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The Effects of Mergers in Open Auction Markets

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The Effects of Mergers in Open Auction Markets

ABSTRACT. A buyer solicits bids from suppliers with different cost distributions. The cost distribution of a supplier is defined by its capacity. The expected market share of each supplier is the ratio of its capacity to the industry capacity. If the buyer's reserve price is fixed, mergers increase industry concentration, increase the expected price, and reduce the buyer's welfare. Moreover, suppliers have an incentive to merge. If the buyer can optimally lower the reserve price, he can partially or fully offset the effects of a merger. However, a merger still reduces the buyer's welfare because he must forego some gains from trade when he lowers the reserve price. The optimal reserve price can undermine the incentive for larger suppliers to merge and result in stable industry structures for which no further mergers would be profitable.

The Effects of Mergers in Open Auction Markets

The effects of mergers have been examined by a number of authors using the traditional Nash-Cournot model with homogeneous products. Several of these papers consider models in which the firms have different capacities and thus differing levels of output in equilibrium.[1] These asymmetric models allow a more realistic assessment of the competitive effects of mergers because one can examine mergers between firms of varying sizes. In this paper we investigate the effects of mergers in an auction market where the firms are asymmetric because they have different distributions for their costs. In the context of this model, we examine the traditional merger questions. Does a merger increase the price and reduce the welfare of buyers? Do firms have an incentive to merge? Do mergers reduce total welfare? The answers to these questions in the context of this auction model will have some similarities and some differences from those found using the asymmetric Nash-Cournot models.

We consider mergers in an asymmetric auction model of procurement in which the firms are interpreted as suppliers bidding to win contracts to supply some input that the buyer needs for the production of a final good. This is typical of an industrial product setting in which auctions are a common method for the purchase of inputs.[2] The model employs a

[1] Perry and Porter (1985) consider the incentive to merge in a Nash-Cournot model with a homogeneous product. In their model, firms have different capacities in that the marginal cost of production for one firm is linearly increasing and shifts horizontally outward with more capacity. Using a linear demand function, they solve for an asymmetric Nash-Cournot equilibrium with two types of firms, one type with twice the capacity of the other type. The merger of two small firms into one large firm increases the price and reduces consumer welfare. There is typically an incentive to merge when the two firms already have large market shares, but there need not be an incentive to merge when the two firms have smaller market shares.

Farrell and Shapiro (1990a and 1990b) examine a general version of the capacity model used by Perry and Porter (1985), and focus on the welfare effects of mergers. They find that large firms operate at a lower output per unit capacity than small firms. This results in an inefficient allocation of a given industry output across the firms. Mergers between large and small firms eliminate some of this inefficiency and thus can increase welfare despite the resulting higher prices. Farrell and Shapiro then provide some general conditions for identifying mergers that would increase welfare. With the linear version of this capacity model, McAfee and Williams (1992) characterize the profitable mergers between two firms that would increase welfare. Their computational results suggest a limited set of circumstances in which mergers between large and small firms are both profitable and increase welfare. Werden (1991) also reexamines the linear model in order to illustrate the relevance of the Herfindahl-Hirschman index (HHI) in assessing the price effects of mergers.

[2] This auction model is unlikely to be an appropriate representation of competition in consumer product markets. In those markets, the more appropriate model would be either the Nash-Cournot model if products are homogeneous or the Nash-Bertrand model if the products are differentiated. In recent years, various economists have proposed and the antitrust agencies have employed empirical methodologies to estimate or simulate the price effects of mergers in differentiated consumer product industries. For an overview of these methodologies and their applications, see the Spring 1997 issue of Antitrust published by the American Bar Association. This empirical work has its theoretical foundations in a paper by Deneckere and Davidson (1985). Deneckere and Davidson investigate the incentive to merge in a model with heterogeneous products. Their model does not have a capacity variable on the cost side. However, the merged firm continues to sell both products. In this sense, a merged firm is larger than the other firms. They find an incentive to merge

second-price auction mechanism. In the second-price auction, suppliers submit sealed bids to a buyer and the input is sold to the lowest bidder at a price equal to the second lowest bid. If the costs of the suppliers are private information and stochastically independent, second-price auctions are strategically equivalent to open auctions, also called descending price oral auctions.

In an open auction, the auction begins at some high price, declines with progressively lower bids, and the contract is awarded to the last bidder active in the auction.[3] The dominant strategy for each supplier in such auctions is to submit a bid equal to its cost. Thus, the supplier with the lowest cost will win the contract and receive a contract price equal to the cost of the supplier with the second lowest cost. We only consider second-price and open auctions since analytical results for the effects of mergers in first-price auction markets are difficult to obtain. The equilibrium in first-price auctions is much less tractable when the cost distributions are not identically distributed.[4]

In this paper we also allow for the possibility that the buyer in this auction can set a reserve price to maximize his expected profit from the employment of the input. A binding reserve price would require that the buyer commit to forego purchasing at a price above the reserve price even when such a price is below the buyer's value for the input.

Several recent papers examine mergers in auction markets with asymmetric firms.[5] Waehrer (1997) examines mergers in both first-price and second-price auction markets. Using the same structure on cost distributions employed in this paper and assuming a fixed reserve price, Waehrer finds that the profit per market share for larger firms is higher than for smaller firms in second-price auction markets, while the opposite is true in first-price auction markets. It follows that non-merging firms benefit from a merger in first-price auction markets, while the non-merging firms are unaffected by a merger in second-price auction markets. This suggests that the incentive to merge may be stronger in second-price auction markets. We return to this point later in the paper.

Using simulations to derive equilibrium bidding strategies for first-price auctions, Dalkir, Logan, and Masson (1998) show that failure to consider the asymmetries generated by merg-

from the ability to coordinate in the pricing of the two products. A merger results in higher prices for both the products of the merged firm and the products of the nonmerging firms.

[3] The second-price auction model might be an appropriate representation of the market even though the buyers do not employ a strict descending price oral auction. For example, buyers might invite an initial sealed bid from the suppliers, suggesting a first-price auction. However, if the buyers subsequently use the low bids to "whipsaw" other suppliers into lowering their bids, a second-price auction model seems more appropriate.

[4] General treatments of asymmetric first-price auctions can be found in Maskin and Riley (1992), Lebrun (1995), and Waehrer (1997).

[5] There is also a related literature on collusion in auction markets. For example, see DeBrock and Smith (1983), Graham and Marshall (1987), Von Ungern-Sternberg (1988), Mailath and Zemsky (1991), McAfee (1994), and McAfee and McMillan (1992).

ers can result in an overstatement of their price effects.[6] Tschantz, Crooke, and Froeb (1997) simulate equilibria for first-price auctions assuming costs follow the extreme value distribution. They find evidence that the anti-competitive effects of mergers are smaller for first-price auctions than for second-price auctions. Thomas (1998) derives analytical solutions for an asymmetric first-price auction by assuming that costs are binomially distributed. Thomas obtains the surprising result that a merger between two weaker firms may in fact benefit the buyer even when there are no efficiencies generated by the combination. However, in such situations the firms have no incentive to merge.

The asymmetric second-price auction is more tractable than the first-price auction because the dominant strategy for a supplier in second-price auctions is to submit a bid equal to its cost. Thus, the expected equilibrium price is simply the expected value of the second-order statistic of the distributions on the costs for all suppliers. Brannman and Froeb (1997) and Froeb, Tschantz, and Crooke (1998) study asymmetric second-price auctions when costs follow the extreme value distribution. They examine the price effects of mergers assuming a fixed reserve price. For a common-value second-price auction, Krishna and Morgan (1997) show that joint bidding can actually benefit the bid-taker.

In this paper, we characterize a family of distributions for the costs of the suppliers that are parameterized by variables which can be interpreted as the sizes, scales, or capacities of the suppliers. The family of cost distributions is assumed to have properties which eliminate both (1) positive or negative externalities across suppliers and (2) economies or diseconomies of scale. Thus, mergers do not affect the cost distribution of non-merging suppliers and generate constant returns to scale for merged suppliers. We then consider a private-value second-price auction with a reserve price. We find that mergers always reduce the welfare of the buyer, even if the buyer optimally adjusts the reserve price in response to the merger. When the reserve price is fixed, mergers result in a higher expected price. However, when the buyer adjusts the reserve price in order to maximize its expected profit, the expected price can fall as a result of a merger.

