A Multi-dimensional Measure of Economic Well-being for the U.S.: The Material Condition Index

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BUREAU OF LABOR STATISTICS
U.S. DEPARTMENT OF LABOR

Economic Well-being: Concepts

Income (Y)

Consumption (C)

Wealth (W)

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Recent Support for Joint Measures

- Commission on the Measurement of Economic Performance and Social Progress (Stiglitz , Sen, Fituoussi, 2009).

- Reports of OECD Expert Group on Micro Statistics on Household Income, Consumption and Wealth (2013)
  - Integrated framework for Y, C, and W
  - Analysis tools, e..g., composite multi-dimensional measures at **micro- or household level** (new field of statistics)

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Objectives of this Research

- Test the feasibility of producing composite multi-dimensional measure
  - Material Condition Index (MCI) defined by Ruiz (2011)
  - For the U.S
- Consistently define income, consumption, and wealth following OECD integrated framework

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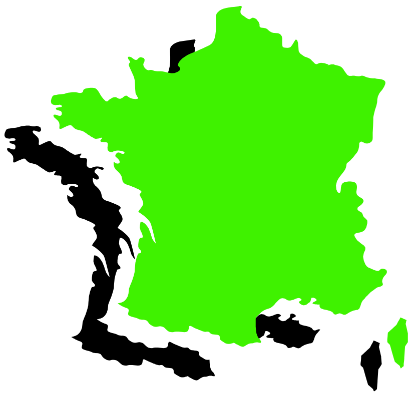
MCI's Produced for…

- France 1995



- Ruiz 2011
- Household Expenditure Survey
- Y, C, financial W

- United States, 2011

- Garner and Short 2013
- Consumer Expenditure Survey
- Y, C, financial and non-financial W

Materials Conditions Index (Ruiz 2011)

- Builds on work on Kolm (1977), Atkinson (1970, 1982) and Foster and colleagues (2005, 2008, 2010)
- Atkinson’s income standard (i.e., equally distributed equivalent income, EDE)
- Foster and colleagues’ multidimensional measures
- Ruiz justifies method based on set of standard properties of aggregation functions and axioms for multidimensional measures

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Materials Conditions Index (Ruiz 2011)

- Combines measures of
  - Central tendency (i.e., *mean* achievements)
  - Dispersion (i.e., *distributions* of achievements)
- Aggregation function uses nested generalized means to summarize achievements within dimensions and across dimensions to single summary index
- *Question addressed*: Does a joint measure modify picture of material living conditions relative to one measure alone?

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Generalized Mean

- Considers whole distribution

- Aversion parameter, *q*, based on a utilitarian welfare concept
  - *q = 1* generalized mean reduces to arithmetic mean
  - *q = 0*, geometric mean
  - *q = -1*, harmonic mean
  - as *q* decreases, greater weight placed on the lower tail of distribution

- Penalties applied for
  - Inequality between individuals, “inter” inequality (*q*)
  - Unbalanced achievement in dimensions, “intra” inequality (*r*)

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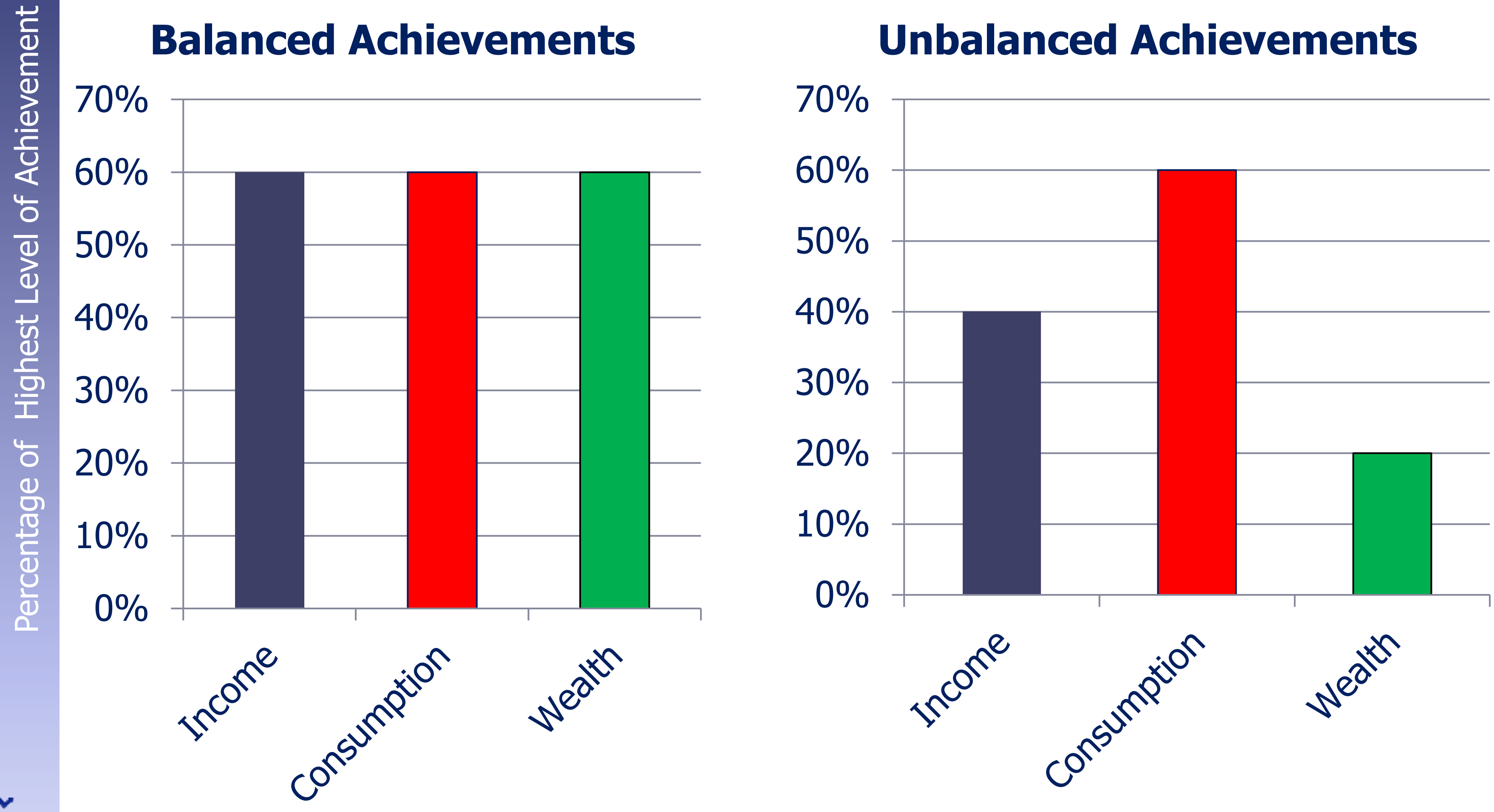
Example of Distribution of Dimensions



- Penalty for unbalanced achievement, as *r* decreases, greater weight placed on the achievements that are less

Construction of MCI

- Ruiz assumes *log transformation* of dimension values (Alkire and Foster 2010)

- Diminishing returns to increases in each dimension
  - e.g., as income increases, diminishing returns to transforming income into material well-being
  - People do not need excessive income, consumption or wealth to ensure decent levels of living

- *Challenge*: 0 and negative values

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Construction of MCI - 2

- *Normalize values* of dimensions to ratio-scale measure (Alkire and Foster, 2010)
  - 0% lowest achievement
  - 100% highest achievement
- Assumption
  - Comparability across Y, C, and W (e.g., 60% achievement in one dimension same as 60% in others)

- Result is Achievement Matrix *(n x 3)* with values 0 to 1

- Order of aggregation

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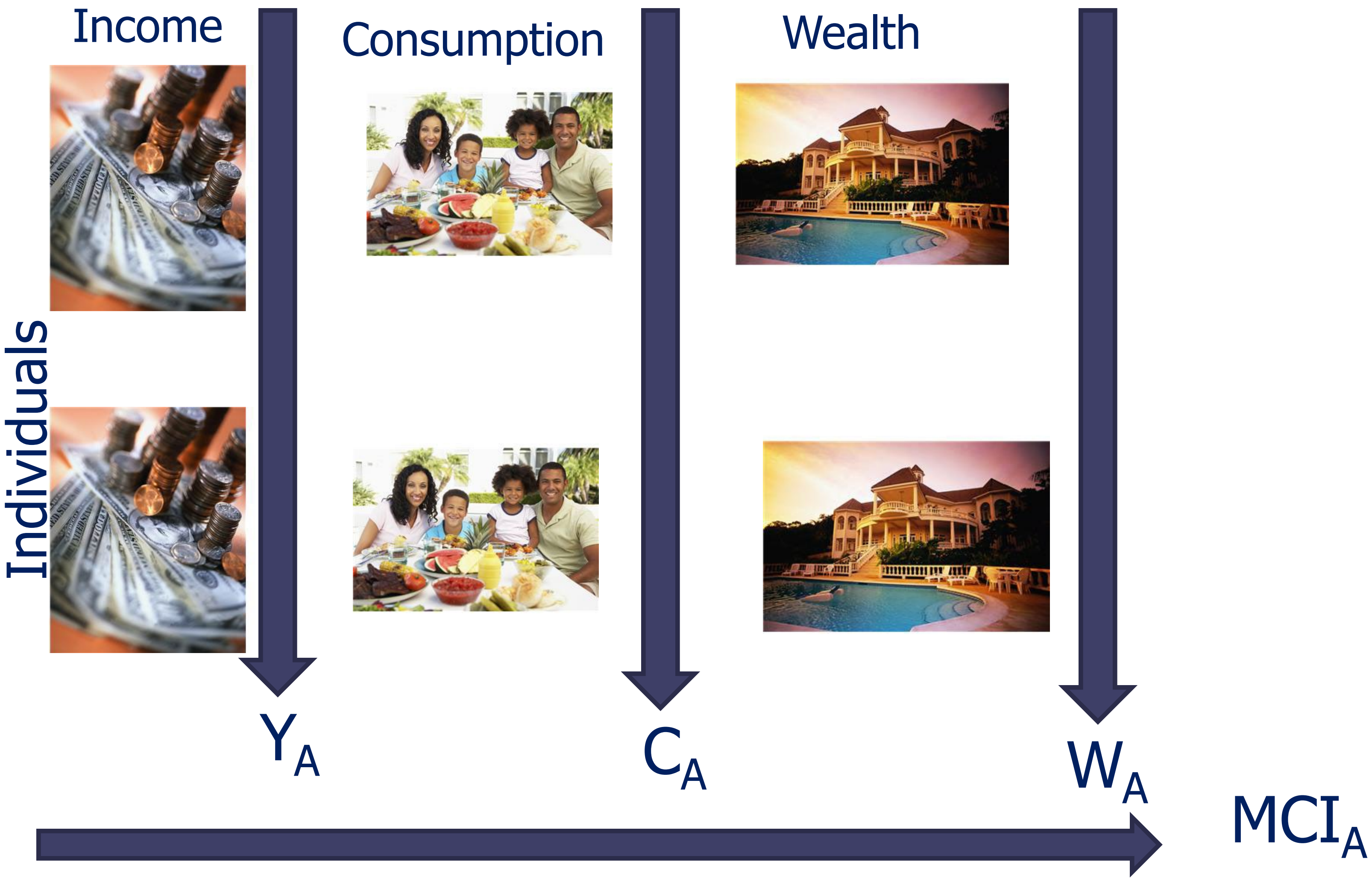
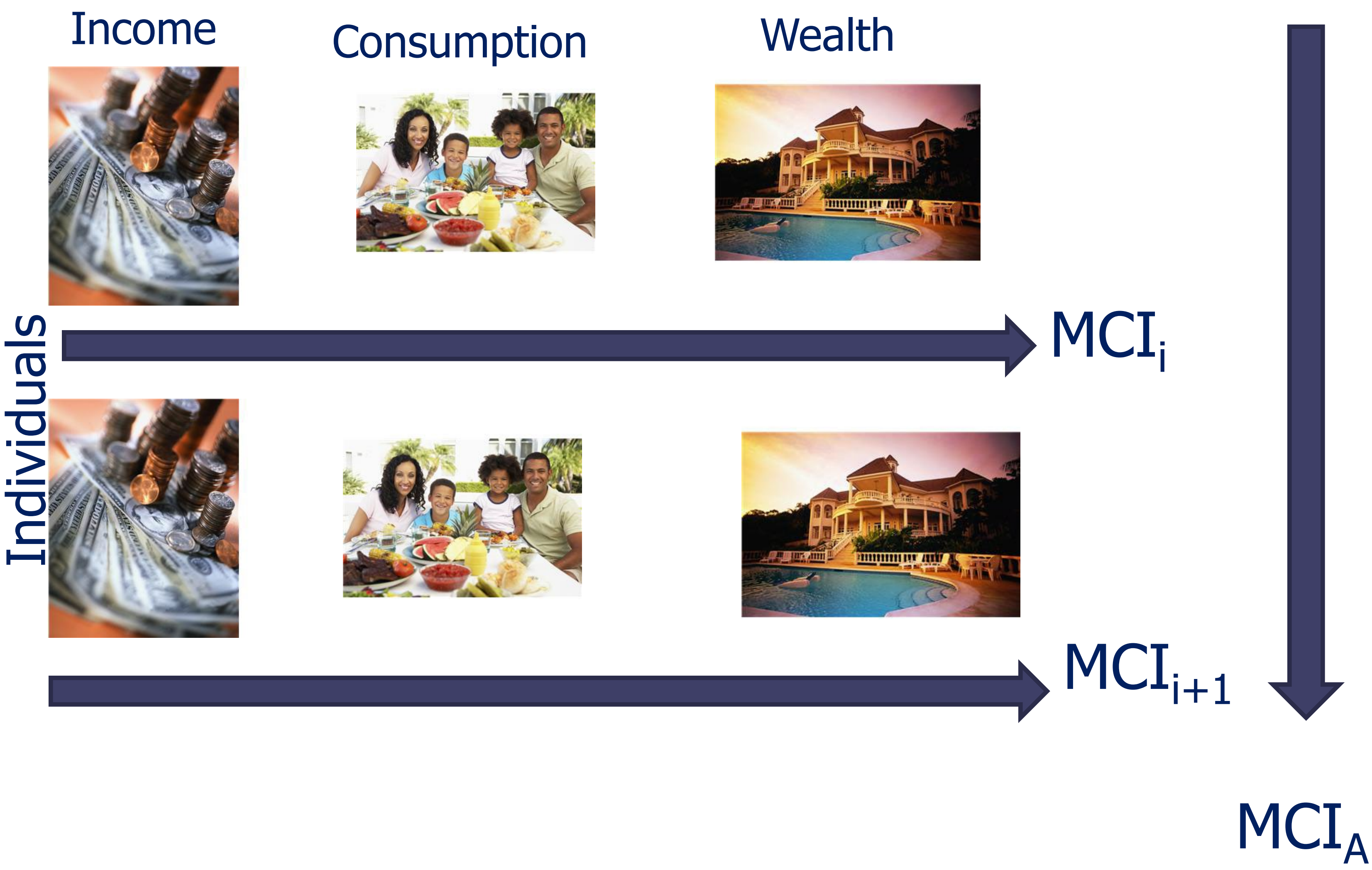
S- Aggregation (Specific: across individuals then across dimensions)



