Estimation and Comparison of Chained CPI-U Standard Errors With Regular CPI-U Results (2000-2001)

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In early 2002, the Bureau of Labor Statistics (BLS) began calculating and publishing the first C-CPI-U set of indexes. This new C-CPI-U (Chained Consumer Price Index – Urban), in its final form, is calculated and published every year, with roughly a one-year lag, using a Tornqvist formula. Its set of weights are updated yearly, so that a unique set of monthly weights are available for both time t as well as for time $t–n$. Thus, the new C-CPI-U index can be labeled "superlative". We briefly outline the Tornqvist formula and then the methodology for estimating a set of standard errors for these new chained indexes. We then compare, over the 24-month period of January 2000 through December 2001, these new superlative index results and their standard errors with the regularly published CPI-U results and their published standard errors.

1. The Chained "Superlative" Index

The official CPI is not a superlative index, and does not use a superlative index formula. The current official CPI claims to know yesterday's prices, even today's prices, and also claims to know yesterday's weights. The CPI is able to collect today's prices but not today's weights, at least not in the same timely way. The CPI calculates and publishes, for example, April's CPI using April's prices in mid-May, while the weights are, at a minimum, two years old. But to call an index "superlative" requires today's weights as well as today's prices. In other words, for an index, or an index formula, to be "superlative", all four ingredients – yesterday's prices, yesterday's weights, today's prices *and* today's weights – must be available. With the Final Tornqvist formula, albeit with a lag time of a year, the BLS has gathered together the four necessary ingredients and, so, has been able to produce a "superlative" index.

The Fisher Ideal formula, which is the geometric mean of a Laspeyres index and a Paasche index, also might have been used as a "superlative" index formula. But for a variety of reasons, after much research and discussion, BLS has chosen the Tornqvist formula over the Fisher Ideal formulation. (In practical terms, the choice is of little consequence, since the Tornqvist and Fisher Ideal estimates are nearly indistinguishable in nearly all simulations.)

The Tornqvist formula is simply the geometric mean of two Geomeans index formulas, one at time t and the other at time $t–n$ (here $t–1$).

(1)

$$PREL^{T}_{I,A,r[t-1;\,t]} = \prod_{i \in I, a \in A} \left(\frac{p^{t}_{i,a,r}}{p^{t-1}_{i,a,r}} \right)^{\left(\frac{s^{t}_{i,a,r} + s^{t-1}_{i,a,r}}{2} \right)}$$

which is a monthly price relative, which is then chained to produce an index,

(2)

$$IX^{T}_{I,A,r[0;\,t]} = PREL^{T}_{I,A,r[t-1;\,t]} \times IX^{T}_{I,A,r[0;\,t-1]}$$

where,

i = elementary item strata (any one of 211 such strata, like Eggs)

I = aggregate item (for example, All-Items)

a = elementary index area (any one of 38 such areas, like Houston)

A = aggregate index area (for example, All-US)

r = replicate (either full or replicate)

t = current calendar month

t–1 = calendar month previous to calendar month *t*

$p^{t}_{i,a,r}$ = elementary price index for calendar month *t* for item *i* in area *a* for replicate *r*

$p^{t-1}_{i,a,r}$ = elementary price index for calendar month *t–1* for item *i* in area *a* for replicate *r*

$s^{t}_{i,a,r}$ = monthly expenditure share in calendar month *t* for item *i* in area *a* for replicate *r* of total monthly expenditures in aggregate item *I* in aggregate area *A* for replicate *r*

$s^{t-1}_{i,a,r}$ = monthly expenditre share in calendar month *t–1* for item *i* in area *a* of total monthly expenditures in aggregate item *I* in aggregate area *A*.