The results also provide a systematic way of ranking different mergers by the magnitude of their effects on the buyers. The incentive to merge and the overall welfare effects depend on whether the reserve price is fixed or set optimally by the buyer. When the reserve price remains the same before and after a merger, total welfare is unaffected by a merger and suppliers always have an incentive to merge. However, when the reserve price is adjusted optimally by the buyer, total welfare decreases as a result of the merger because the buyer's optimal response is to reduce the reserve price. In this case suppliers may or may not have an incentive to merge and the expected price may increase or decrease. Numerical examples

[6] Also using simulations, Marshall, et.al. (1994) find evidence that the bid-taker's payoff is higher in first-price auctions than in second-price auctions when cost distributions are independent and asymmetric.

suggest that smaller suppliers are more likely to have an incentive to merge than larger suppliers.

The paper is organized as follows. In Section I, we define the second-price auction with asymmetric capacities. In Section II, we then solve for the expected probability of winning each contract by the suppliers and the expected price paid by the buyer. In Section III, we define industry concentration and explain its relation to the HHI. In Section IV, we examine the welfare questions of whether mergers increase the expected price and reduce the expected profit of the buyer. Finally, in Section V, we examine the incentive to merge and the effects of a merger on total welfare.

1. Model

We assume that a buyer requests bids from suppliers who can provide an input necessary for production of a final good. The buyer employs an open auction to select the winning supplier. More generally, the market would be composed of a series of such auctions by a number of buyers.

The buyer has a value $v > \underline{c}$ for the input where v is known to the buyer and to all of the suppliers. The buyer would clearly reject a bid price for the input above v. However, we will also consider cases in which the buyer can reject bid prices below his value v by committing to a single reserve price $r < v$. With a reserve price $r < v$, the buyer is committing not to purchase the input from any supplier at a bid p where $r < p$, either during the auction or after an auction in which no supplier offered a bid less than r.[7] When the reserve price is set below v and below the upper support on the cost distributions of the suppliers, an ex post inefficiency could arise from the lack of trade at some price less than the buyer's value but greater than some supplier's cost. Note that if the buyer cannot commit to a reserve price less than its value, we will simply assume that $r = v$.

Let $N = \{1, \ldots, n\}$ denote the set of potential suppliers of the input. We assume that each supplier i has a capacity parameter t_i and draws his cost c_i of producing the input from the distribution $G(\cdot|t_i)$ with a support of $[\underline{c}, \bar{c}]$ common to all suppliers. Denote the profile of capacities for the suppliers as $\mathbf{t} = (t_1, \ldots, t_n)$ and define $\hat{t} = \sum_{i=1}^{n} t_i$ as the industry capacity. We assume that the suppliers' costs are independently distributed.[8] Furthermore, each supplier obtains his cost of production prior to submitting a bid to the buyer and need

[7] We only consider the case where the buyer uses a single reserve price that applies equally to all of the suppliers regardless of size. When the suppliers have different capacities, the buyer may be able to gain by setting different reserve prices for the different suppliers.

[8] This independence assumption would be violated if the costs of suppliers depended on the uncertain future prices of raw materials and if each supplier received a correlated signal about those future prices. Our model assumes that no such correlation exists. Thus, if raw material prices are relevant, we assume that these costs are known and common to the bids of all suppliers. Any remaining uncertainty about the costs of each supplier would depend only on characteristics unique to that supplier.

not incur this cost unless he wins the auction to supply the input.

We assume that the suppliers are asymmetric in that they have different capacity parameters which define different distributions for their cost of producing the input. With different distributions on costs, the equilibrium will necessarily have the property that suppliers will have different expected probabilities of winning the auction, and thus, have different market shares over a large number of auctions. When two suppliers merge, they have a larger combined capacity, and thus, the cost of producing the input is drawn from a new distribution that places a higher probability on low costs. This is the sense in which the merger results in one new larger supplier who will have a higher probability of winning the auction.

In defining the notion of size, scale, or capacity which generates the differences in the cost distributions for suppliers, we want to impose three properties. First, we assume that there are no positive or negative externalities across suppliers. In particular the cost distribution of each supplier is independent of the capacities of the other suppliers. As such, we eliminate any real or pecuniary externalities from mergers which affect the costs of the other suppliers.

Second, we assume that capacity is a homogeneous parameter. In particular, any suppliers with the same capacity have the same cost distribution. As a result, the equilibrium will depend only on the current capacity of each of the suppliers, and not the history of mergers.

Third, we assume that there are no economies or diseconomies of scale in these cost distributions. In order to accomplish this, we assume that the probability distribution of the minimum cost draw of all the suppliers does not depend on how capacity is distributed across suppliers. That is, as long as total capacity is constant, the distribution of the minimum cost is the same whether there are only a few large firms or many small firms. This property is equivalent to assuming that the cost distribution of a merged supplier is the same as the distribution of the minimum cost of the two suppliers before the merger. By eliminating economies or diseconomies, we can focus on the price and welfare effects that result solely from increasing concentration after a merger.[9]

The three properties of the cost distributions can be formally expressed as follows.

Property 1 (No Externalities): The distribution of each supplier's cost depends only on its own capacity and not on the capacities of other suppliers.

Property 2 (Homogeneity): If two suppliers have the same capacity, then they also have the same cost distribution.

[9] Brannman and Froeb (1997), Waehrer (1997), Dalkir, Logan, and Masson (1998), and Froeb, Tschantz, and Crooke (1998) also model mergers in this way. This type of merger is equivalent to efficient collusion as discussed by Mailath and Zemsky (1991) and McAfee and McMillan (1992).

The quadratic cost model in Perry and Porter (1985) possesses this property of constant returns. Define $x(c, s)$ as the quantity of output that can be produced at or below a marginal cost of c, given the capacity s. The quadratic cost function implies that $x(c, s_i + s_j) = x(c, s_i) + x(c, s_j)$.

Property 3 (Constant Returns): The probability distribution of the lowest cost draw out of all of the suppliers depends only on total capacity. In particular, it does not depend on the number of suppliers or how total industry capacity is distributed across suppliers. That is, for any two capacity profiles $t_1, \ldots, t_n$ and $s_1, \ldots, s_m$ such that $\sum_{i=1}^{n} t_i = \sum_{i=1}^{m} s_i$ and for any $c \in [\underline{c}, \overline{c}]$,

$$1 - \prod_{i=1}^{n} [1 - G(c|t_i)] = 1 - \prod_{i=1}^{m} [1 - G(c|s_i)] . \tag{1}$$

For mergers, we assume that if two suppliers with capacities t_i and t_j merge, then the capacity of the resulting supplier is $t_m = t_i + t_j$. Thus, Property 3 implies that the cost distribution of a merged supplier is the minimum of the two cost draws from the original suppliers. That is,

$$G(c|t_i + t_j) = 1 - [1 - G(c|t_i)] [1 - G(c|t_j)] .$$

The following result characterizes Properties 1, 2, and 3 in terms of the form the distribution must take.

Theorem 1. *Properties 1, 2, and 3 hold if and only if there exists a distribution function F with a support of $[\underline{c}, \overline{c}]$ such that for $c \in [\underline{c}, \overline{c})$,*

$$G(c|t_i) = 1 - [1 - F(c)]^{t_i} .$$

Proof in the Appendix.

According to Theorem 1, any cost structure that satisfies Properties 1, 2, and 3 must take the form described in the theorem. In other words, there is no cost structure more general than this form that satisfies Properties 1, 2, and 3. Alternatively, for any distribution F, when the cost distribution G takes the form described, then Properties 1, 2, and 3 must be satisfied. Fortunately, this cost structure simplifies the analysis. The remainder of our analysis will be conducted assuming that Properties 1, 2, and 3 hold and thus, making use of the functional form provided by Theorem 1.[10]

If f is the density function associated with the distribution F, then the resulting density function for $G(\cdot|t_i)$ can be written as $g(c|t_i) = t_i [1 - F(c)]^{t_i - 1} f(c)$. The distribution F

[10] Waehrer (1997) makes use of the same family of distributions. The structure here generalizes the cost structure assumed by Marshall, et.al. (1994) and Dalkir, Logan, and Masson (1998) in that those analyses assume that F is the uniform distribution. Brannman and Froeb (1997) and Froeb, Tschantz, and Crooke (1998) employ the extreme value distribution for which a parameter similar to t can be defined. However, the analysis is limited to this family of distributions and the convenience of this family requires that positive probability be assigned to any value or cost in $(-\infty, \infty)$.

defines a family of cost distributions G for suppliers with different capacity parameters. If a supplier has a capacity of one, then its cost distribution is simply $G(\cdot|1) = F$. For a supplier with a capacity parameter greater than one, the supplier would have a more favorable cost distribution in that there would be a higher probability of obtaining a cost below c. That is, for any $c \in (\underline{c}, \overline{c})$ and $t_i > 1$, $G(c|t_i) > G(c|1)$.