I- Aggregation (Individualistic: across dimensions then across individuals)



Does Aggregation Matter?

- Do results equal for I- and S-aggregations?
  - Path independence
  - Nested generalized mean of curvatures q and r path independent only when $q=r$

Does Aggregation Matter?

- Do results equal for I- and S-aggregations?
  - Path independence
  - Nested generalized mean of curvatures *q* and *r* path independent only when $q=r$

- More flexible form of index desirable
  - $q \neq r$ allows for greater concern for inequality between individuals within dimensions (*q*) and less for correlation across dimensions (*r*)

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U.S. Consumer Expenditure Survey, Interview

- Collection period: 2009 Q2 - 2012 Q1
- *Y* and *C* cover same 12 months, *W* at end of period
- Sample: 14,948 unique consumer units

- Adjustments to *Y*, *C*, and *W*
  - 2011 $U.S.
  - Modified OECD equivalence scale

- Results population weighted, not adjusted for attrition across quarters
- No standard errors
- Results PRELIMINARY

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Variables

- Adjusted disposable income
- Consumption expenditures
- Wealth = assets – liabilities

- All consistently defined using OECD framework

- New for U.S. measures of economic well-being: vacation properties and time shares

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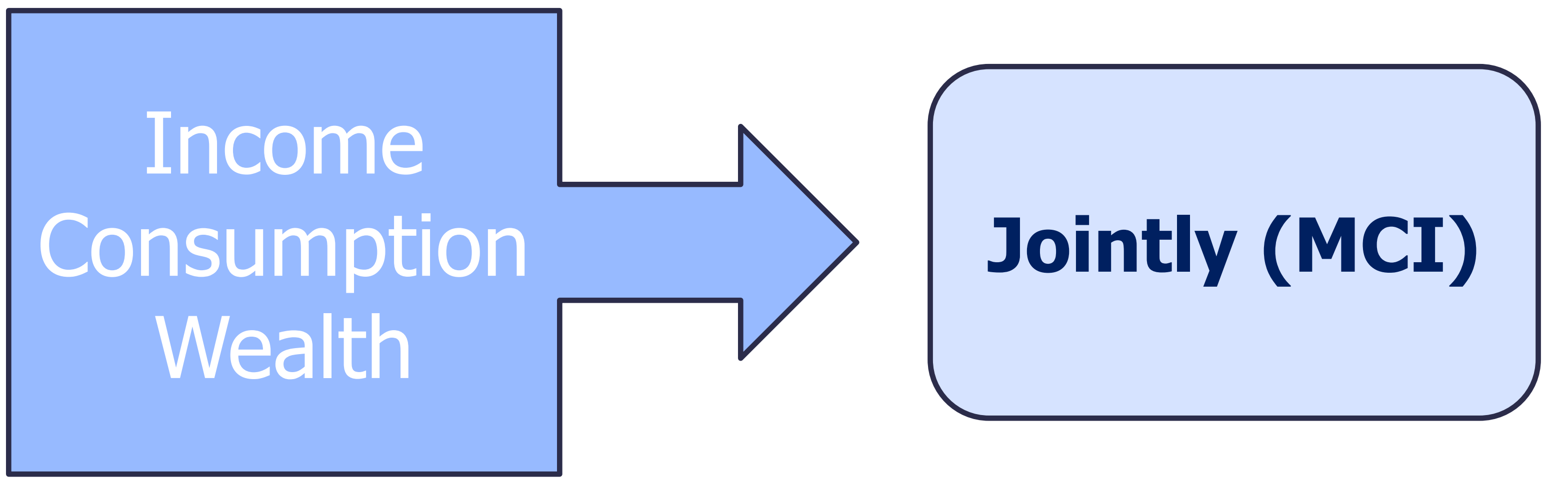
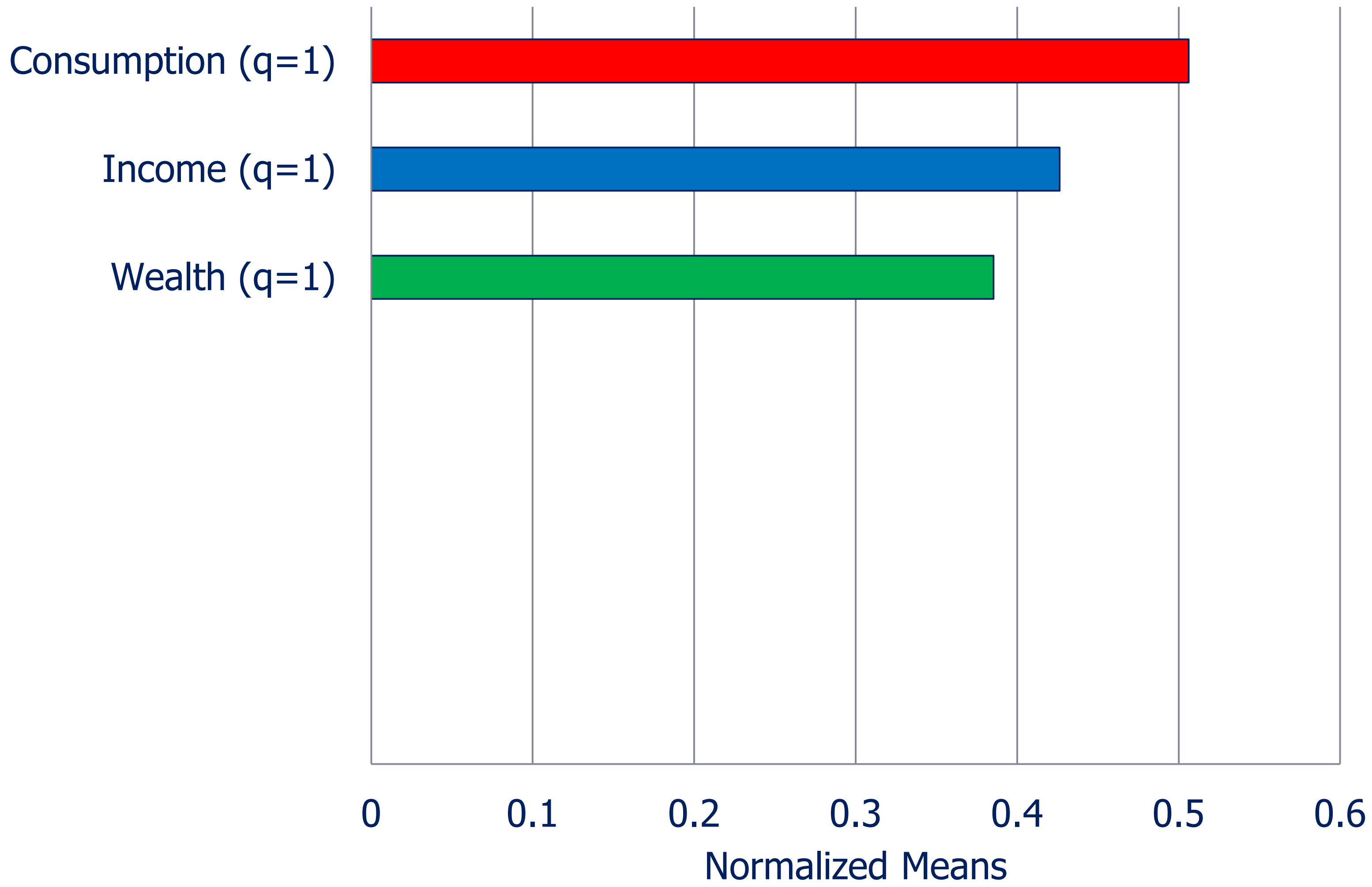
Results Presented by



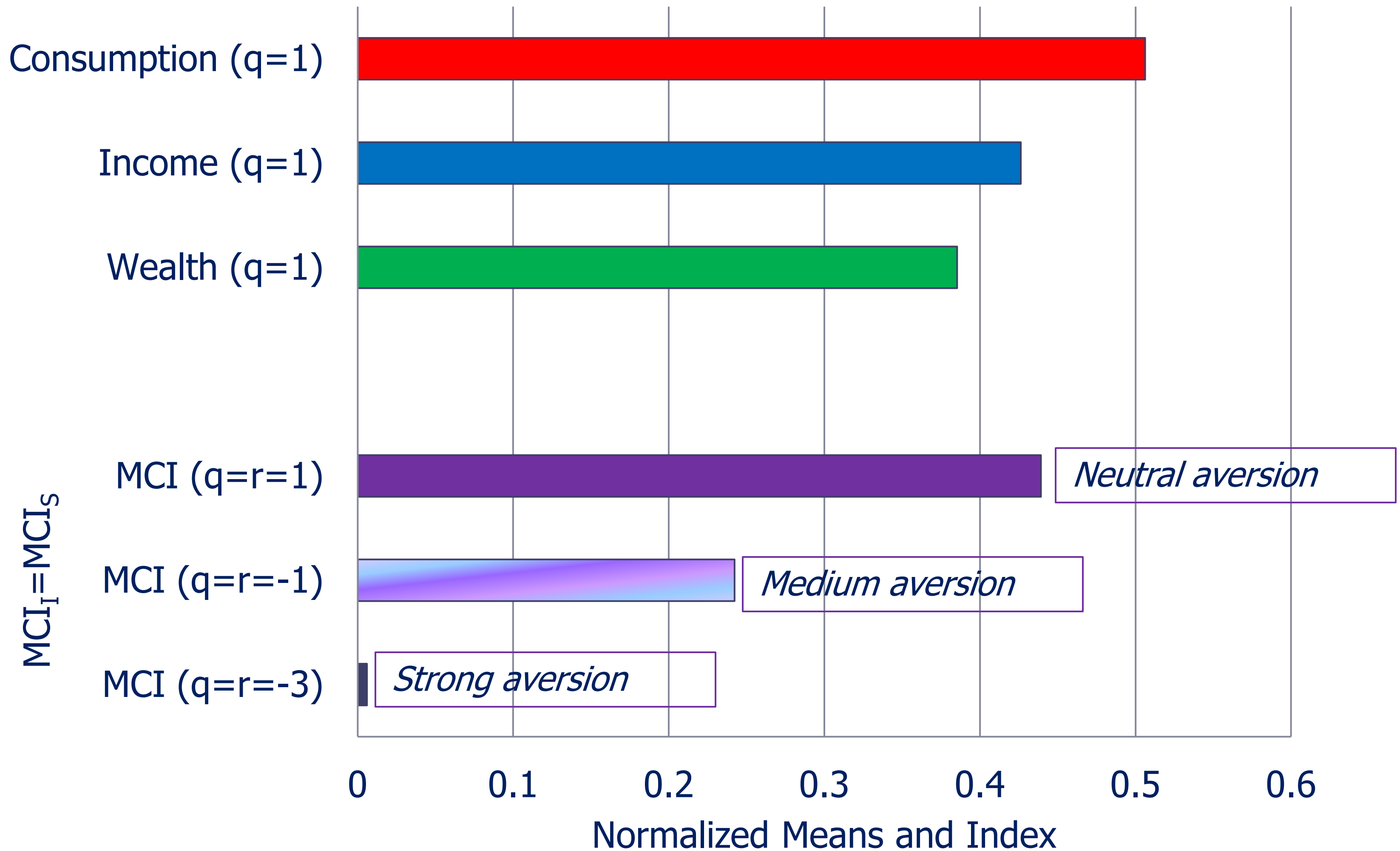
- Overall, income deciles, housing tenure
- Aversion to inequality and un-balancement in achievement
- Aggregation order
- Weighting dimensions

