigma_{22.34}$
1	1.6872 (0.0955)	0.6483 (0.0714)	1.5966 (0.0879)
2	1.6881 (0.0980)	0.6462 (0.0684)	1.5946 (0.0868)
3	1.6873 (0.0995)	0.6480 (0.0721)	1.5983 (0.0891)
4	9.0875 (0.6082)	8.0376 (0.6126)	8.9716 (0.6030)
5	9.0306 (0.6216)	7.9850(0.6207)	8.9232 (0.6121)
6	9.0611 (0.6155)	8.0078 (0.6188)	8.9588 (0.6118)

Table 4: Simulation Results: Mean (Standard Deviation) of the Four Elements of $\Sigma_{12.34Z0}$ ($Z_1 = 0$ only) for Cases 1-6

Case	$\sigma_{11.34Z0}$	$\sigma_{12.34Z0} = \sigma_{21.34Z0}$	$\sigma_{22.34Z0}$
1	0.3829 (0.2045)	0.2483 (0.1423)	0.2494 (0.1307)
2	0.3799 (0.2032)	0.2495 (0.1414)	0.2493 (0.1320)
3	0.3813 (0.2111)	0.2475 (0.1446)	0.2466 (0.1308)
4	0.3803 (0.2027)	0.2492 (0.1431)	0.2508 (0.2354)
5	0.3820 (0.2226)	0.2505 (0.1558)	0.2514 (0.1425)
6	0.3734 (0.2022)	0.2459 (0.1438)	0.2481 (0.1388)

Table 5: Simulation Results: Mean (Standard Deviation) of the Four Elements of $\sum_{12.34Z1}$ ($Z_1 = 1$ only) for Cases 1-6

Case	$\sigma_{11.34Z1}$	$\sigma_{12.34Z1} = \sigma_{21.34Z1}$	$\sigma_{22.34Z1}$
1	1.8230 (0.1034)	0.6483 (0.0714)	1.5966 (0.0878)
2	1.8237 (0.1053)	0.6827 (0.0845)	1.7315 (0.0922)
3	1.8220 (0.1056)	0.6840 (0.0778)	1.7352 (0.0947)
4	1.8217 (0.1061)	0.6885 (0.0741)	1.7328 (0.0911)
5	1.8197 (0.1075)	0.6849 (0.0781)	1.7315 (0.0928)
6	1.8207 (0.1076)	0.6780 (0.0774)	1.7297 (0.0927)

Table 6: Simulation Results: Mean (Standard Deviation) of the Four Elements of $\sum_{12.34Z0R}$ ($Z_1 = 0$ and R=1) for Cases 1-6

Case	$\sigma_{11.34Z0R}$	$\sigma_{12.34Z0R} = \sigma_{21.34Z0R}$	$\sigma_{22.34Z0R}$
1	0.7916 (0.4280)	0.5126 (0.2935)	0.5154 (0.2721)
2	0.7569 (0.4089)	0.4971 (0.2832)	0.4968 (0.2651)
3	0.7482 (0.4191)	0.4868 (0.2884)	0.4850 (0.2616)
4	0.7872 (0.4323)	0.5169 (0.3055)	0.5196 (0.2875)
5	0.7649 (0.4499)	0.5019 (0.3137)	0.5041 (0.2884)
6	0.7352 (0.4024)	0.4849 (0.2856)	0.4891 (0.2754)

Table 7: Simulation Results: Mean (Standard Deviation) of the Four Elements of $\sum_{12.34Z1R}$ ($Z_1 = 1$ and R=1) for Cases 1-6

Case	$\sigma_{11.34Z1R}$	$\sigma_{12.34Z1R} = \sigma_{21.34Z1R}$	$\sigma_{22.34Z1R}$
1	2.4931 (0.1506)	0.9369 (0.1070)	2.3719 (0.1338)
2	2.4671 (0.1473)	0.9236 (0.1006)	2.3426 (0.1314)
3	2.4459 (0.1471)	0.9184 (0.1056)	2.3294 (0.1324)
4	2.4964 (0.1519)	0.9395 (0.1034)	2.3745 (0.1326)
5	2.4619 (0.1513)	0.9266 (0.1067)	2.3425 (0.1293)
6	2.4422 (0.1495)	0.9095 (0.1043)	2.3200 (0.1294)

Table 8: Simulation Results: Mean (Standard Deviation) of Estimated Coefficients for Multivariate Regression of (X_1, X_2) and (X_3, X_4) (All sample units with $Z_1 = 0$)