(3)

$$s^{t}_{i,a,r} = \frac{E^{t}_{i,a,r}}{\sum_{i \in I, a \in A} E^{t}_{i,a,r}}$$

where, $E^{t}_{i,a,\mathbf{r}}$ is the estimated monthly expenditure in calendar month *t* for item *i* in area *a* for replicate *r*

(4)

$$\mathbf{E}^{t}_{i,a} = E^{t}_{i} \times \left(\frac{\sum_{n=0}^{11} E^{t-n}_{i,a}}{\sum_{a \in A} \sum_{n=0}^{11} E^{t-n}_{i,a}} \right)$$

where,

E = expenditure

t = month

i = elementary item

a = elementary area

A = U.S. City Average aggregate area

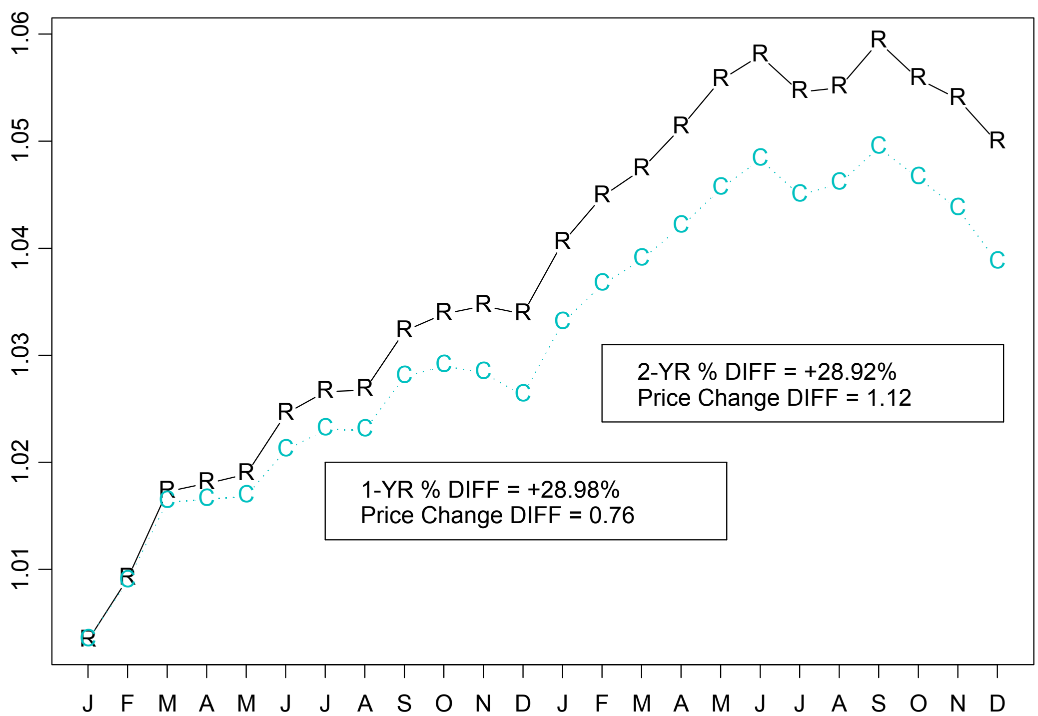
The new "superlative" set of indexes also include *Initial* and *Interim* indexes, but only the Final Tornqvist index, detailed above, can be thought of as "superlative". The official title is Chained CPI-U. The *Initial* C-CPI-U index is published in real time along with the regular official CPI-U. Both CPI-U and the *Initial* C-CPI-U use the same set of lower-level indexes. These lower-level indexes are price relative updates from a Hybrid system, where most of the price relative calculations are Geomeans and the remaining 30% are Laspeyres. Regular CPI-U aggregrates these lower-level hybrid indexes, using a Laspeyres formulation, to produce all its higher-level indexes. *Initial* C-CPI-U, on the other hand, aggregates these same lower-level hybrid indexes using a Geomeans formula. The *Interim* C-CPI-U indexes update the *Initial* indexes with a simple factor to reflect the more recently calculated Final Tornqvist results. In our study here we are only interested in the Final Tornqvist C-CPI-U results, in particular as they and their standard errors compare with official CPI-U results.