Normalized Arithmetic Means of Economic Well-Being Dimensions



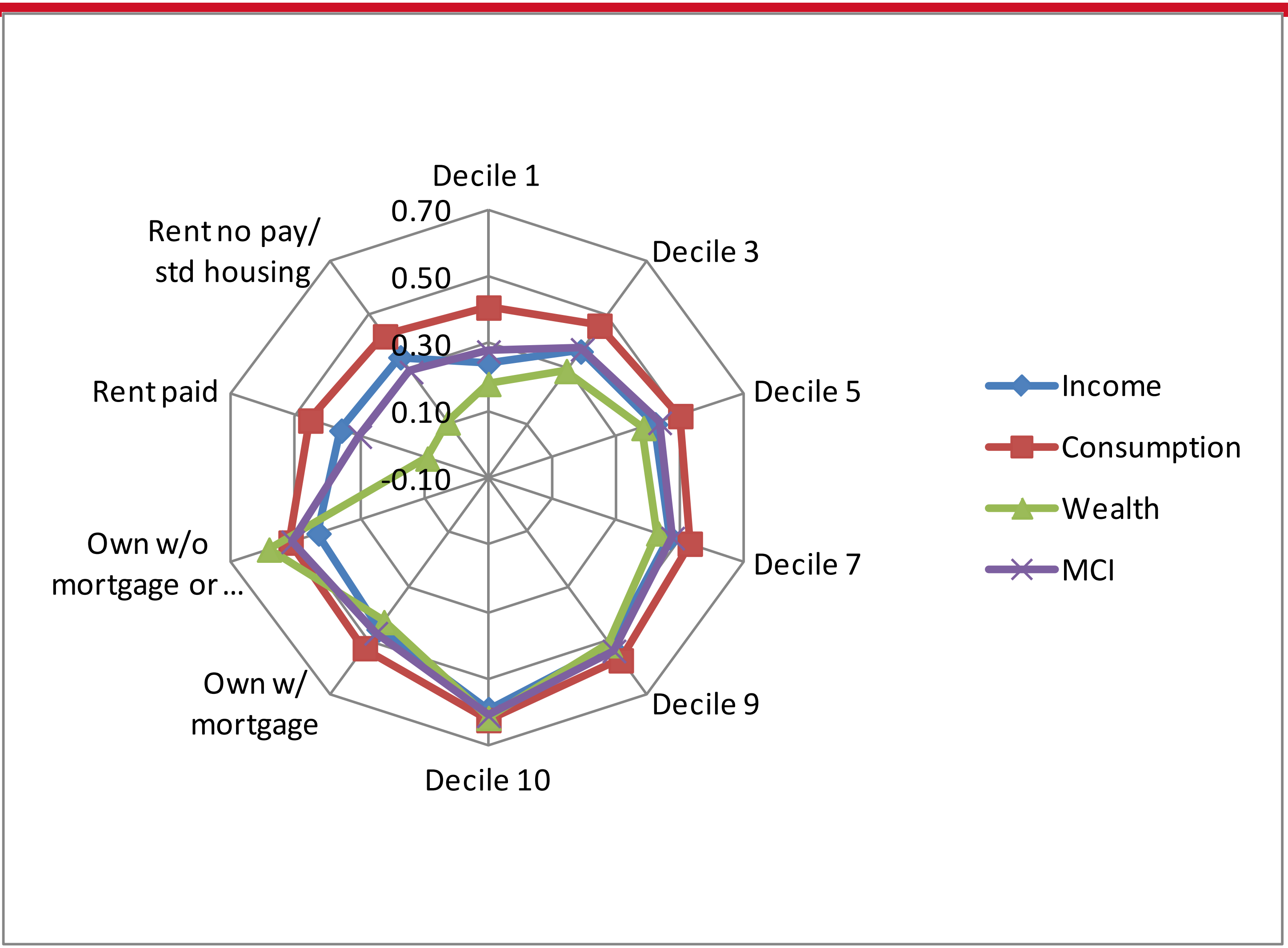
Data source: U.S. Consumer Expenditure Interview Survey, based on values in 2011 dollars U.S.

Normalized Arithmetic Dimension Means and MCI under Different Aversions to Inequality



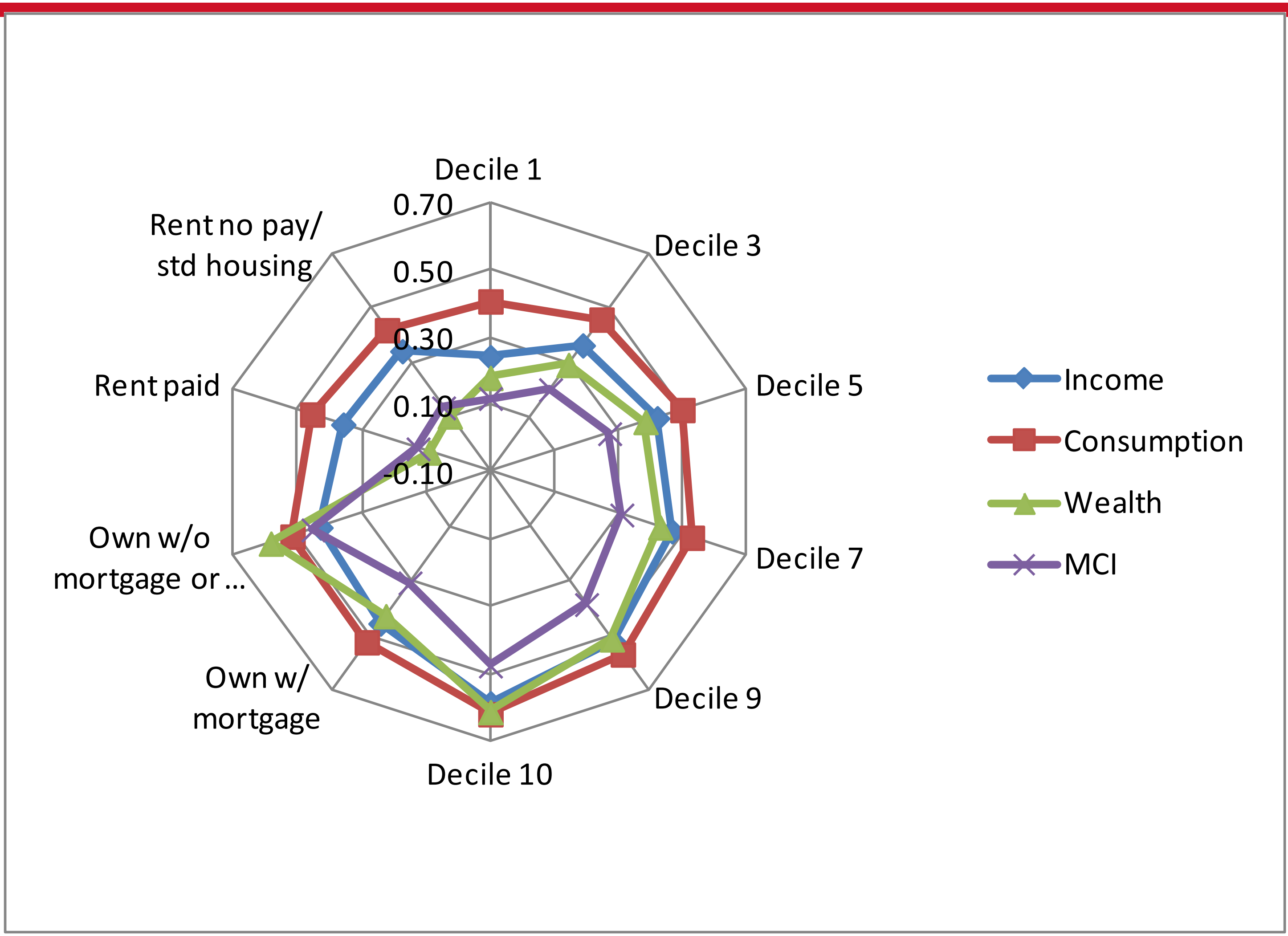
Data source: U.S. Consumer Expenditure Interview Survey, based on values in 2011 dollars U.S.

Neutral Aversion to Inequality *(q=r=1)* by Income Deciles and Housing Tenure



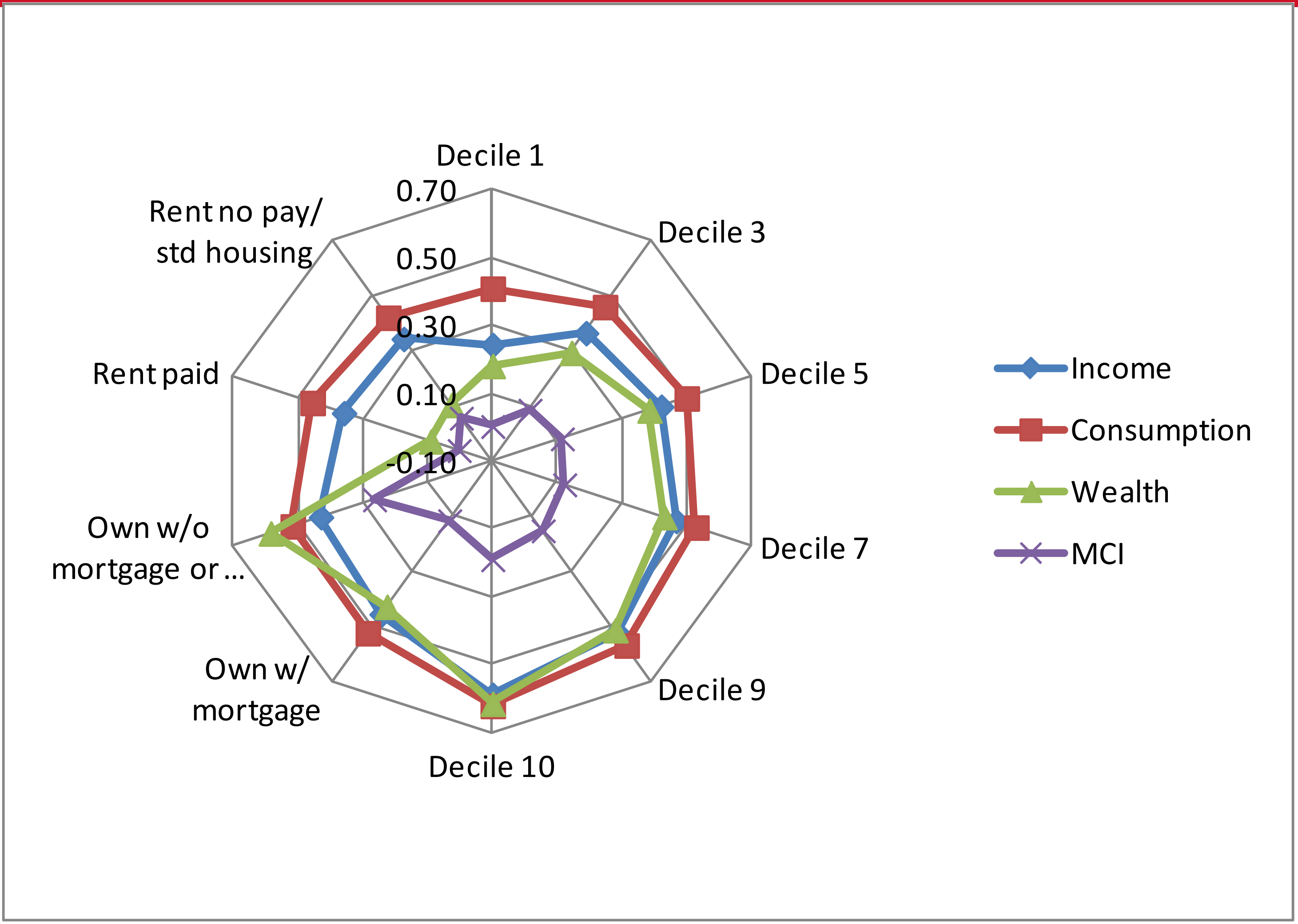
$MCI_I = MCI_S$ (path independent)

MCI with Medium Aversion to Inequality *(q=r=-1)* by Income Deciles and Housing Tenure



Neutral aversion for dimensions, $MCI_I = MCI_S$ (path independent)

MCI with Strong Aversion to Inequality *(q=r=-3)* by Income Deciles and Housing Tenure