The capacity parameter t_i can be interpreted as the number of draws from a cost distribution F, where the supplier uses the lowest draw for its cost of producing the input. As such, $G(\cdot|t_i)$ is the distribution function of the order statistic for the lowest cost from among t_i independent draws from F. Similarly, the capacity parameter t_i can be interpreted as the number of plants from which the supplier could produce the input, each plant having a cost distribution of F. This is the sense in which the parameter t_i can be interpreted as measuring the size, scale, or capacity of a supplier.[11] Even though t_i does not measure capacity in the sense of an absolute physical limit on production, we will refer to t_i as the capacity of the ith supplier since it is a measure of size.

2. Market Equilibria and Expected Price

In a second-price auction, the supplier with the lowest bid wins and sells at a price equal to the lower of the reserve price or the second lowest bid. When setting the reserve price, the buyer knows the capacities of the suppliers but does not know their actual cost draws. In equilibrium, each supplier has a dominant strategy to submit a bid equal to its cost. If the seller submits a bid below its cost, then there is a possibility that the supplier will be required to sell at a price below its cost. On the other hand, if a supplier submits a bid above its cost, then there is a possibility that the supplier will forego an opportunity to sell the input at a price above its cost. Conditional on winning, a supplier's bid has no effect on the price it receives. Therefore, there is no gain to any supplier from raising its bid above its cost. Thus, in equilibrium, each supplier submits a bid equal to its cost.

For any given contract, there would be at most one winning supplier at a given price. However, in order to assess the industry performance, we examine the expected market share. The expected market share of a supplier is its probability of winning a given contract conditional on some supplier winning the contract. We can now state the following lemma.

Lemma 1. *Suppose the capacity profile is* $\mathbf{t}$. *(a) The probability distribution for the lowest cost in the industry is* $1 - [1 - F(c)]^{\hat{t}}$. *(b) The ex ante expected market share of each supplier is the ratio of its capacity and the total industry capacity,* $t_i/\hat{t}$.

[11] Piccione and Tan (1996) obtain the same functional form from a property which they call "complete stochastic ordering". In Piccione and Tan (1996) and an earlier paper, Tan (1992), the exponent is interpreted as research and development activity. Both papers examine the symmetric equilibrium in research and development expenditures by firms competing in an auction for a production contract.

Proof in the Appendix.

We can now derive expressions for the suppliers' profits, the buyer's profit, and the expected price paid by the buyer conditional on the contract being awarded. Let $\pi^i(c_i, r|\mathbf{t})$ denote supplier i's expected profit in equilibrium when i has costs c_i, the reserve price is r, and the capacity profile is $\mathbf{t}$. The distribution function of the lowest cost of supplier i's rivals is $1 - [1 - F(c_i)]^{\hat{t}-t_i}$. Thus, we have

$$\begin{aligned}
\pi^i(c_i, r|\mathbf{t}) &= \left[\int_{c_i}^{r} [\xi - c_i](\hat{t} - t_i)[1 - F(\xi)]^{\hat{t}-t_i-1} f(\xi)d\xi + (r - c_i)[1 - F(r)]^{\hat{t}-t_i}\right] \cdot 1_{\{c_i \leqslant r\}} \\
&= \left[\int_{c_i}^{r} [1 - F(\xi)]^{\hat{t}-t_i} d\xi\right] \cdot 1_{\{c_i \leqslant r\}} \qquad \text{(by integration by parts).}
\end{aligned}$$

The indicator function, $1_{\{c_i \leqslant r\}}$, is equal to one when the condition within the braces is true and zero otherwise. Supplier i's *ex ante* expected profit is $\Pi^i(r|\mathbf{t}) = E[\pi^i(c_i, r|\mathbf{t})]$ where the expectation is taken over supplier i's cost.

$$\begin{aligned}
\Pi^i(r|\mathbf{t}) &= \int_{\underline{c}}^{r} \int_{c_i}^{r} [1 - F(\xi)]^{\hat{t}-t_i} t_i [1 - F(c_i)]^{t_i - 1} f(c_i) d\xi dc_i \\
&= \int_{\underline{c}}^{r} \left\{[1 - F(c_i)]^{\hat{t}-t_i} - [1 - F(c_i)]^{\hat{t}}\right\} dc_i \qquad \text{(by integration by parts).} \qquad (2)
\end{aligned}$$

Holding the reserve price constant, a supplier's expected profit (both $\pi^i(c_i, r|\mathbf{t})$ and $\Pi^i(r|\mathbf{t})$) depends only on its capacity t_i and the industry capacity $\hat{t}$. Expected profit does not depend on how the capacity $\hat{t} - t_i$ is distributed among the rival suppliers. Thus, a merger does not alter the expected profits of the non-merging suppliers when the buyer's reserve price remains unchanged. Similarly, a merger would not increase the expected profits of potential entrants since potential entrants can be thought of as potential non-merging suppliers.[12] It is also easy to see that for $r \in (\underline{c}, \overline{c})$, a supplier's expected profit is increasing in the reserve price. A lower reserve price does not alter a supplier's market share, but it does reduce the probability of purchase by the buyer.

Note that the total expected profit to the suppliers can be written as

$$\sum_{i=1}^{n} \Pi^i(r|\mathbf{t}) = (p(r|\mathbf{t}) - E[z_1|z_1 \leqslant r]) \Pr\{z_1 \leqslant r\} \qquad (3)$$

where $p(r|\mathbf{t})$ is the expected price paid by the buyer conditional on a sale taking place and

[12] As Waehrer (1997) points out, this would not be true in a first-price auction. A merger in a first-price auction would increase the expected profit of both non-merging suppliers and entrants. Similarly, this is not true in the asymmetric Nash-Cournot model. A merger in that model increases the profits of the non-merging firms because the higher price after the merger induces them to expand output.

where $z_1 = \min\{c_1, \ldots, c_n\}$. Making use of Lemma 1, it is straightforward to see that

$$\Pr\{z_1 \leqslant r\} = 1 - [1 - F(r)]^{\hat{t}} \qquad \text{and} \qquad E\left[z_1 | z_1 \leqslant r\right] = \frac{\int_{\underline{c}}^{r} c\, \hat{t}[1 - F(c)]^{\hat{t}-1} f(c)\, dc}{1 - [1 - F(r)]^{\hat{t}}}.$$

Hence, the right-hand side of (3) is the expected price net of the expected cost, times the probability of purchase. Rearranging (3) yields an expression for expected price

$$p(r|\mathbf{t}) = \frac{\sum_{i=1}^{n} \Pi^i(r|\mathbf{t})}{\Pr\left\{z_1 \leqslant r\right\}} + E\left[z_1 | z_1 \leqslant r\right]. \tag{4}$$

An expression for the profit of the buyer $U(r|\mathbf{t})$ can be written as the total gains from trade minus the expected profit to the suppliers. That is,

$$U(r|\mathbf{t}) = \left(1 - [1 - F(r)]^{\hat{t}}\right)\left(v - E\left[z_1 | z_1 \leqslant r\right]\right) - \sum_{i=1}^{n} \Pi^i(r|\mathbf{t}) \tag{5}$$

The total gains are the probability of purchase times the difference between the value of the input to the buyer and the expected minimum cost draw conditional on it being less than the reserve price.

If the reserve price is fixed, then the capacity profile enters $p(r|\mathbf{t})$ and $U(r|\mathbf{t})$ through its effect on the total expected profit of the suppliers. We model mergers as changes in how capacity is distributed across suppliers. The effect of a merger on the expected price and the expected profit of the buyer will follow directly from its effect on the suppliers' total expected profits. If the reserve price is optimally set by the buyer, then the effects of a merger on the expected price and profits of the buyer are more complicated. Section IV discusses the effect of concentration and mergers on the buyer. Our analysis is based on the definitions of concentration defined in the following section.