Case	$\beta_{11.34Z0}$	$\beta_{12.34Z0}$	$\beta_{21.34Z0}$	$\beta_{22.34Z0}$
1	0.4728 (0.5442)	0.4758 (0.5449)	0.9744 (0.4461)	-0.0148 (0.4463)
2	0.4580 (0.5562)	0.4899 (0.5559)	0.9769 (0.4657)	-0.0163 (0.4645)
3	0.5004 (0.6035)	0.4500 (0.6023)	0.9982 (0.4659)	-0.0376 (0.4653)
4	0.4961 (0.5889)	0.4502 (0.5893)	0.4502 (0.5893)	-0.0355 (0.4661)
5	0.4582 (0.5802)	0.4884 (0.5787)	0.9888 (0.4632)	-0.0308 (0.4628)
6	0.4671 (0.5563)	0.4817 (0.5536)	0.9910 (0.4269)	-0.0310 (0.4266)

Table 9: Simulation Results: Mean (Standard Deviation) of Estimated Coefficients for Multivariate Regression of (X_1, X_2) and (X_3, X_4) (All sample units with $Z_1 = 1$)

Case	$\beta_{11.34Z1}$	$\beta_{12.34Z1}$	$\beta_{21.34Z1}$	$\beta_{22.34Z1}$
1	0.4240 (0.0345)	0.4195 (0.0347)	0.4734 (0.0327)	0.3799(0.0327)
2	0.4216 (0.0348)	0.4216 (0.0346)	0.4704 (0.0348)	0.3833 (0.0339)
3	0.4225 (0.0331)	0.4218 (0.0335)	0.4742 (0.0332)	0.3792 (0.0339)
4	0.4221 (0.0359)	0.4201 (0.0350)	0.4725 (0.0345)	0.3818 (0.0345)
5	0.4236 (0.0479)	0.4205 (0.0347)	0.4735 (0.0333)	.03814 (0.0345)
6	0.4239 (0.0348)	0.4195 (.00340)	0.5737 (0.0333)	0.3820 (0.0342)

Table 10: Simulation Results: Mean (Standard Deviation) of Estimated Coefficients for Multivariate Regression of (X_1, X_2) and (X_3, X_4) (All sample units with $Z_1 = 0$ and R=1)

Case	$\beta_{11.34Z0R1}$	$\beta_{12.34Z0R1}$	$\beta_{21.34Z0R1}$	$\beta_{22.34Z0R1}$
1	0.4725 (0.5456)	0.4761 (0.5461)	0.9740 (0.4471)	-0.0146 (0.4474)
2	0.4578 (0.5571)	0.4900 (0.5567)	0.9755 (0.4662)	-0.0160 (0.4649)
3	0.4671 (0.5884)	0.4813 (0.5557)	0.9909 (0.4285)	-0.0314 (0.4284)
4	0.4961 (0.5898)	0.4501 (0.5900)	0.9949 (0.4660)	-0.0356 (0.4668)
5	0.4580 (0.5793)	0.4885 (0.5793)	0.9884 (0.5779)	-0.0305 (0.4626)
6	0.4725 (0.5456)	0.4761 (0.5461)	0.9740 (0.4471)	-0.0146 (0.4474)

Table 11: Simulation Results: Mean (Standard Deviation) of Estimated Coefficients for Multivariate Regression of (X_1, X_2) and (X_3, X_4) (All sample units with $Z_1 = 1$ and R=1)

Case	$\beta_{11.34Z1R1}$	$\beta_{12.34Z1R1}$	$\beta_{21.34Z1R1}$	$\beta_{22.34Z1R1}$
1	0.4240 (0.0345)	0.4194 (0.0347)	0.4734 (0.0318)	0.3799 (0.0326)
2	0.4216 (0.0348)	0.4216 (0.0346)	0.4704 (0.0337)	0.3833 (0.0240)
3	0.4224 (0.0306)	0.4218 (0.0335)	0.4742 (0.0332)	0.3791 (0.0339)
4	0.4221 (0.0346)	0.4201 (0.0350)	0.4725 (0.0345)	0.3818 (0.0346)
5	0.4236 (0.0344)	0.4205 (0.0347)	0.4735 (0.0333)	0.3813 (0.0345)
6	0.4249 (0.0348)	0.4194 (0.0340)	0.4736 (0.0333)	0.3819 (0.0342)