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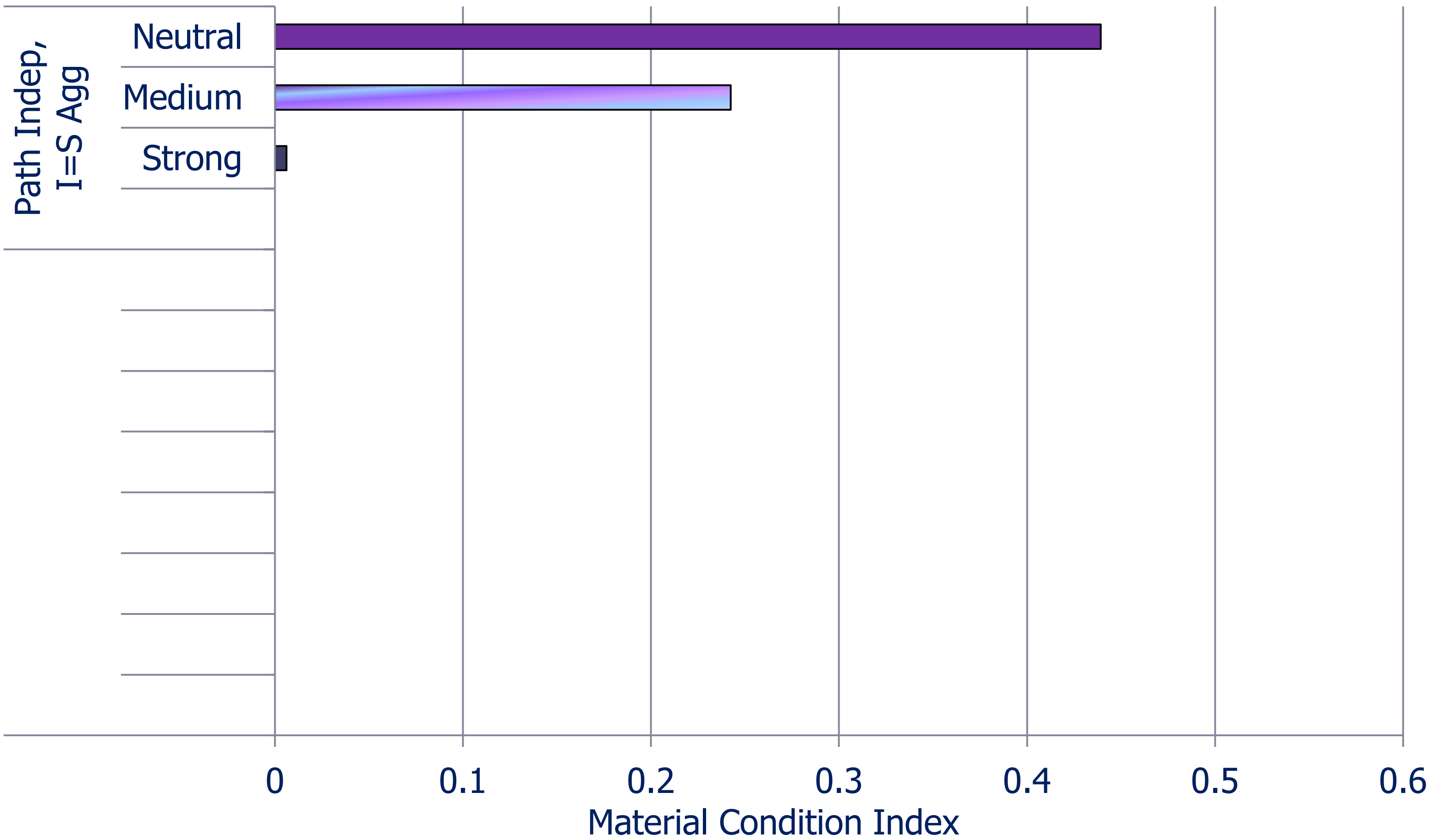
MCI, Aversion and Path Dependency



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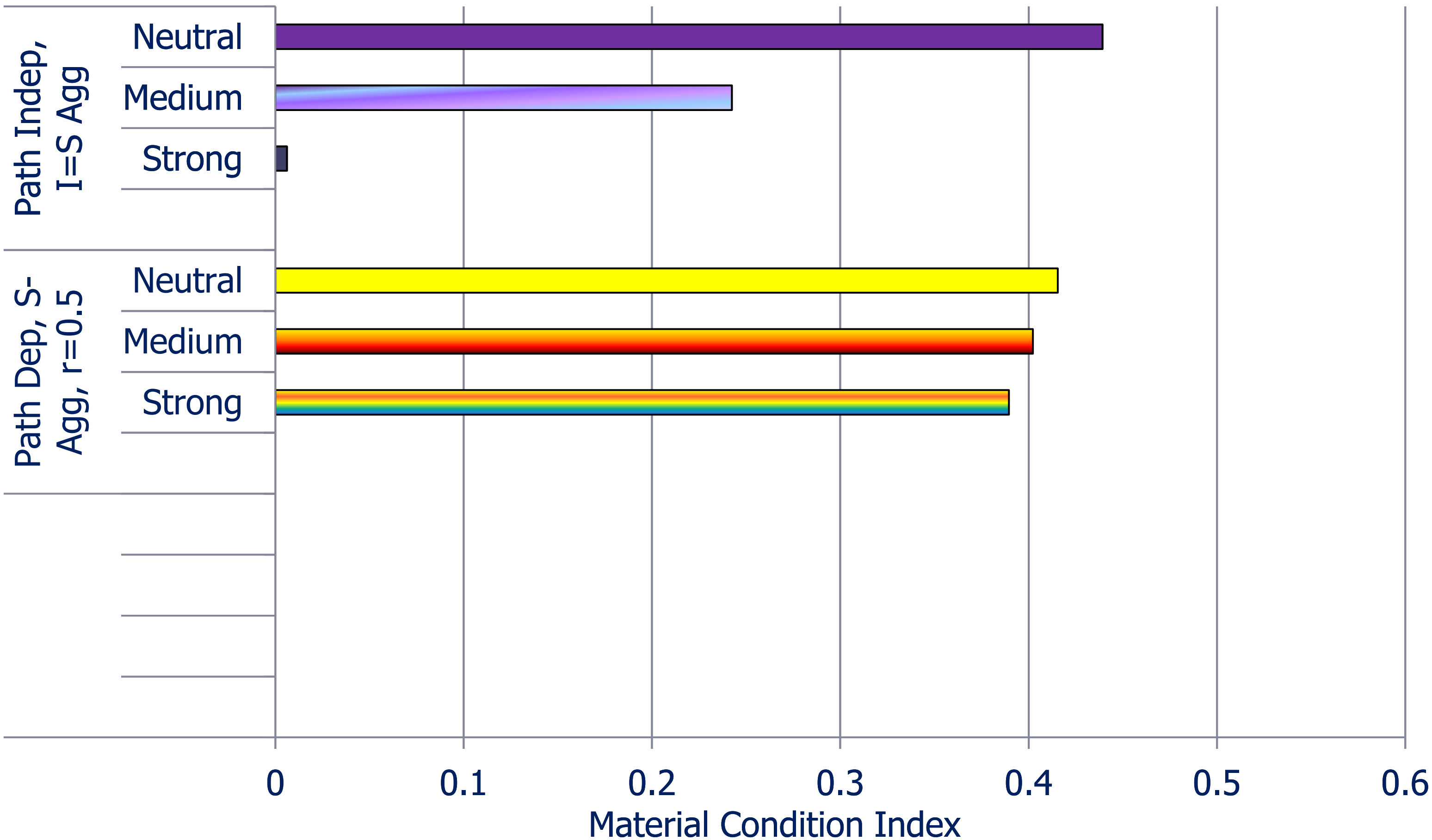
MCI, Aversion and Path Dependency



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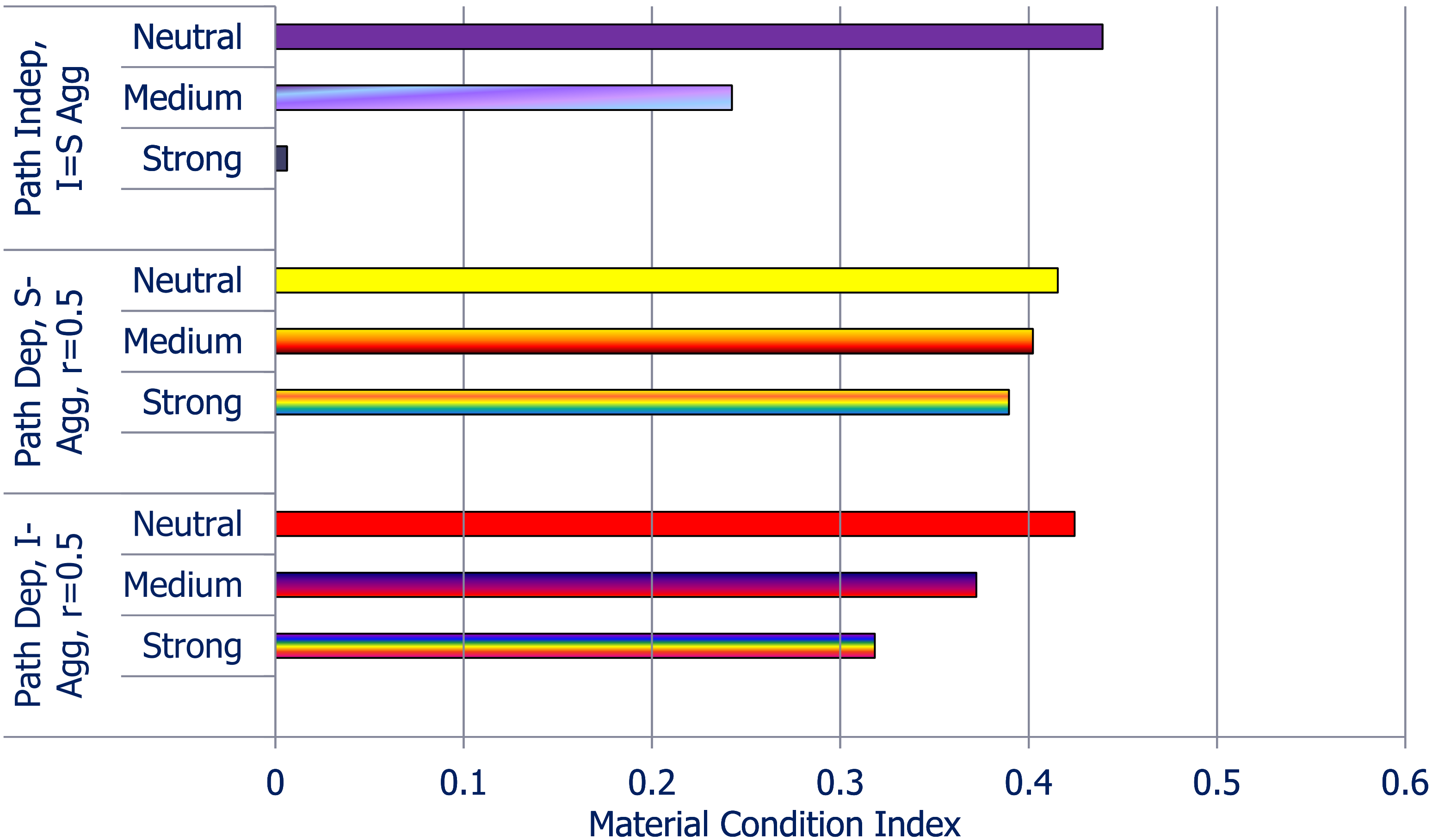
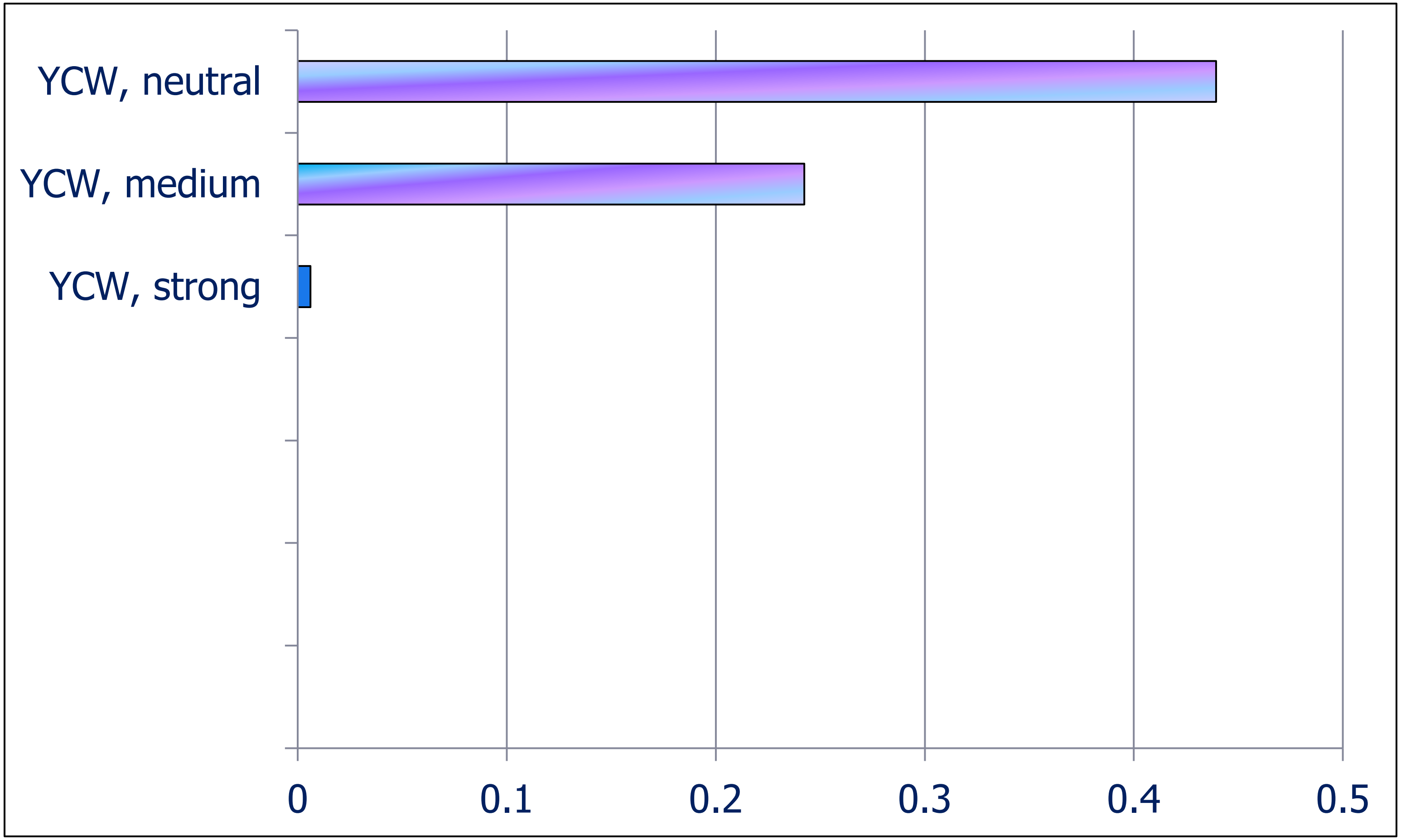
MCI, Aversion and Path Dependency