The superlative nature of the Final Tornqvist comes from its use of a set of unique monthly weights, detailed above at the item-area level, for *both* time period *t* and for time period *t–1*. These $\mathbf{E}^{t}_{i,a}$'s are smoothed weights, but they do represent a unique monthly weight for that particular month for that particular item in one particular area. The "smoothed" aspect of these weights mitigates somewhat the purity of this uniqueness, but the "superlative" character of the Final Tornqvist formally remains intact. The two weights ($E^{t}_{i,a}$ *and* $E^{t-1}_{i,a}$) are unique, but roughly 90% of the information content of the one is shared by the other. The other obvious mitigating factor is the non-"superlative" nature of the lower-level indexes that are used in the Final Tornqvist formula.

2. Comparison of Chained vs. Regular CPI

With the first two full years of Final Tornqvist results in hand, we can begin to tell if and how much the new Chained CPI indexes diverge from the corresponding regular CPI indexes.

FIG 1. Regular INDEX vs. Chained INDEX (Jan'00-Dec'01)



Price Change (PC) = (INDEX – 1) * 100

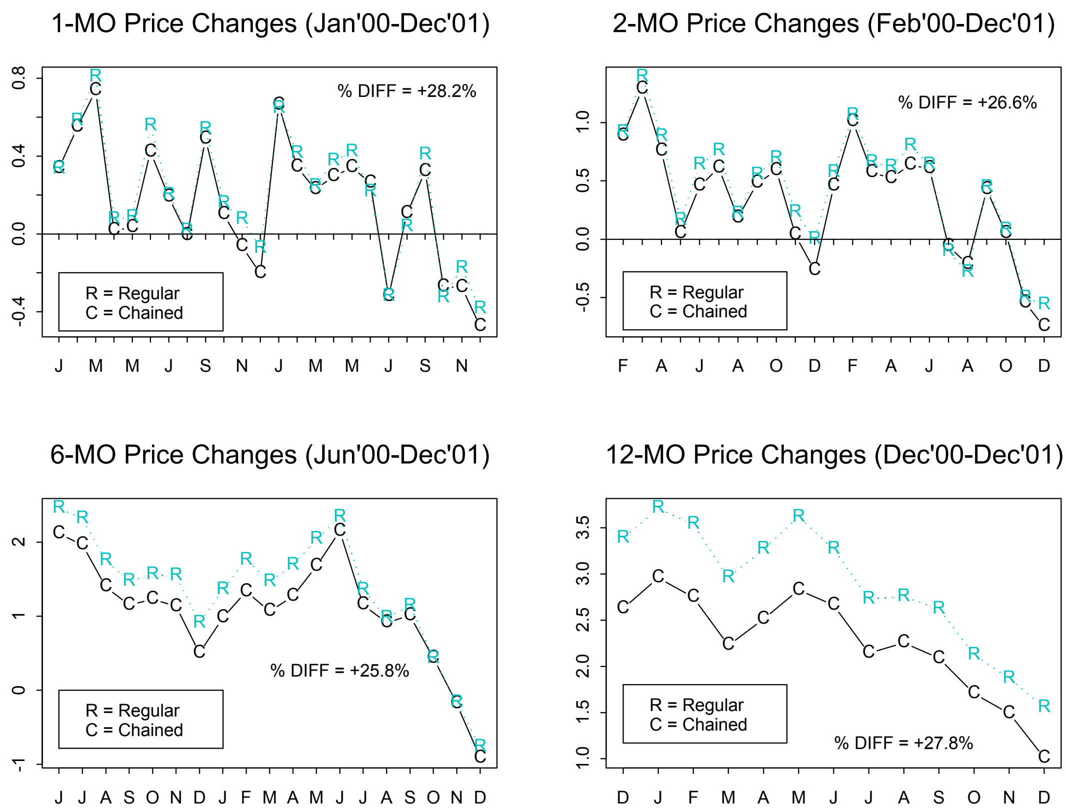
Price Change DIFF = $PC_R - PC_C$

% DIFF = $(PC_R / PC_C - 1) * 100$

As ongoing indexes, the Regular and the Chained Indexes are clearly diverging from each other. The percent difference of the inflationary divergence seems to be holding steady at around 29%, even as the price change difference itself grows steadily wider.