3. Changes in the Concentration of Market Capacity

In this section, we define a concentration ordering which allows us to compare industry performance for different capacity profiles. This ordering facilitates our examination of the welfare effects of mergers and the incentive for mergers.[13] Consider two capacity profiles $\mathbf{t}$ and $\mathbf{s}$. The profiles $\mathbf{t}$ and $\mathbf{s}$ are equivalent if $\mathbf{s}$ is simply a rearrangement of the elements of $\mathbf{t}$. For parts of the analysis, it is useful to order the capacities in a profile by size. Thus, we define the index notation $t_{(i)}$ such that $t_{(1)} \geqslant \cdots \geqslant t_{(n)}$. When the suppliers $\{1, \ldots, n\}$ follow their equilibrium strategies, the profiles $(t_1, \ldots, t_n)$ and $(t_1, \ldots, t_n, 0)$ will result in the same outcomes. This follows from the fact that a supplier with zero capacity has no chance of being the lowest cost supplier and winning the auction. Hence, it is always possible to add

[13] Encaoua and Jacquemin (1980) discuss the application of concentration measures to entry and mergers.

suppliers with zero capacity to a market without affecting the equilibrium outcomes. This is convenient because our definitions of concentration will require that the profiles being compared have the same number of suppliers.

An equalizing transfer between two suppliers is a transfer of capacity from one supplier to another such that the absolute difference in capacities between the two suppliers is reduced. Let $\mathbf{t}$ be the pre-transfer capacity profile and $\mathbf{s}$ be the post-transfer capacity profile. More formally, an *equalizing transfer* of capacity from $\mathbf{t}$ to $\mathbf{s}$ for firms $j, k \in N$ requires that (i) $|\ t_j - t_k| > |s_j - s_k|$, (ii) $t_j + t_k = s_j + s_k$, and (iii) $t_i = s_i$, for all $i \in N \backslash \{j, k\}$.[14] In applying this definition, it is important to note that any profile can be arbitrarily reindexed without materially changing its properties.

Definition 1. Capacity profile $\mathbf{t}$ *is more concentrated than* $\mathbf{s}$ *by the transfer principle* ($\mathbf{t} \succ_T \mathbf{s}$) if and only if $\mathbf{s}$ can be constructed from $\mathbf{t}$ by applying a finite series of equalizing transfers.

It is not difficult to see that if $t_1 = \cdots = t_m$, then there is no profile $\mathbf{s}$ such that $\mathbf{t} \succ_{\mathrm{T}} \mathbf{s}$. Thus, the symmetric capacity profile is the least concentrated profile for any given total industry capacity. The concentration ordering implied by the transfer principle is irreflexive, transitive, and incomplete. Irreflexivity follows from the fact that an equalizing transfer is defined with a strict inequality. Transitivity follows from the fact that the order is defined in terms of a series of equalizing transfers. The transfer principle is incomplete because not all profiles can be ranked. This fact is illustrated in Example 4 below.

While our results on concentration will make use of the transfer principle, at times it is useful to employ the following equivalent definition.

Definition 2. Capacity profile $\mathbf{t}$ *is more concentrated than* $\mathbf{s}$ *by second-order dominance* ($\mathbf{t} \succ_{SD} \mathbf{s}$) if and only if for all $m = 1, \ldots, n$, $\sum_{i=m}^{n} t_{(i)} \leqslant \sum_{i=m}^{n} s_{(i)}$ with the inequality strict for at least one m.

In words, $\mathbf{t}$ is more concentrated than $\mathbf{s}$ by second-order dominance if for all $m = 1, \ldots, n$, the sum of the capacities of the $(n-m)$th smallest suppliers is not greater for $\mathbf{t}$ than for $\mathbf{s}$.[15] In order to apply this ordering to capacity profiles with a positive capacity for a different total number of suppliers, suppliers with zero capacity can be added to the shortest profile

[14] The Pigou-Dalton condition holds that inequality should increase when income is transferred from a poorer individual to a richer individual. While stated differently, the Pigou-Dalton transfer condition implies a concentration ordering that is equivalent to the transfer principle defined here.

[15] Shorrocks and Foster (1987) and Foster and Sen (1997) describe a number of concentration or inequality concepts such as generalized Lorenz dominance that are equivalent to second-order dominance (also referred to as second-order stochastic dominance).

so that the total number of suppliers in the two profiles is equal. Note that second-order dominance can order two profiles even if the total capacity of each profile is different. If $\hat{t} > \hat{s}$, then the capacity profile $\mathbf{t}$ cannot be more concentrated than $\mathbf{s}$ by second-order dominance. However, $\mathbf{s}$ can be more concentrated than $\mathbf{t}$ by second-order dominance.

In practice, it is common to calculate and compare the shares of the largest firms.[16] Second-order dominance allows this alternative approach when the profiles being compared have the same total capacity. If $\hat{t} = \hat{s}$, then it is clear that for all $m = 2, \ldots n$, $\sum_{i=m}^{n} t_{(i)} \leqslant \sum_{i=m}^{n} s_{(i)}$ if and only if $\sum_{i=1}^{m-1} t_{(i)} \geqslant \sum_{i=1}^{m-1} s_{(i)}$. Therefore, when $\hat{t} = \hat{s}$, $\mathbf{t} \succ_{\mathrm{SD}} \mathbf{s}$ is equivalent to the criteria that, for all m, the sum of the capacities of the m largest firms for $\mathbf{t}$ is not less than for $\mathbf{s}$.

Finally, we define the Herfindahl-Hirschman Index (HHI), which the Justice Department and the Federal Trade Commission use in the Merger Guidelines.[17]

DEFINITION 3. Capacity profile $\mathbf{t}$ *is more concentrated than* $\mathbf{s}$ *by the Herfindahl-Hirschman Index* ($\mathbf{t} \succ_H \mathbf{s}$) if and only if $\sum_{i=1}^{n} t_i^2 > \sum_{i=1}^{n} s_i^2$ and $\hat{t} = \hat{s}$.

Note that a merger clearly results in a more concentrated industry under all three definitions. The following result relates the three definitions of concentration for arbitrary capacity profiles.

Proposition 1. *For capacity profiles* $\mathbf{t}$ *and* $\mathbf{s}$ *such that* $\hat{t} = \hat{s}$,

(a) $\mathbf{t} \succ_{\mathrm{T}} \mathbf{s}$ *if and only if* $\mathbf{t} \succ_{\mathrm{SD}} \mathbf{s}$

(b) *If* $\mathbf{t} \succ_{\mathrm{T}} \mathbf{s}$, *then* $\mathbf{t} \succ_{\mathrm{H}} \mathbf{s}$.

These results are known from the literatures on mergers and income inequality.[18] Part (a) of Proposition 1 is an equivalence result for second-order dominance and the transfer principle when comparing market structures with equal total capacities. Part (b) of Proposition 1 states that whenever the transfer principle provides a ranking of capacity profiles by concentration, the HHI will provide the same ranking. However, the reverse implication is not true. A capacity profile which is more concentrated by the HHI need not be more concentrated by the transfer principle. Consider the following example.

Example 1. Suppose that $\mathbf{t} = (0.77, 0.18, 0.05)$ and $\mathbf{s} = (0.75, 0.25, 0.0)$. Since $\sum_{i=1}^{n} t_i^2 = 0.6278$ and $\sum_{i=1}^{n} s_i^2 = 0.625$, $\mathbf{t} \succ_H \mathbf{s}$. However, it is not true that $\mathbf{t} \succ_T \mathbf{s}$. By Proposition 1,

[16] It is typically easier to identify the large firms in an industry and estimate their capacity or market share.

[17] See U.S. Department of Justice/Federal Trade Commission, Horizontal Merger Guidelines, April 2, 1992 (Revised: April 8, 1997).

[18] See for example Encaoua and Jacquemin (1980) and Shorrocks and Foster (1987).

$\mathbf{t} \succ_T \mathbf{s}$ only if for each $m = \{1, 2, 3\}$, $\sum_{i=m}^n t_i \leqslant \sum_{i=m}^n s_i$. However, for $m = 3, \sum_{i=m}^n t_i = 0.05 > 0 = \sum_{i=m}^n s_i$.