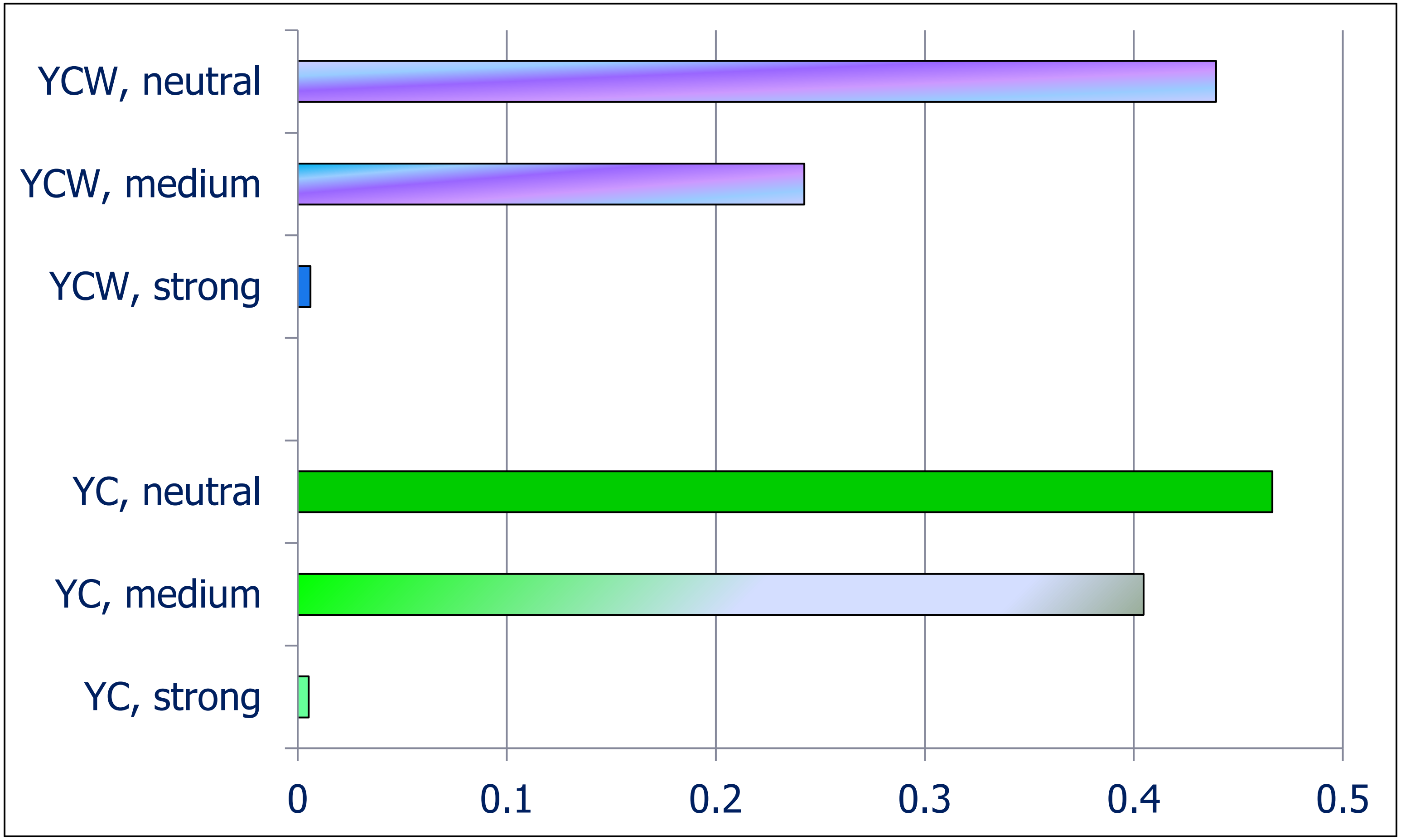
Data source: U.S. Consumer Expenditure Interview Survey, based on values in 2011 dollars U.S.

MCI's Based on Income, Consumption and Wealth vs. Income and Consumption (*q*=*r*)



Data source: U.S. Consumer Expenditure Interview Survey, based on values in 2011 dollars U.S.

MCI's Based on Income, Consumption and Wealth vs. Income and Consumption (*q=r*)



Data source: U.S. Consumer Expenditure Interview Survey, based on values in 2011 dollars U.S.

Summary

- Goals met
  - Produced Y, C, and W measures using OECD Framework with U.S. data, with limits
  - Applied Ruiz method to produce MCI, with caveats

- Preliminary results
  - MCI provides different picture of economic well-being than does any dimension alone
  - What matters
    - Aversions to inequality and un-balancement
    - Aggregation
    - Dimensions

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Future Directions

- Relax assumptions for construction of MCI
  - Normalization
  - Treatment of negatives and extreme values
- Refine definition of dimensions
  - Expenses associated with owned housing
  - Imputations
    - Vehicles
    - Income taxes
    - More social transfers in-kind

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Numerical Example: Arithmetic Mean

$$X = (I, C, W) = \begin{pmatrix} 3 & 1 & 2 \\ 6 & 2 & 5 \\ 7 & 3 & 10 \\ 7 & 5 & 14 \\ 10 & 5 & 24 \end{pmatrix}$$

$$X^N = \begin{pmatrix} \frac{3-0}{10-0} & \frac{1-0}{5-0} & \frac{2-0}{24-0} \\ \frac{6-0}{10-0} & \frac{2-0}{5-0} & \frac{5-0}{24-0} \\ \frac{7-0}{10-0} & \frac{3-0}{5-0} & \frac{10-0}{24-0} \\ \frac{7-0}{10-0} & \frac{5-0}{5-0} & \frac{14-0}{24-0} \\ \frac{10-0}{10-0} & \frac{5-0}{5-0} & \frac{24-0}{24-0} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix}$$

S-aggregation

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 0.7 & 0.5 \end{pmatrix}$$

$$\downarrow$$

$$0.6$$

I-aggregation

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1 \end{pmatrix} \rightarrow 0.6$$

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Numerical Example:
I- Aggregation, Curvature: *q=r=-2*

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} \left[\frac{1}{3}\left(0.3^{-2}+0.2^{-2}+0.1^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.6^{-2}+0.4^{-2}+0.2^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.7^{-2}+0.6^{-2}+0.4^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.7^{-2}+1^{-2}+0.6^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(1^{-2}+1^{-2}+1^{-2}\right)\right]^{\frac{1}{-2}} \end{pmatrix}$$

$$\begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} 0.1 \\ 0.3 \\ 0.5 \\ 0.7 \\ 1 \end{pmatrix} \rightarrow \left[\frac{1}{5}\left(0.1^{-2}+0.3^{-2}+0.5^{-2}+0.7^{-2}+1^{-2}\right)\right]^{\frac{1}{-2}} = 0.3$$

One-dimensional Index

- Assuming ($x_1, \dots, x_n$) is the distribution of income, for example, over n units of observations, the generalized mean of curvature q of this distribution

$$\mu_q\left(x_1,\dots,x_n\right)=\left[\frac{1}{n}\sum_{i=1}^{n}x_i^q\right]^{\frac{1}{q}}\ \forall q\neq 0$$

$$=\prod_{i=1}^{n}x_i^{\frac{1}{n}}\ \text{for q} = 0$$

- $q = 1$ the generalized mean simply reduces to the arithmetic mean
- $q = 0$, this case is the geometric mean
- $q = -1$, the harmonic mean
- as q decreases, greater weight on lower end of income (e.g.,) distribution so generalized mean refers this (Foster, Lopez-Calva, and Szekely 2005)

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Examples of Multi-dimensional Index

- S-Aggregation

$$W^S(X) = \left[\frac{1}{m} \sum_{j=1}^{m} \left[\left[\frac{1}{n} \sum_{i=1}^{n} \left(x_i^j \right)^q \right]^{\frac{1}{q}} \right]^r \right]^{\frac{1}{r}} \quad \forall q < 1, \forall r < 1, q \neq 0, r \neq 0$$

- I-Aggregation

$$W^I(X) = \left[\frac{1}{n} \sum_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j \right)^r \right]^{\frac{1}{r}} \right]^q \right]^{\frac{1}{q}} \quad \forall q < 1, \forall r < 1, q \neq 0, r \neq 0$$

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Example: MCI_{PI}*

- When $q=r=1$, $MCI=0.6$
- When $q=r=-2$, $MCI=0.3$

- Reduction in MCI result of discounting for 2 forms of inequality
  - Within dimensions between individuals, q
  - Between dimensions, r