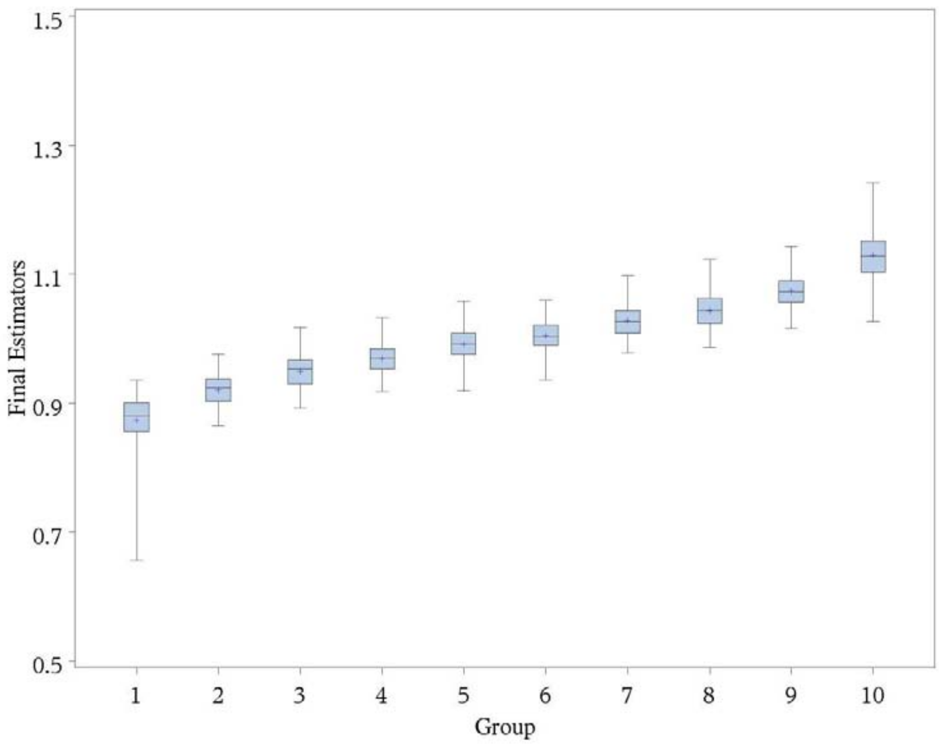

Figure 4: Boxplots of final estimators by group, for Case 1

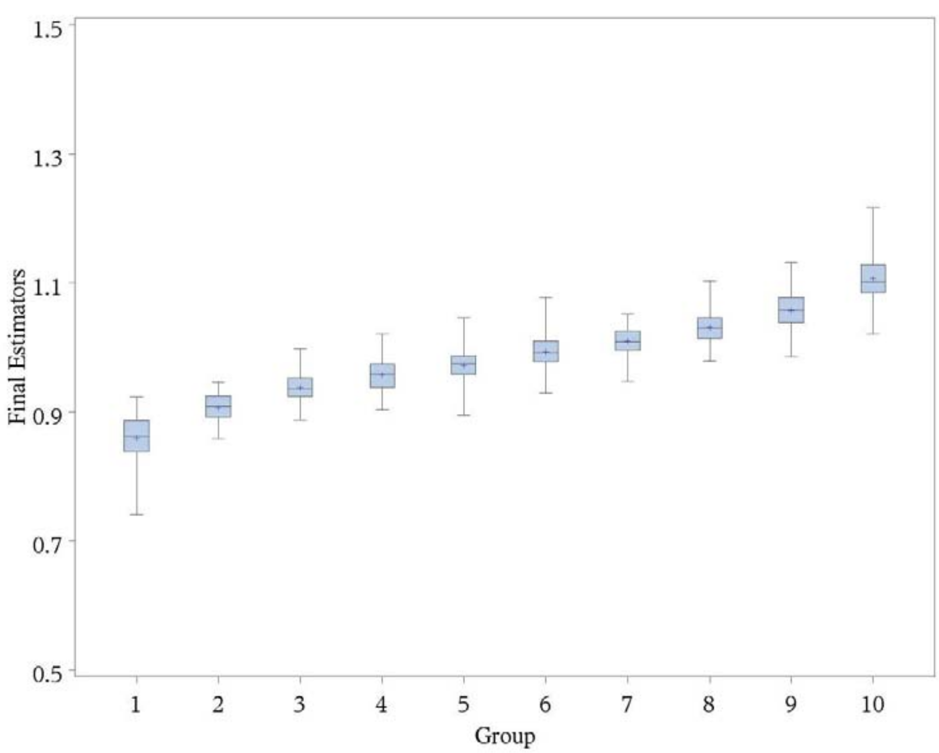

Figure 5: Boxplots of final estimators by group, for Case 3