Although second-order dominance and the transfer principle define the same ordering when the total industry capacities are equal, each generates a different computational method for comparing two capacity profiles. With the transfer principle, one would look for an appropriate set of transfers of capacity. On the other hand, with second-order dominance, one would compare the sums of capacity for subsets of the suppliers from the supplier with the smallest capacity on up or the largest capacity on down. Depending on the two capacity profiles being compared, one method or the other may be easier to apply.

It is important to note that neither the transfer principle nor second-order dominance can compare every pair of capacity profiles even when the total industry capacity is fixed. However, we show in the following section that when two profiles can be ordered, that ordering unambiguously predicts how the buyer's expected profit and other variables of interest will compare for the two profiles.

4. The Effects of Concentration and Mergers on Buyers

In this section, we examine the effect of industry concentration on the expected price and the welfare of buyers. The following lemma states that the aggregate expected profits of the suppliers is higher for capacity profiles that are more concentrated by the transfer principle. This lemma makes the proof for many of the subsequent results follow quite easily.

Lemma 2. *If $\mathbf{t} \succ_T \mathbf{s}$, then for any $r > \underline{c}$, $\sum_{i=1}^n \Pi^i(r|\mathbf{t}) > \sum_{i=1}^n \Pi^i(r|\mathbf{s})$.*

Proof in the Appendix.

The next proposition is the key result. It states that increases in concentration by the transfer principle decrease the expected profit of the buyer and increase the expected price when the reserve price is held constant. It also states that an increase in concentration lowers the optimal reserve price.

Proposition 2. *Suppose $\mathbf{t} \succ_T \mathbf{s}$. Then for any $r > \underline{c}$, (a) $U(r|\mathbf{t}) < U(r|\mathbf{s})$, (b) $p(r|\mathbf{t}) > p(r|\mathbf{s})$, and (c) $U(r|\mathbf{t}) - U(r|\mathbf{s})$ is decreasing in r. Further suppose that $r^*_{\mathbf{t}}$ and $r^*_{\mathbf{s}}$ are the profit-maximizing reserve prices for the buyer under capacity profiles $\mathbf{t}$ and $\mathbf{s}$. Then (d) $U(r^*_{\mathbf{t}}|\mathbf{t}) < U(r^*_{\mathbf{s}}|\mathbf{s})$ and (e) $r^*_{\mathbf{t}} \leqslant r^*_{\mathbf{s}}$.*

Proof. (a) Making use of (5), for any given $r > \underline{c}$, we have

$$U(r|\mathbf{t}) - U(r|\mathbf{s}) = \sum_{i=1}^n \Pi^i(r|\mathbf{s}) - \sum_{i=1}^n \Pi^i(r|\mathbf{t}) < 0.$$

The inequality follows from Lemma 2.

(b) Similarly, making use of (4), for any given $r > \underline{c}$, we have

$$p(r|\mathbf{t}) - p(r|\mathbf{s}) = \frac{\sum_{i=1}^{n} \Pi^i(r|\mathbf{t}) - \sum_{i=1}^{n} \Pi^i(r|\mathbf{s})}{\Pr\{z_1 \leqslant r\}} > 0.$$

(c) Note that,

$$\begin{aligned}
\frac{\partial\left[U(r|\mathbf{t}) - U(r|\mathbf{s})\right]}{\partial r} &= \frac{\partial\left[\sum_{i=1}^{n} \Pi^i(r|\mathbf{s}) - \sum_{i=1}^{n} \Pi^i(r|\mathbf{t})\right]}{\partial r} \\
&= -\int_0^{\alpha} \frac{\partial^2 \Delta(r|\xi)}{\partial r \partial \xi} d\xi \\
&= -\int_0^{\alpha} \left\{[1-F(r)]^{\hat{t}-t_k-\xi} - [1-F(r)]^{\hat{t}-t_j+\xi}\right\} \ln[1-F(r)] d\xi < 0.
\end{aligned}$$

(d) When the reserve price is adjusted to maximize the expected profit of the buyer, $U(r_{\mathbf{s}}^*|\mathbf{s}) \geqslant U(r_{\mathbf{t}}^*|\mathbf{s}) > U(r_{\mathbf{t}}^*|\mathbf{t})$. The first inequality follows from the fact that $r_{\mathbf{s}}^*$ is the profit-maximizing reserve price when the buyer is faced with capacity profile $\mathbf{s}$. The second inequality follows from part (a) of the proposition.

(e) The result follows from part (c) of this proposition. *Q.E.D.*

Proposition 2 is a general result about the effect of concentration on the expected price and the expected profits of the buyer. An immediate implication is that for a given number of suppliers, a given total industry capacity, and a given reserve price, the expected price paid by the buyer is minimized when all of the firms have the same capacity.[19]

In the remainder of this section, we examine various merger scenarios and consider their impact on concentration, the expected price, the expected profits of the buyer, and the optimal reserve price.

Proposition 3. *A merger of two or more suppliers results in a more concentrated market by the transfer principle. Thus, when the reserve price is fixed, a merger results in a higher expected price and a lower expected profit to the buyer. When the reserve price is set to maximize the expected profit of the buyer, a merger results in a lower reserve price*[20] *and a lower expected profit to the buyer.*

Proof. Suppose that suppliers j and k merge. For an initial profile $\mathbf{s}$, the resulting capacity profile is can be defined as $t_j = s_j + s_k$, $t_k = 0$, and $t_i = s_i$, for all $i \in N \backslash \{j, k\}$.

[19] This result is also true in the asymmetric Nash-Cournot model. See Farrell and Shapiro (1990a) and Werden (1991).

[20] Graham and Marshall (1987) and Mailath and Zemsky (1991) prove a similar result relating the optimal reserve price. They consider efficient collusion among bidders of different sizes. Both show that as the number of colluding bidders rises the bid-taker sets a more aggressive reserve price.

It is straightforward to see that $\mathbf{t} \succ_{\mathrm{T}} \mathbf{s}$. The proof is completed by applying Proposition 2. *Q.E.D.*

With a fixed reserve price, buyers will obtain a lower expected profit because of the higher expected price of the input even though there is no change in the probability of purchase. The higher expected price arises because for cases in which the two subsidiaries of the merged supplier have the two lowest costs among all the suppliers, the price then becomes the third highest cost. By adjusting the reserve price optimally, the buyer may be able to reduce the adverse effects of the merger, but the envelope theorem implies that the merger reduces the expected profit of the buyer after the buyer adjusts the reserve price optimally.[21]

When the buyer is allowed to set the reserve price optimally, the buyer possesses some bargaining power as a monopsonist. A decrease in the reserve price is analogous to a monopsonist reducing his consumption to lower the price. When the reserve price is set optimally by the buyer, the effect of a merger on the expected price is ambiguous. Higher levels of concentration have a positive effect on the expected price, while the lower reserve price has a negative effect on the expected price. The following two examples demonstrate that when the reserve price is set optimally, expected price can rise or fall following a merger.

Example 2. Suppose that $\mathbf{s} = (\frac{1}{2}, \frac{1}{2})$ and F is the uniform distribution over the unit interval. Now consider the merger of suppliers 1 and 2. Thus, $\mathbf{t} = (1, 0)$. When $v = 1$, the optimal reserve prices are $r_s^* = 5/9$ and $r_t^* = 1/2$, and the expected prices are $p(r_s^*|\mathbf{s}) = 47/90$ and $p(r_t^*|\mathbf{t}) = 1/2$. Thus, expected price falls after the merger.

Example 3. Assume the same setup as in Example 2 except that $v = 1.8$. The optimal reserve prices are $r_s^* = 1$ and $r_t^* = 9/10$. Under capacity profile $\mathbf{s}$, it is optimal to set the reserve price greater than or equal to the upper bound on the support of the cost distributions. The expected prices are $p(r_s^*|\mathbf{s}) = 5/6$ and $p(r_t^*|\mathbf{t}) = 9/10$. Thus, expected price rises after the merger.

The fact that the expected price can fall after the merger should not be interpreted as an increase in the buyer's welfare. Proposition 3 demonstrates that the expected profits of the buyer decline after a merger even though the buyer optimally lowers the reserve price. The buyer is using the reserve price to moderate his welfare reduction after the merger. Thus, in auction markets with a reserve price, one cannot simply examine the price effect in order to assess the welfare effects of mergers on buyers.[22]

[21] Thomas (1998) finds some support for the conjecture that allowing two small firms to merge may benefit the buyer since they will be better able to compete with the larger firm. Since Proposition 2 does not distinguish between small and large firms, this model clearly does not support such a result.