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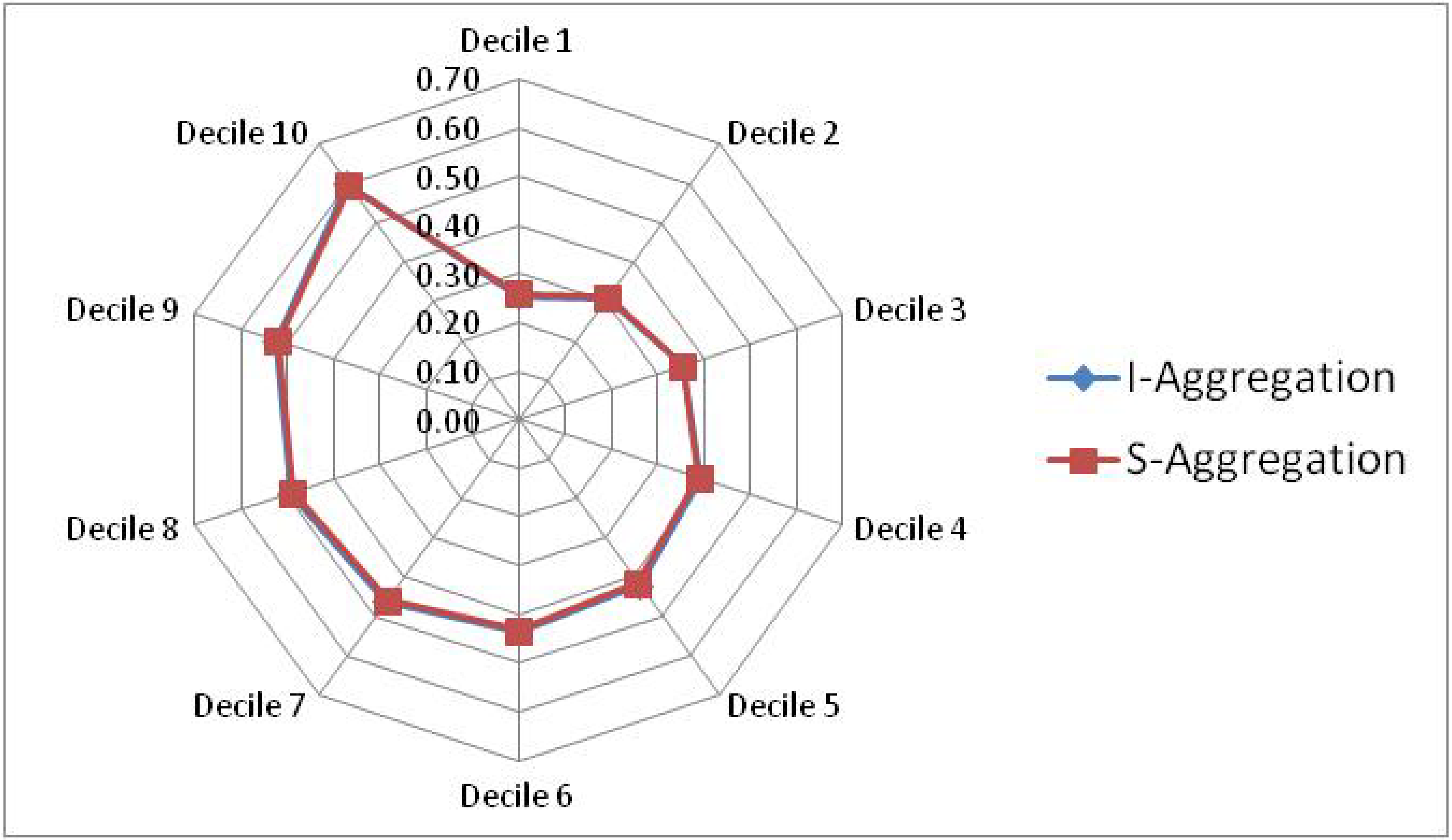
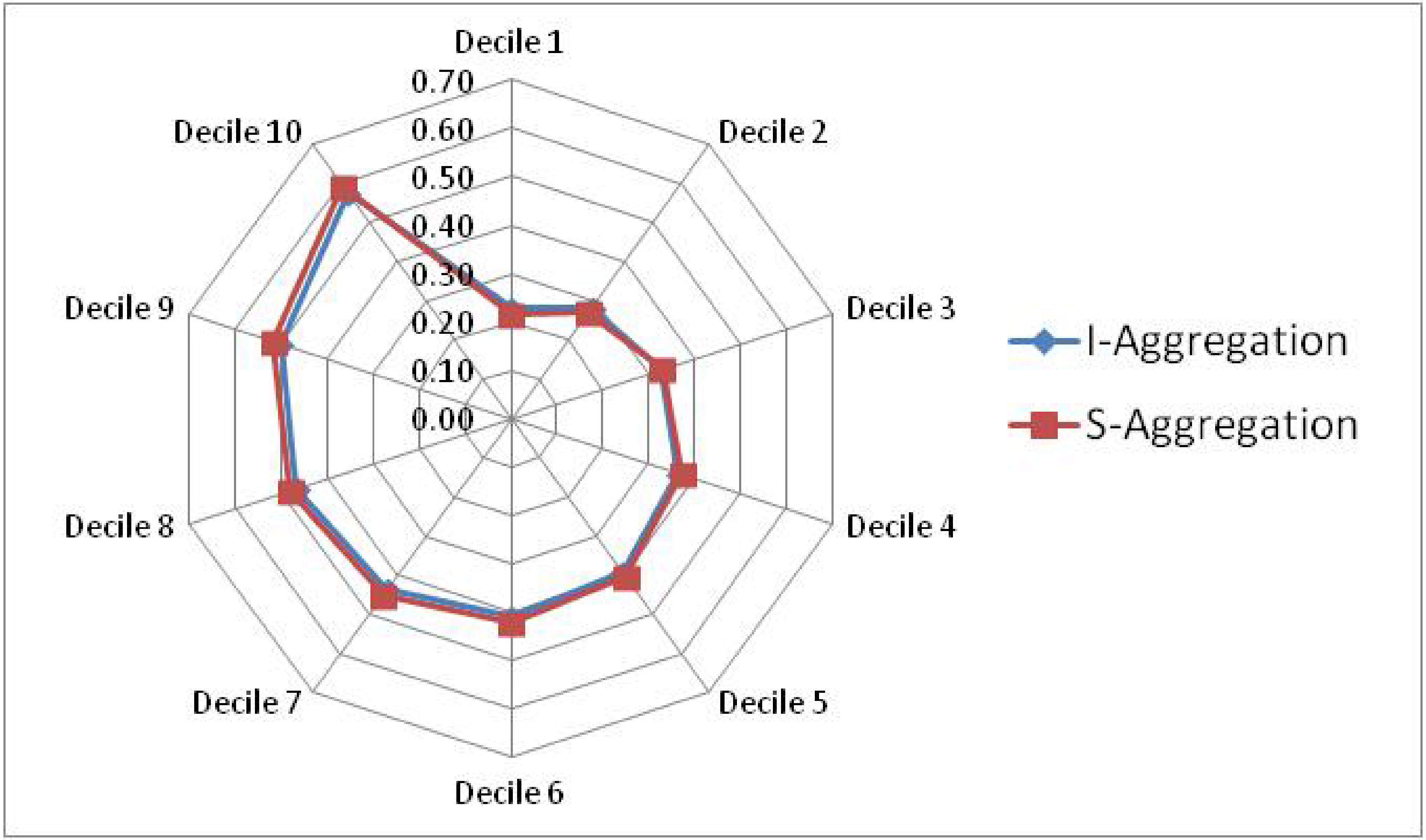
* PI = path independent assumption

MCI: Flexible Form *q =1, r=0.5*



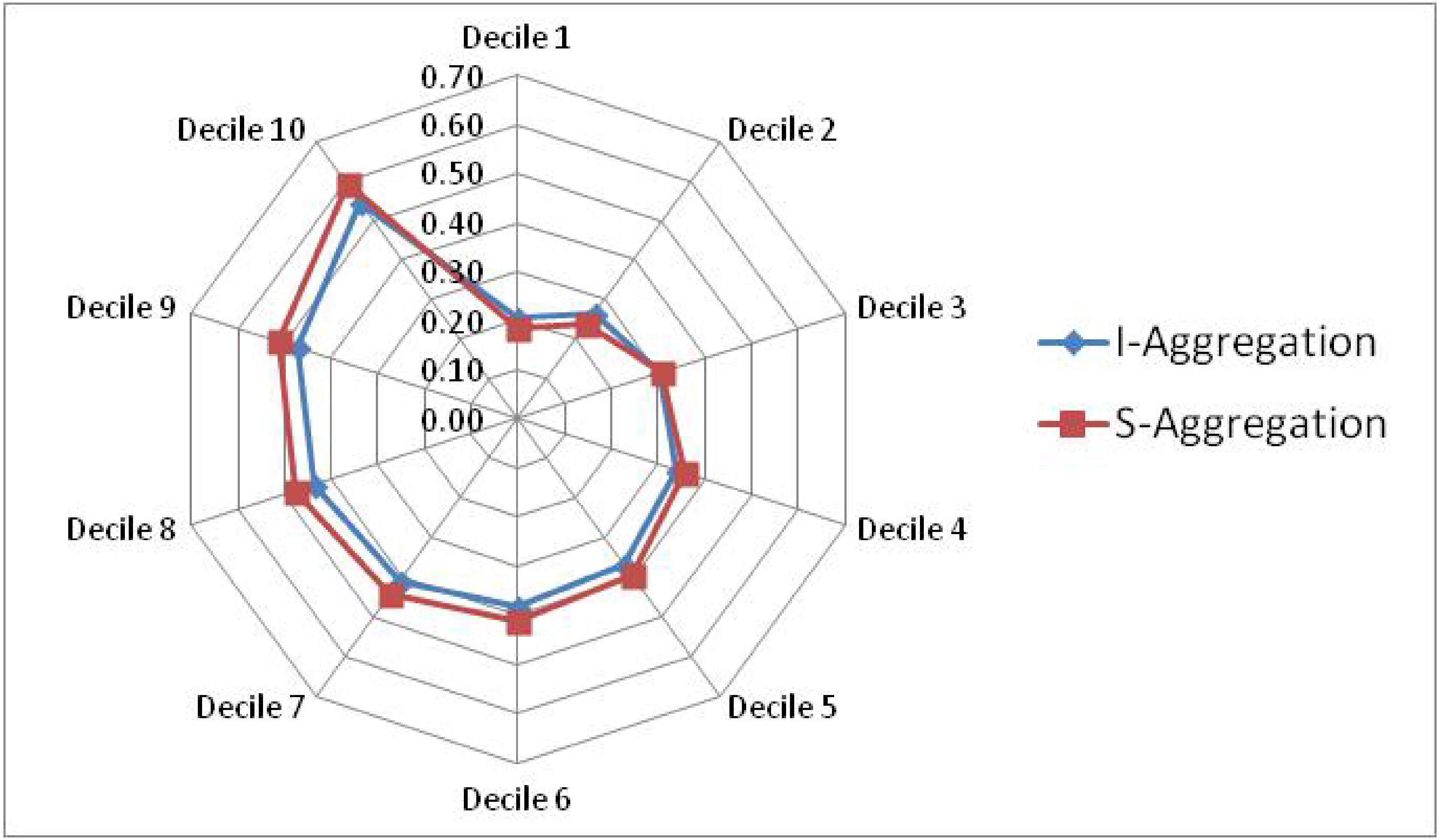
Neutral aversion to inequality across individuals; weak aversion across dimensions

MCI: Flexible Form *q =-1, r=0.5*



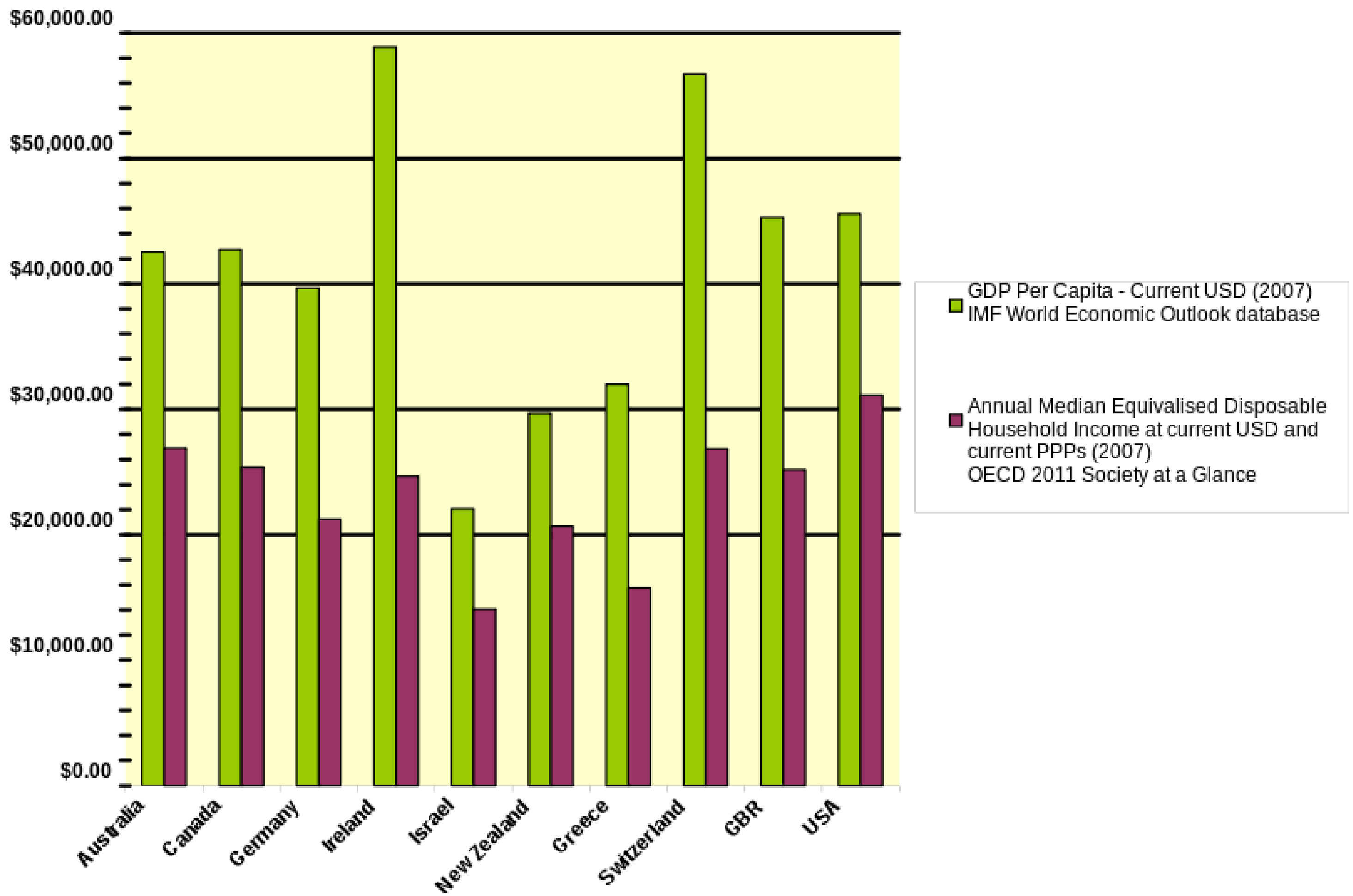
Medium aversion to inequality across individuals; weak aversion across dimensions

MCI: Flexible Form *q =-3, r=0.5*



Strong aversion to inequality across individuals; weak aversion across dimensions