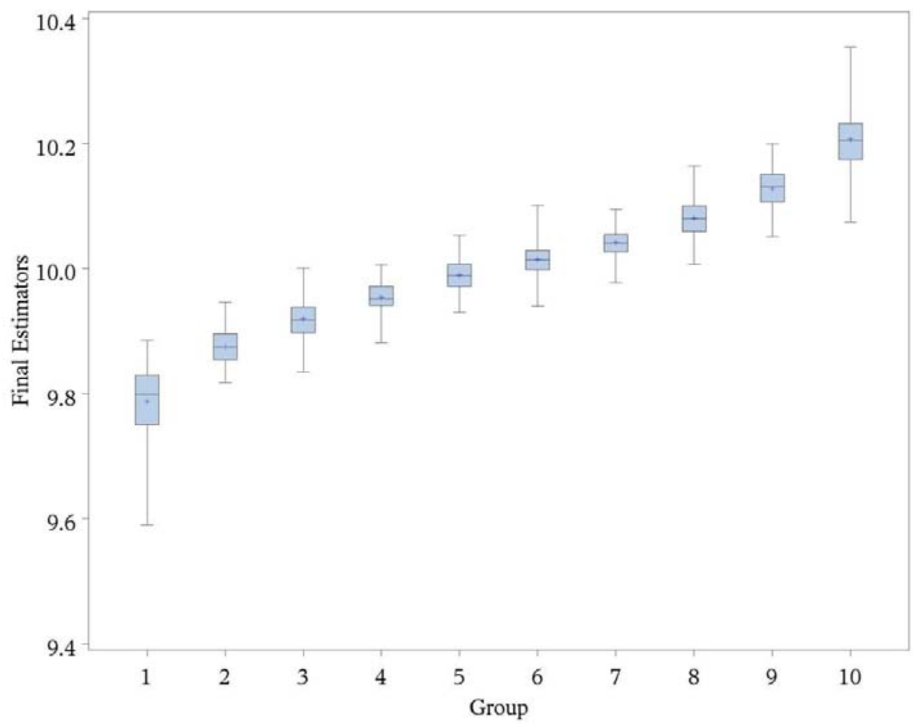

Figure 6: Boxplots of final estimators by group, for Case 4

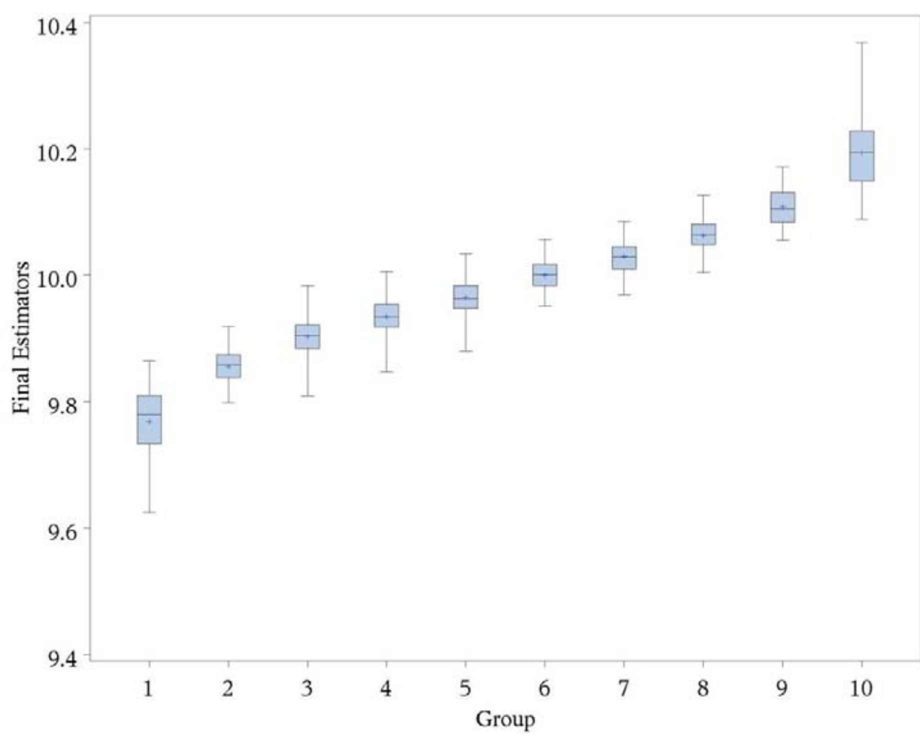

Figure 7: Boxplots of final estimators by group, for Case 6