[22] In the asymmetric Nash-Cournot model, mergers unambiguously increase the equilibrium price and reduce consumer welfare.

We can also use the transfer principle definition of market concentration to compare the effects of different mergers. The following proposition allows us to compare a variety of mergers between differing suppliers in the industry.

Proposition 4. *For any initial capacity profile* $\mathbf{t}$, *if* $t_j > t_k$ *and* $t_g \geqslant t_h$, *then a merger between suppliers* j *and* g *results in a more concentrated industry by the transfer principle than a merger between suppliers* k *and* h. *Furthermore, a merger between firms* j *and* g *results in a higher expected price (when the reserve price is fixed), a lower expected profit to the buyer, and a lower optimal reserve price than a merger between suppliers* k *and* h.

Proof. We first show that for any $j, k, g \in N$ such that $t_j > t_k$, a merger between suppliers j and g results in a more concentrated market than a merger between suppliers k and g. Let $\mathbf{s}^1$ denote the capacity profile after the merger of j and g, and let $\mathbf{s}^2$ denote the capacity profile after the merger of k and g. Specifically, $s_g^1 = s_g^2 = 0$, $s_j^1 = t_g + t_j$, $s_k^2 = t_g + t_k$, $s_k^1 = t_k$, $s_j^2 = t_j$, and for all $i \in N \backslash \{j, k\}$, $s_i^1 = s_i^2 = t_i$. Note that $\sum_{i=1}^n s_i^1 = \sum_{i=1}^n s_i^2$. Therefore, $\mathbf{s}^1 \succ_{\mathrm{T}} \mathbf{s}^2$ if $\left|s_j^1 - s_k^1\right| > \left|s_j^2 - s_k^2\right|$. This inequality is equivalent to $|t_g + t_j - t_k| > |t_g + t_k - t_j|$ which follows from $t_j > t_k$.

Now consider the capacity profiles that result from the mergers of suppliers j and g and suppliers k and h. For the case where $t_g = t_h$, the proof is complete. Now suppose $t_g > t_h$. With an initial profile $\mathbf{t}$, let $\mathbf{s}^3$ denote the capacity profile after the merger of firms k and h. Note that $\mathbf{s}^2 \succ_{\mathrm{T}} \mathbf{s}^3$ follows from the result established in the first part of the proof. Therefore, by transitivity $\mathbf{s}^1 \succ_{\mathrm{T}} \mathbf{s}^3$. The proof is completed by applying Proposition 2. *Q.E.D.*

Proposition 4 provides one straightforward conclusion for industries in which some suppliers are interested in acquiring other suppliers. First, if a supplier is shopping for another supplier to acquire, the acquisition of a larger supplier will result in a more concentrated industry than the acquisition of a smaller supplier. Conversely, if a supplier is for sale, the acquisition of this supplier will result in a less concentrated industry if it is acquired by a smaller rather than larger supplier.

In order to illustrate the implications of Proposition 4 consider four suppliers from a capacity profile $\mathbf{t}$ such that $t_1 > t_2 > t_3 > t_4$. There are three possible merger comparisons between two distinct pairs of suppliers: (a) a merger of suppliers 1 and 2 versus a merger of suppliers 3 and 4, (b) a merger of suppliers 1 and 3 versus a merger of suppliers 2 and 4, and (c) a merger of suppliers 1 and 4 versus a merger of suppliers 2 and 3. According to Proposition 4, we easily can rank the mergers in (a) and (b). That is, a merger of suppliers 1 and 2 results in a more concentrated market than a merger between suppliers 3 and 4, and a merger of suppliers 1 and 3 results in a more concentrated market than a merger between suppliers 2 and 4.

The comparison of the merger between suppliers 1 and 4 versus the merger between suppliers 2 and 3 cannot be deduced from Proposition 4. However, with the additional condition that $t_2+t_3 \geqslant t_1+t_4$, we can conclude that a merger of suppliers 2 and 3 results in a more concentrated market than a merger of suppliers 1 and 4. These two mergers require a comparison of the following two capacity profiles (t_2+t_3, t_1, t_4) and (t_1+t_4, t_2, t_3). The former is more concentrated than the latter under second-order dominance because $t_3 > t_4$ and $t_2+t_3 \geqslant t_1+t_4$.

If, on the other hand, $t_1+t_4 > t_2+t_3$, then neither merger results in a market that is more concentrated than the other and either merger may result in a higher price than the other. Consider the following example.

Example 4. Suppose that $\mathbf{t} = (9, 5, 3, 1)$. Now compare the merger of suppliers 1 and 4 with the merger of suppliers 2 and 3. Note that $t_1+t_4 > t_2+t_3$. Hence, $\mathbf{s}^1 = (10, 5, 3, 0)$ and $\mathbf{s}^2 = (9, 8, 1, 0)$. Neither $\mathbf{s}^1 \succ_T \mathbf{s}^2$ nor $\mathbf{s}^2 \succ_T \mathbf{s}^1$. This ambiguity follows from an application of second-order dominance. To see this, note that $s_1^1 > s_1^2$ while $s_1^1 + s_2^1 < s_1^2 + s_2^2$.

Even though the merger of suppliers 1 and 4 creates a larger firm than the merger suppliers 2 and 3, it need not result in a lower expected profit for the buyer.[23]

5. The Incentive to Merge and Total Welfare

The incentive to merge and the effects of a merger on total welfare depend on how the reserve price is affected. If the buyer sets the reserve price optimally given the capacity profile for the industry, then a merger decision will presumably take into account this reaction by the buyer. When the reserve price is fixed, the incentive to merge is always present. However, when the buyer sets the reserve price optimally, the negative effect on supplier profits from the lower reserve price implies that it is possible for suppliers to have no incentive to merge.[24]

Two suppliers have an incentive to merge if their combined profits after the merger are higher than the sum of their individual profits before the merger. For the fixed reserve price case, Lemma 2 implies that the total profits of all suppliers increases after a merger. Inspection of (2) indicates that non-merging suppliers are unaffected by a merger because

[23] For cases where the transfer principle fails to order capacity profiles, it may be tempting to use the HHI. However, in such cases, the HHI cannot be trusted to predict correctly which profile is associated with lower expected profits for the buyer. It is straightforward to construct examples where the HHI indicates higher concentration but the buyer's expected profit increases. Using the profiles in Example 1, $\mathbf{t} \succ_H \mathbf{s}$, even through a straightforward calculation shows that $U(1|\mathbf{t}) > U(1|\mathbf{s})$, when $v = 1$ and F is the uniform distribution on $[0, 1]$.

[24] Our presumption is that when the buyer is able to set the reserve price optimally, he sets the reserve price to whatever level is optimal for the capacity profile that he faces. However, suppose that the buyer announces to the market that any merger will be met with an adjustment of the reserve price to $\underline{c}$. If the suppliers believe that such a reaction would occur, no merger would take place because any merger would result in zero profits for all suppliers.

neither their capacity nor total industry capacity is affected. Therefore, when the reserve price is fixed, the expected profits of merging suppliers must increase after the merger.

Total welfare $W(r|\mathbf{t})$ can be written as

$$W(r|\mathbf{t}) = \left(1 - [1 - F(r)]^{\hat{t}}\right)(v - E[z_1|z_1 \leqslant r]).$$

Total welfare does not depend directly on the allocation of market capacity among firms, but only on the total market capacity $\hat{t}$ and the reserve price r. This is a direct implication of our assumption of constant economies of scale. Therefore, when the reserve price is constant, a merger has no effect on total welfare. Hence, for the case of a constant reserve price, the gains from a merger are simply a transfer of expected profits from the buyer to the merging suppliers. The non-merging suppliers are unaffected.

For $r < \min\{\bar{c}, v\}$, total welfare is increasing in the reserve price. Therefore, for the case where the buyer can set the reserve price optimally, any increase in concentration that reduces the reserve price, also reduces total welfare. The preceding discussion is summarized by the following proposition.