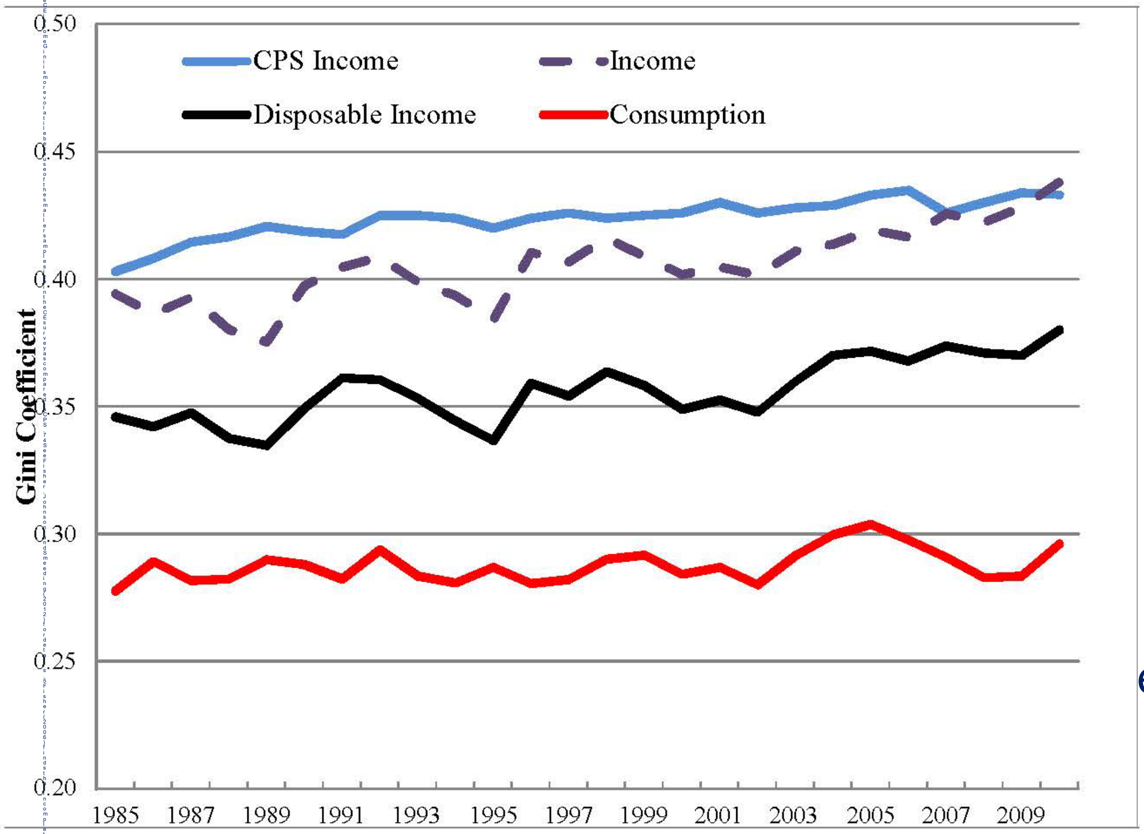
Ways to Measure: Levels or Means in Bar Graph e.g., GPD Per Capita or Median Household Income



Source; http://en.wikipedia.org/wiki/File:GDPPC_vs_Median_Income.svg

Ways to Measure: Distributions and Inequality as an Index, e.g., Gini Coefficient

Figure 1: Inequality using the Gini Coefficient (1985-2010)



Lower Gini = more equal the distribution

Source: Fisher, Jonathan and David S. Johnson, "Measuring the Trends in Inequality of Individuals and Families: Income and Consumption," AEA Session, January 2013.

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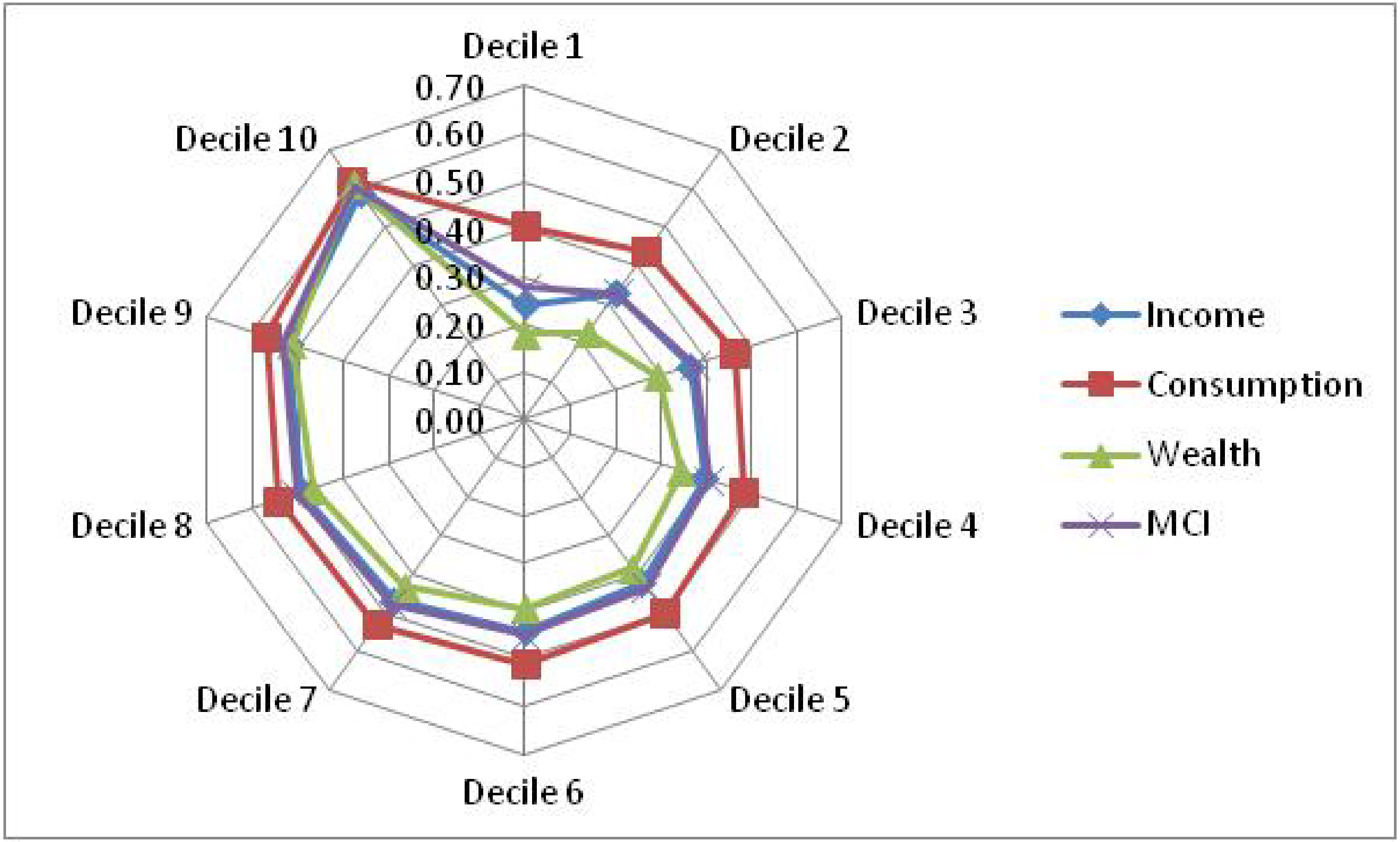
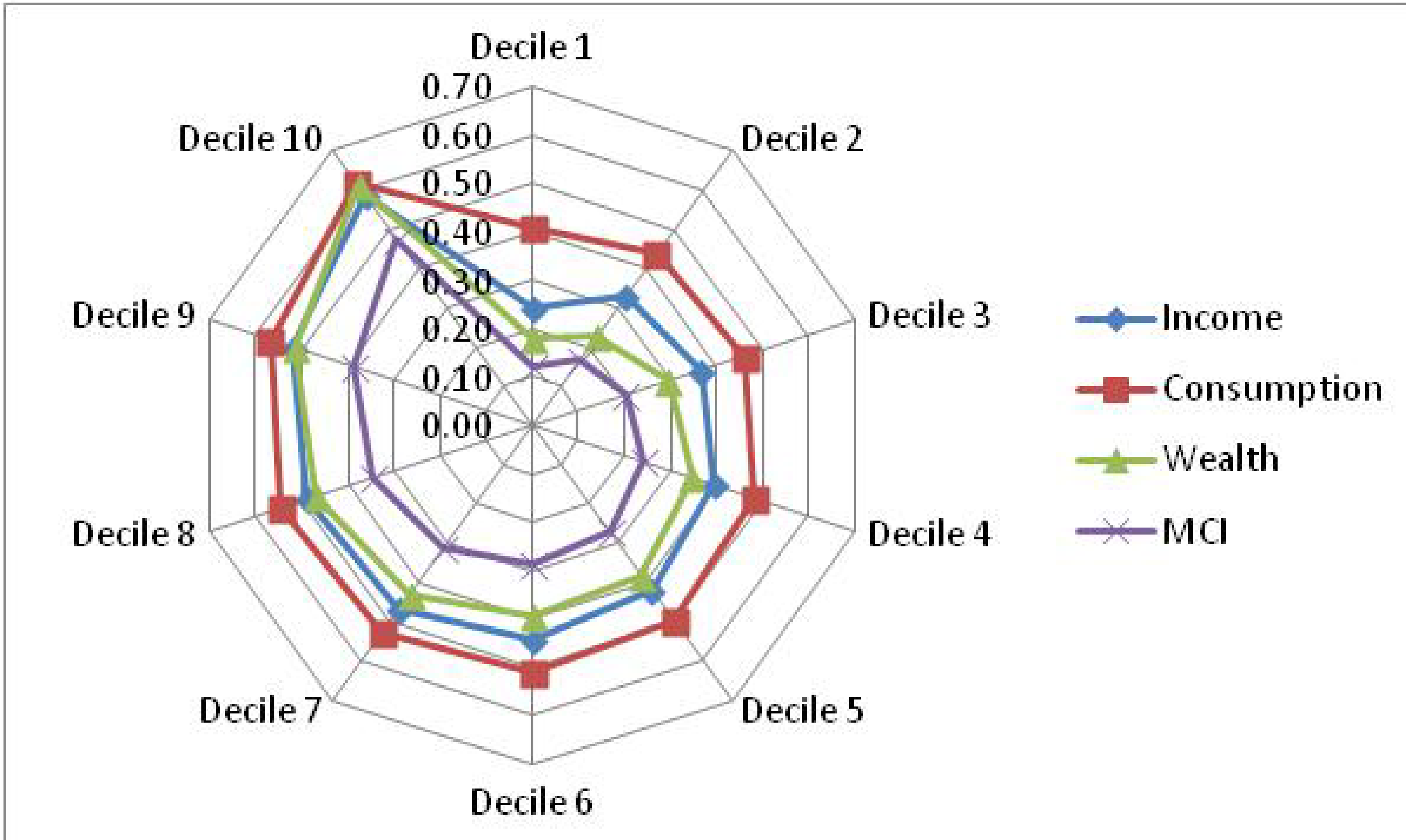
Neutral Aversion to Inequality *(q=r=1)* by Income Deciles



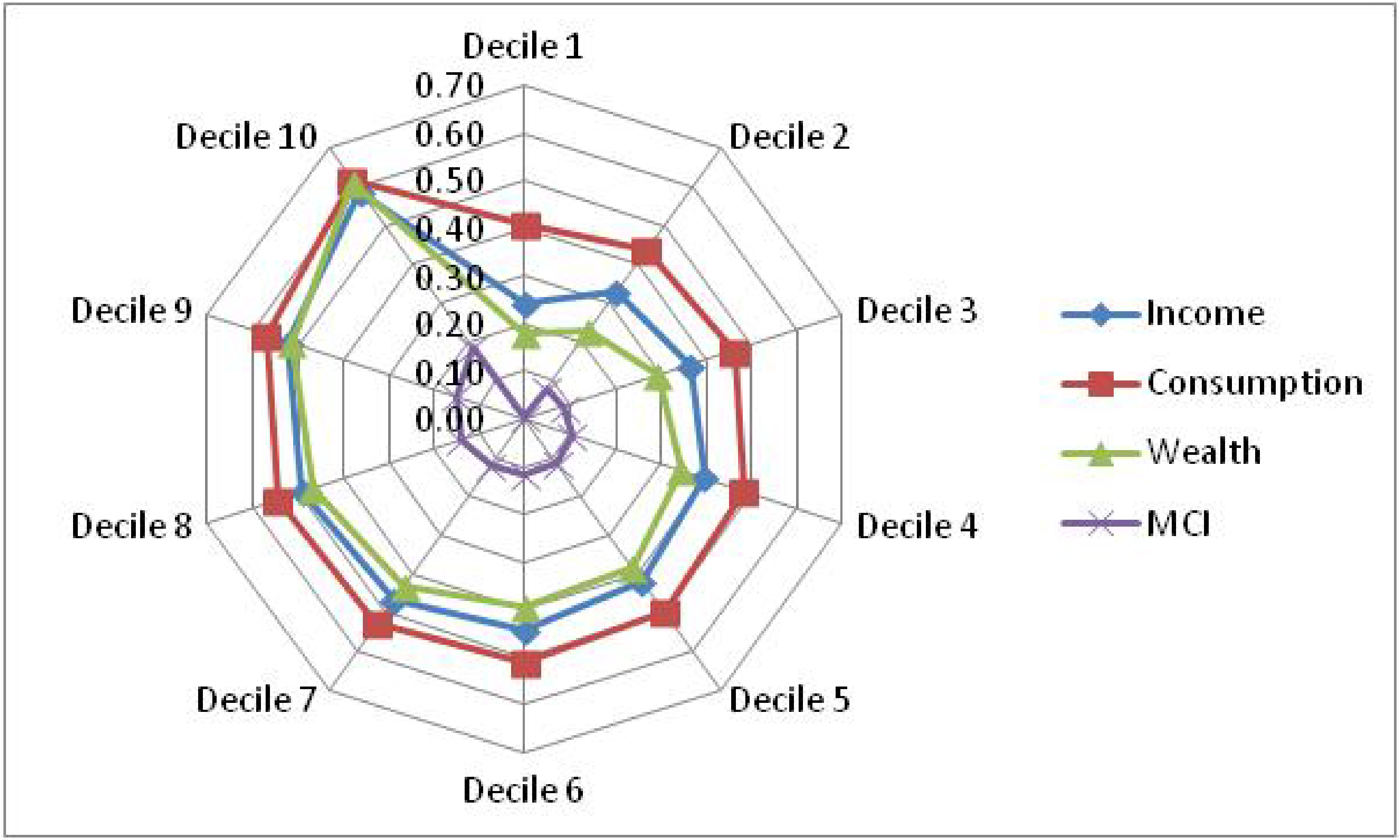
$MCI_I = MCI_S$ (path independent)