Proposition 5. *If the reserve price is unaffected by a merger, then there is always an incentive to merge, but total welfare and the profits of the non-merging suppliers are unaffected. However, if the buyer sets the reserve price optimally given the capacity profile and if the optimal reserve price is initially below $\bar{c}$ or falls below $\bar{c}$ after the merger, then mergers reduce total welfare and the profits of the non-merging suppliers.*

The implication of Proposition 5 for the non-merging suppliers is interesting. If the reserve price is constant, the non-merging suppliers are unaffected by the merger. However, if the buyer sets the reserve price optimally, then the buyer reacts to the merger by lowering the reserve price making the non-merging suppliers worse off. They have the same expected market share but lower expected profits because the probability of purchase by the buyer is lower. The buyer lowers the reserve price to mitigate the increased concentration and this harms the non-merging suppliers.[25]

When the buyer optimally sets the reserve price below $\bar{c}$, there may or may not be an incentive for suppliers to merge. Consider the following two examples. In Example 5, there is an incentive to merge, while in Example 6 there is no incentive to merge.

[25] This result is different from the asymmetric Nash-Cournot model. In that model, non-merging suppliers obtain higher profits by expanding production in response to the higher market price after the merger. This insight has been used in Antitrust cases to argue that non-merging suppliers cannot successfully challenge a merger because they would have no damages. This auction model provides an explanation of why non-merging suppliers, in addition to buyers, would be harmed by a merger.

Example 5. Suppose that $\mathbf{s} = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$ and F is the uniform distribution over the unit interval. Now consider the merger of suppliers 2 and 3. Thus, $\mathbf{t} = (\frac{1}{2}, \frac{1}{2}, 0)$. When $v = 1$, the optimal reserve prices are $r_s^* = (25 - 3\sqrt{5})/32$ and $r_t^* = 5/9$. The expected profits for the merging suppliers, pre- and post-merger are $\Pi^2(r_s^*|\mathbf{s}) + \Pi^3(r_s^*|\mathbf{s}) \approx 0.0671$ and $\Pi^2(r_t^*|\mathbf{t}) = 11/162 \approx 0.0679$. Thus, suppliers 2 and 3 have an incentive to merge.

Example 6. Using the same scenarios as in Example 2, suppose that $\mathbf{s} = (\frac{1}{2}, \frac{1}{2})$ and F is the uniform distribution over the unit interval. Now consider the merger of suppliers 1 and 2. Thus, $\mathbf{t} = (1, 0)$. When $v = 1$, the optimal reserve prices are $r_s^* = 5/9$ and $r_t^* = 1/2$, and the expected profits of merging suppliers are $\Pi^1(r_s^*|\mathbf{s}) + \Pi^2(r_s^*|\mathbf{s}) = 11/81$ and $\Pi^1(r_t^*|\mathbf{t}) = 1/8$. Thus, suppliers 1 and 2 have no incentive to merge.

These two examples provide some intuition about the incentive to merge. In Example 6, the negative effects of the lower reserve price fall completely on the merged supplier since it is the only supplier in the market after the merger. On the other hand, in Example 5, the negative effect of the lower reserve price on the suppliers in the market is shared by the merged supplier and the non-merging supplier.

The surprising result of Example 6 is that merger to monopoly is not profitable for a symmetric duopoly.[26] In this auction model the buyer's optimal reserve price facing a monopoly completely undermines the incentive of the duopolists to merge. Although Examples 5 and 6 are calculated using the uniform distribution for F, they suggest that the optimal reserve price of the buyer can undermine a merger wave.[27] As such, the buyer's ability to commit to a reserve price r cannot only mitigate his loss in expected profits for a given industry structure, it can also maintain a "stable" industry structure in which there would be no incentive for mergers that would further concentrate the industry.

However, it is not always the case that merger to monopoly is unprofitable. Consider the following example.

Example 7. Maintain the same assumptions as in Example 6 except let $v = 1.8$. That is, we consider the same scenario as in Example 3. The optimal reserve prices are $r_s^* = 1$

[26] Using a models that are mathematically Thomas (1998) and McAfee (1994) also find that in some cases there is no incentive for merger to monopoly. These results from the auction models are clearly at variance with the symmetric or asymmetric Nash-Cournot models. The symmetric model of Salant, Switzer, and Reynolds (1983) finds an incentive for symmetric duopolists to merge even though there is no incentive for oligopolists to merge. In Perry and Porter (1985), larger firms are more likely to have an incentive to merge, but asymmetric duopolists would always have an incentive to merge. All the gains from a merger to monopoly by duopolists in these models would be internalized.

[27] Perry and Porter (1985) find cases such that if a merger between two firms with the same capacity is profitable, all subsequent mergers between firms of the same size will also be profitable. Examples 4 and 5 illustrate that the opposite conclusion may arise in this auction model.

and $r_t^* = 9/10$, and the expected profits of merging suppliers are $\Pi^1(r_s^*|\mathbf{s}) + \Pi^2(r_s^*|\mathbf{s}) = 1/3$ and $\Pi^1(r_t^*|\mathbf{t}) = 81/200$. Thus, suppliers 1 and 2 have an incentive to merge.

With a higher value for the input, the reserve price adjustment is not large enough to eliminate the incentive to merge. In Example 6, we showed that under certain assumptions the industry profile of $(\frac{1}{2}, \frac{1}{2})$ is stable. In the following example we consider what other industry profiles are stable.

Example 8. As in Examples 5 and 6, let F be the uniform distribution over the unit interval and $v = 1$. Starting from a symmetric industry structure, we search for the industry structures that can result after all profitable merger opportunities have been exhausted.[28] The following table gives the stable profiles that can be reached from an initial symmetric industry with n firms each with a capacity of $1/n$, considering all possible merger paths.

n	*Stable Profiles*
3	$(1/3, 1/3, 1/3)$
4	$(1/2, 1/2)$
5	$(2/5, 2/5, 1/5)$
6	$(1/2, 1/2)$ *and* $(1/3, 1/3, 1/3)$
7	$(4/7, 3/7)$
8	$(1/2, 1/2)$ *and* $(3/8, 3/8, 1/4)$
9	$(5/9, 4/9)$ *and* $(1/3, 1/3, 1/3)$
10	$(1/2, 1/2)$, $(2/5, 2/5, 1/5)$, *and* $(2/5, 3/10, 3/10)$

The numerical calculations reported in Example 8 provide some interesting insights. First, small suppliers have an incentive to merge but large firms may not. Second, different stable capacity profiles can arise from the same initial profile through a different sequence of mergers. Third, in the particular example considered, the buyer's optimal reserve price eliminates the incentive for suppliers to merge once they approach half the capacity of the industry. As v becomes larger than $\overline{c}$, the buyer is more reluctant to set a reserve price below $\overline{c}$ and reduce the probability of purchase. If v is sufficiently large, the buyer's ability to set the reserve price optimally does not even prevent merger to monopoly.

6. Conclusion

Using this auction model, we have found both similarities and differences from the traditional asymmetric Nash-Cournot model of mergers. Consider the simplest case in which the

[28] *An effective but inefficient program written in Mathematica checks all possible merger paths reporting the resulting stable profile when all profitable mergers have been exhausted along a particular path. Our program is available upon request.*

reserve price is constant. As in the traditional model, we find that the welfare of buyers declines because mergers increase the expected price paid by the buyers. However, unlike the traditional model, total welfare is unchanged because there is no inefficiency in production and no change in the probability of purchase. The lowest cost supplier wins the contract before and after the merger, but the merger results in a higher price to the buyer when the merging suppliers are the two lowest cost suppliers. As a result, there is always an incentive for any merger. The non-merging suppliers are unaffected by the merger because they receive the same expected price when they have the lowest cost.

The differences from the traditional asymmetric Nash-Cournot model are more accentuated when the buyer can optimally commit to a reserve price. As in the traditional model, we find that the welfare of buyers declines with a merger. In addition, total welfare declines because the buyer reduces the reserve price after the merger, thereby decreasing the probability of purchase. However, unlike the traditional model, the expected price paid by the buyers need not increase. The buyer's optimal reserve price may offset the price effect of the merging suppliers. The non-merging suppliers are harmed by a merger. Even though the non-merging suppliers have the same market share as before the merger, the probability of purchase is now lower because the reserve price has fallen. In addition, the lower reserve price may reduce the price received when a non-merging supplier wins the contract.