MCI with Medium Aversion to Inequality *(q=r=-1)* by Income Deciles



Neutral aversion for dimensions, $MCI_I = MCI_S$ (path independent)

MCI with Strong Aversion to Inequality *(q=r=-3)* by Income Deciles



Neutral aversion for dimensions, $MCI_I = MCI_S$ (path independent)

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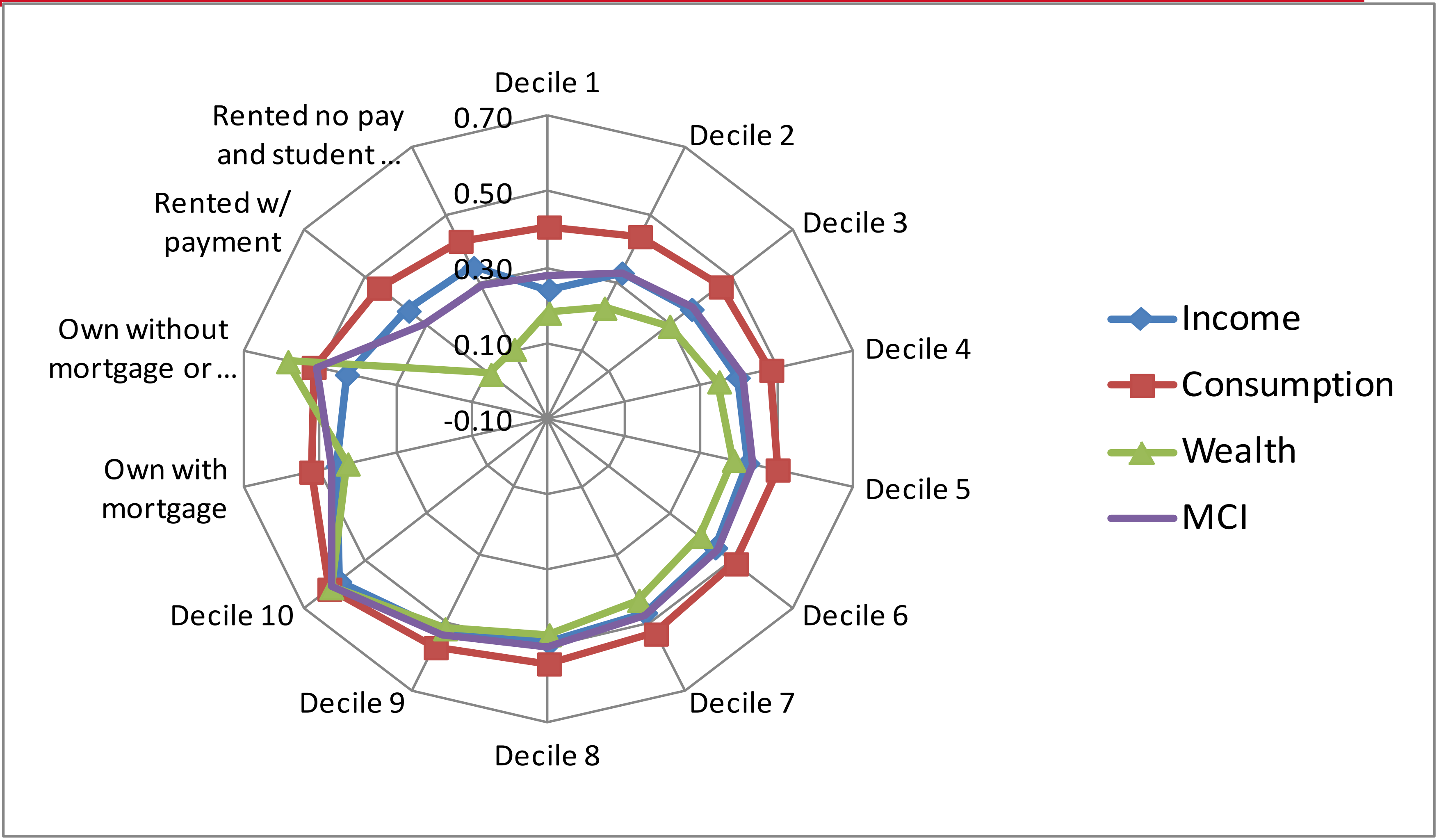
Neutral Aversion to Inequality *(q=r=1)* by Income Deciles and Housing Tenure



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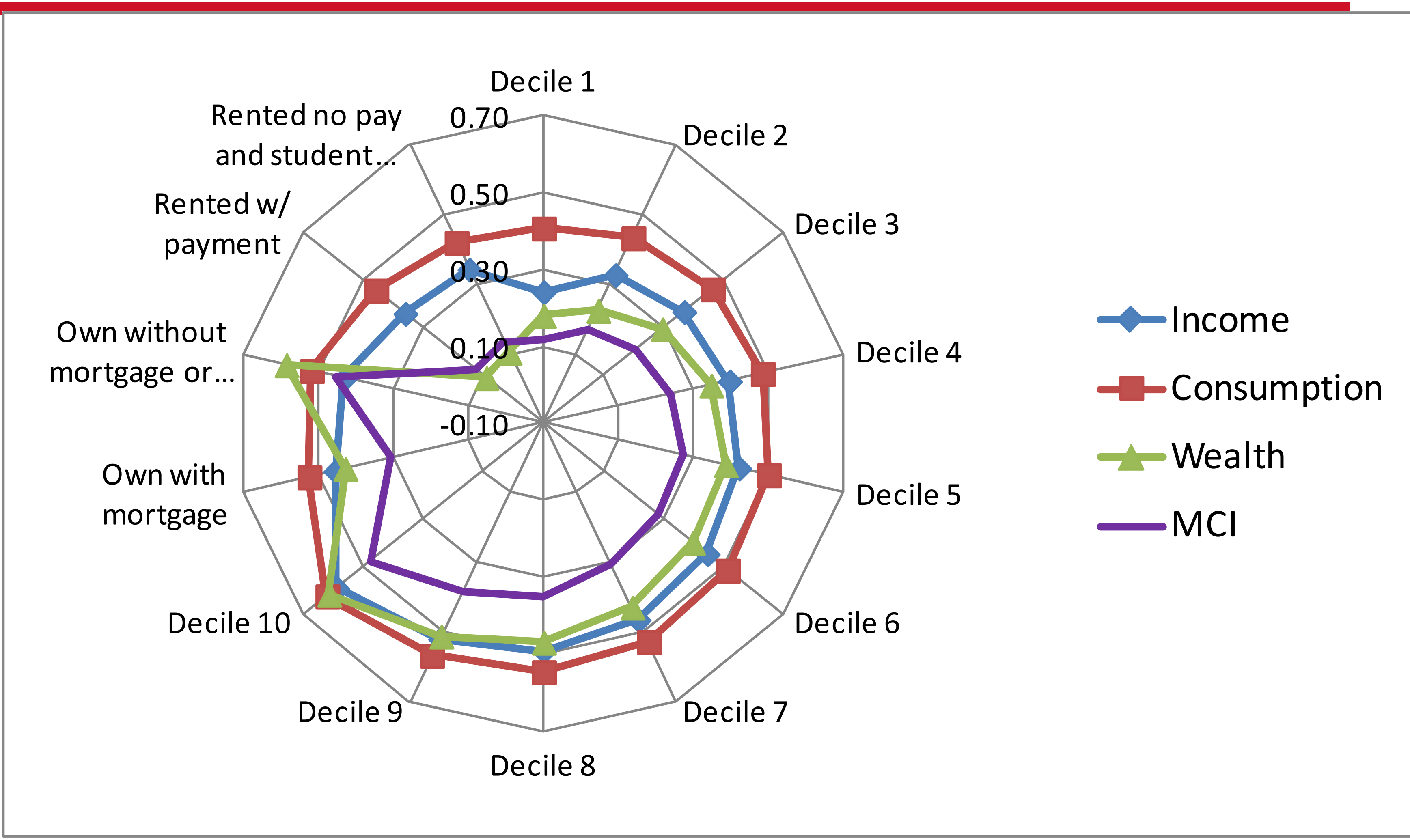

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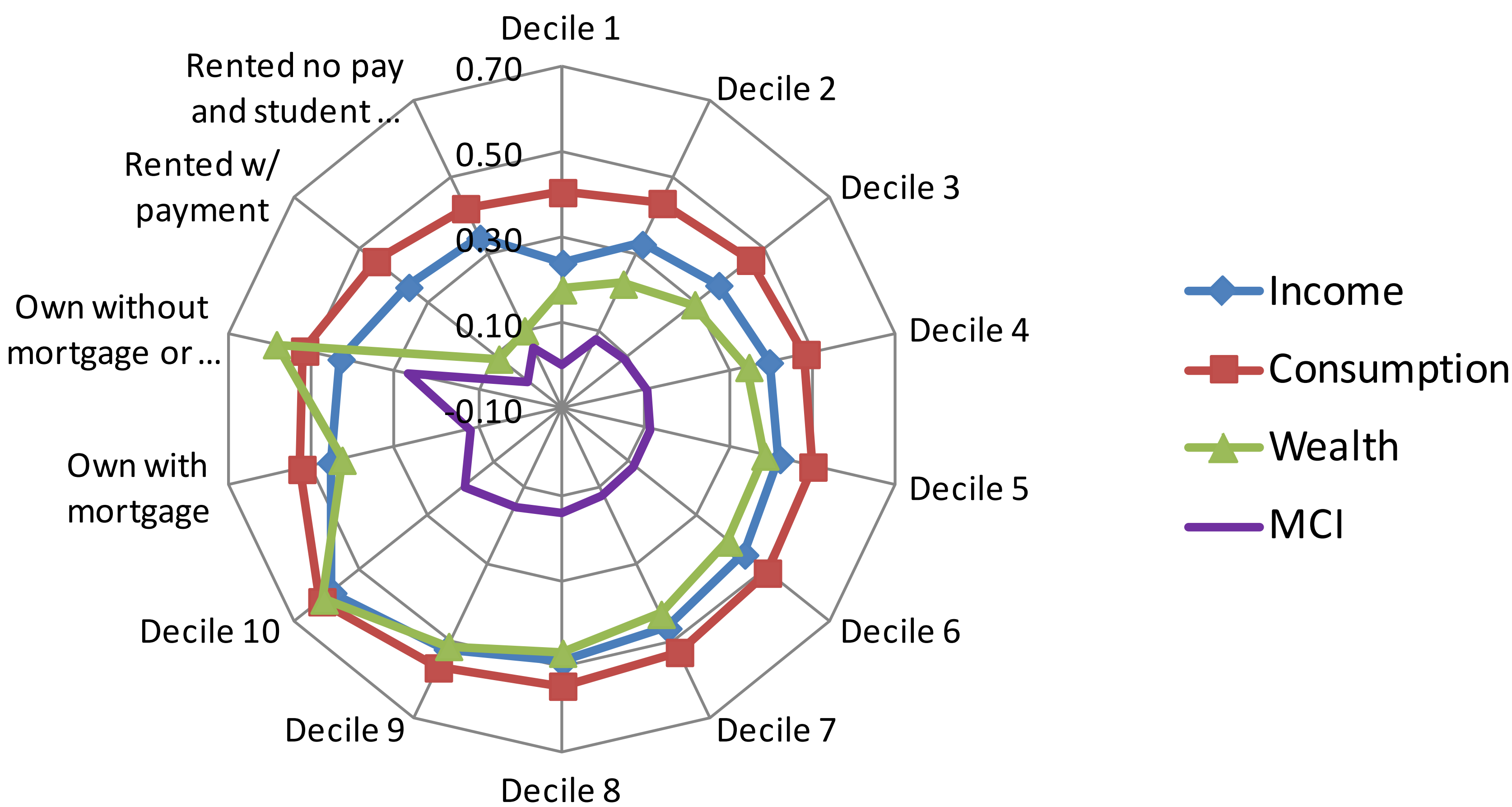

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