This auction model need not always result in an incentive to merge when the buyer optimally reduces the reserve price. In the traditional model, an incentive to merge need not arise because the non-merging suppliers expand production in response to the higher price. Thus, an incentive to merge is most likely to exist for two large suppliers rather than two small suppliers. In this auction model, the buyer dulls the incentive to merge by reducing the reserve price. The suppliers with larger capacities may have no incentive to merge because they bear most of the burden of the reduction in the reserve price. On the other hand, suppliers with smaller capacities typically have an incentive to merge even when larger firms do not. The examples suggest that the optimal use of the reserve price can create a stable oligopolistic industry structure for which no further mergers are profitable. Unfortunately these stable industry structures may be relatively concentrated.

Appendix

Proof of Theorem 1. (If): It is easy to verify that the distribution function as defined in the theorem satisfies P1, P2, and P3.

(Only if): For arbitrary $t_1, t_2 \geqslant 0$, define the profiles $\mathbf{t} = (t_1, t_2)$ and $\mathbf{s} = (t_1 + t_2)$. Note $\mathbf{s}$ contains only one supplier with a capacity of $t_1 + t_2$. Making use of these profiles in equality (1), Properties 1, 2, and 3 imply that for all $t_1, t_2 \geqslant 0$ and all $c \in [\underline{c}, \overline{c}]$,

$$G(c|t_1 + t_2) = 1 - [1 - G(c|t_1)][1 - G(c|t_2)] \tag{6}$$

First we prove that for any $c \in [\underline{c}, \overline{c}]$, $G(c|\cdot)$ is nondecreasing in its second argument. For $t_i > t_j \geqslant 0$,

$$\begin{aligned} G(c|t_i) - G(c|t_j) &= 1 - [1 - G(c|t_i - t_j)][1 - G(c|t_j)] - G(c|t_j) \\ &= G(c|t_i - t_j)[1 - G(c|t_j)] \geqslant 0. \end{aligned}$$

The first equality in the expression above follows from (6) since $G(c|t_i) = 1 - [1 - G(c|t_i - t_j)][1 - G(c|t_j)]$.

In order to show that $G(c|t_i)$ is differentiable in its second argument for all $t_i > 0$, choose an arbitrary $(c, t_i) \in [\underline{c}, \overline{c}] \times \mathbb{R}_{++}$. There exists a $t_k > t_i$ such that $G(c|\cdot)$ is differentiable in its second argument at t_k. Such a t_k exists since being monotonic, $G(c|\cdot)$ is differentiable almost everywhere. Define $t_j > 0$ such that $t_k = t_i + t_j$. By choice of t_k, $G(c|\cdot)$ is differentiable in its second argument at t_k, and thus, $G(c|t_i + t_j)$ is differentiable in t_i. Since equality (6) with t_i and t_j replacing t_1 and t_2 holds for all $t_i, t_j \geqslant 0$, the fact that the left-hand side is differentiable in t_i implies that the right-hand side and, thus, $G(c|t_i)$ is differentiable in t_i.

Replacing t_1 with t_i and t_2 with t_j and differentiating (6), noting that $\partial G(c|t_i + t_j)/\partial t_i = \partial G(c|t_i + t_j)/\partial t_j$, we have, for any $c \in [\underline{c}, \overline{c})$,

$$\frac{\partial G(c|t_i)/\partial t_i}{[1 - G(c|t_i)]} = \frac{\partial G(c|t_j)/\partial t_j}{[1 - G(c|t_j)]}.$$

By Property 1, the left-hand side of the equality does not depend on t_j. Therefore, since the equality holds for all $t_i, t_j \geqslant 0$, the ratio $[\partial G(c|t_i)/\partial t_i] \,/\, [1 - G(c|t_i)]$ is constant with respect to t_i. Therefore, $G(c|\cdot)$ must satisfy a differential equation of the form $\partial G(c|t_i)/\partial t_i = [1 - G(c|t_i)]\, k(c)$ where $k(c)$ can depend on c but not on t_i. This is a first-order linear differential equation which has a unique solution. In fact, the solution can be written as

$$G(c|t_i) = 1 - \sigma(c)\,[1 - F(c)]^{t_i}$$

where $F(c) = 1 - e^{-k(c)}$ and $\sigma(c)$ is a constant term with respect to t_i.

The fact that $G(c|0) = 0$ for all $c \in [\underline{c}, \overline{c})$ follows immediately from expression (6). Therefore, $\sigma(c) = 1$. Since G is a distribution function with a support of $[\underline{c}, \overline{c}]$, F must also be a distribution function with a support of $[\underline{c}, \overline{c}]$. *Q.E.D.*

Proof of Lemma 1. (a) a straightforward calculation of the probability yield the result.

(b) The probability that all of supplier i's rivals have costs higher than c_i is $\prod_{j\neq i}[1 - F(c_i)]^{t_j} = [1 - F(c_i)]^{\hat{t}-t_i}$. Since each supplier submits a bid equal to its cost and the lowest bidder wins, supplier i's *ex ante* probability of winning the auction is equal to the probability that all of the other suppliers have a higher cost and its cost is below the reserve price. Supplier i's *ex ante* probability of winning is equal to

$$\begin{aligned}\int_{\underline{c}}^{r} [1 - F(c_i)]^{\hat{t}-t_i} t_i [1 - F(c_i)]^{t_i - 1} f(c_i) dc_i &= t_i \int_{\underline{c}}^{r} [1 - F(c_i)]^{\hat{t}-1} f(c_i) dc_i \\ &= \frac{t_i}{\hat{t}} \left(1 - [1 - F(r)]^{\hat{t}}\right).\end{aligned}$$

Hence, supplier i's *ex ante* probability of winning conditional on some bidder winning is $\frac{t_i}{\hat{t}}\left(1 - [1 - F(r)]^{\hat{t}}\right)\left(\sum_{j=1}^{n} \frac{t_j}{\hat{t}}\left(1 - [1 - F(r)]^{\hat{t}}\right)\right)^{-1} = \frac{t_i}{\hat{t}}$. *Q.E.D.*

Proof of Lemma 2. Consider two capacity profiles $\mathbf{t}$ and $\mathbf{s}$ such that for some $j, k \in N$, $t_j > s_j \geqslant s_k > t_k$, $t_j + t_k = s_j + s_k$, and $t_i = s_i$, for all $i \in N\backslash\{j, k\}$. Let $\alpha = t_j - s_j$. Recall that capacity profiles can be reindexed without materially affecting the market outcomes. Notice that $\mathbf{t} \succ_{\mathrm{T}} \mathbf{s}$ because $\mathbf{s}$ can be constructed from $\mathbf{t}$ by taking $\alpha > 0$ capacity from supplier j and giving it to supplier k. It is sufficient to prove the lemma for $\mathbf{t}$ and $\mathbf{s}$, because any two profiles that can be ordered by the transfer principle differ by a finite series of transfers.

Define Δ such that

$$\begin{aligned}\Delta(r|\alpha) &= \sum_{i=1}^{n} \Pi^i(r|\mathbf{t}) - \sum_{i=1}^{n} \Pi^i(r|\mathbf{s}) \\ &= \int_{\underline{c}}^{r} \left([1 - F(c)]^{\hat{t}-t_j} - [1 - F(c)]^{\hat{t}-t_j+\alpha} + [1 - F(c)]^{\hat{t}-t_k} - [1 - F(c)]^{\hat{t}-t_k-\alpha}\right) dc.\end{aligned}$$

The second equality follows after making use of (2) and canceling terms. Note that $\Delta(r|0) = 0$. Therefore, $\Delta(r|\alpha) > 0$ and, hence, the conclusion of the lemma follows if $\partial\Delta(r|\xi)/\partial\xi > 0$, for $\xi \in (0, \alpha)$. Differentiating the expression for Δ yields

$$\frac{\partial\Delta(r|\xi)}{\partial\xi} = \int_{\underline{c}}^{r} \left\{[1 - F(c)]^{\hat{t}-t_k-\xi} - [1 - F(c)]^{\hat{t}-t_j+\xi}\right\} \ln[1 - F(c)] dc.$$

The desired inequality follows as long as for $\xi \in (0, \alpha)$, $\hat{t} - t_k - \xi > \hat{t} - t_j + \xi$ or equivalently,

$t_j - t_k - 2\xi > 0$. Notice that for $\xi \in (0, \alpha)$,

$$t_j - t_k - 2\xi > t_j - t_k - 2\alpha = s_j - s_k \geqslant 0.$$

The first equality follows from the fact that $\alpha = t_j - s_j = s_k - t_k$. *Q.E.D.*

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