In the four graphs below, the price change (*not* the price relative) differences are graphed for the 1-, 2-, 6- and 12-month results.

FIGs 2-5. CHAINED vs. REGULAR CPI ALL-US–ALL-ITEMS



The one summary statistic that is displayed is a percent difference (% DIFF) between the mean of the Regular CPI price changes and the mean of the Chained CPI price changes. The consistency of these percent differences across the four sets of price changes (≈ 26%) is somewhat notable and is comparable with the 29% price change difference in the index graph. The magnitude of the differences is clearly of interest and note, and may need to be investigated more closely in further study. A chained index using a geometric (albeit Tornqvist) formula throughout was expected to produce a consistently lower index than regular CPI which uses a Laspeyres formula at this highest level of aggregation, but the difference was not expected to be quite this large. Simple paired t-tests for the four sets of paired price changes find that all four of these differences are significant (p-values all less than 0.001, with the differences growing as the tests move from 1- to 2- to 6- to 12-month results). The actual mean differences of the four sets are:

1-month price change mean difference = 0.045
2-month price change mean difference = 0.090
6-month price change mean difference = 0.276
12-month price change mean difference = 0.626

These mean differences are nearly exactly linear with respect to the length of the (time) chain: The 2-month difference is twice the 1-month, the 6-month is six times the 1-month, and the 12-month difference is just over 12 times as large. Thus, the current results show that the yearly inflation rate using the Official CPI is running more than one-half of a percentage point higher than the Chained Tornqvist "Superlative" Index.

One important way to determine how significant are these differences is to calculate standard errors for the new Tornqvist indexes. We already have published standard error results for Regular CPI, but we need to know if and how much more variable are these new "superlative" indexes, and if that variability alone might account for these observed differences between the two indexes at the All-US–All-Items level.

3. Standard errors for the Chained CPI

Official BLS standard errors for the new Final Tornqvist Indexes have not yet been calculated for publication, but the general methodological requirements have been written and approved. The standard errors for the new C-CPI-U that will be calculated and analyzed here follow these requirements. The regular CPI-U standard errors are calculated every month alongside the CPI-U itself, and then published, at the end of the year each year *for* the whole preceding year, in the CPI Detailed Report (usually the February issue).

The standard errors for the CPI-U utilize a Stratified Random Groups methodology. The standard errors for the new C-CPI-U borrow this methodology and fairly closely mimic it. Essentially, the Stratified Random Groups Method employs replicate price change values at the Item-Area level, which are then subtracted off from their corresponding full-sample price change value.

Each replicate PC value, in its simplest form, is the full-sample PC with one of the area's component value contributing a replicate value in place of its regular full-sample value. The differences are then squared and summed up and finally appropriately divided through by the $N_a(N_a-1)$ number of replicates in each Item-Area group. Thus,

$$(5)\quad VAR(I,A,t,t-k) = \sum_{a\in A} \frac{1}{N_a\left(N_a-1\right)} \times \sum_{r=1}^{N_a} \left(PC\left(I,A,t,t-k,a,r\right) - PC\left(I,A,t,t-k,0\right)\right)^2$$

This formula applies equally to the new C-CPI-U variances and to the regular CPI variances. The differences arise in the Price Change (PC) formulas used for each. For the CPI-U,

$$PC_{CPI\text{-}U} = (\sum CW_t / \sum CW_{t\text{-}k} - 1) * 100$$

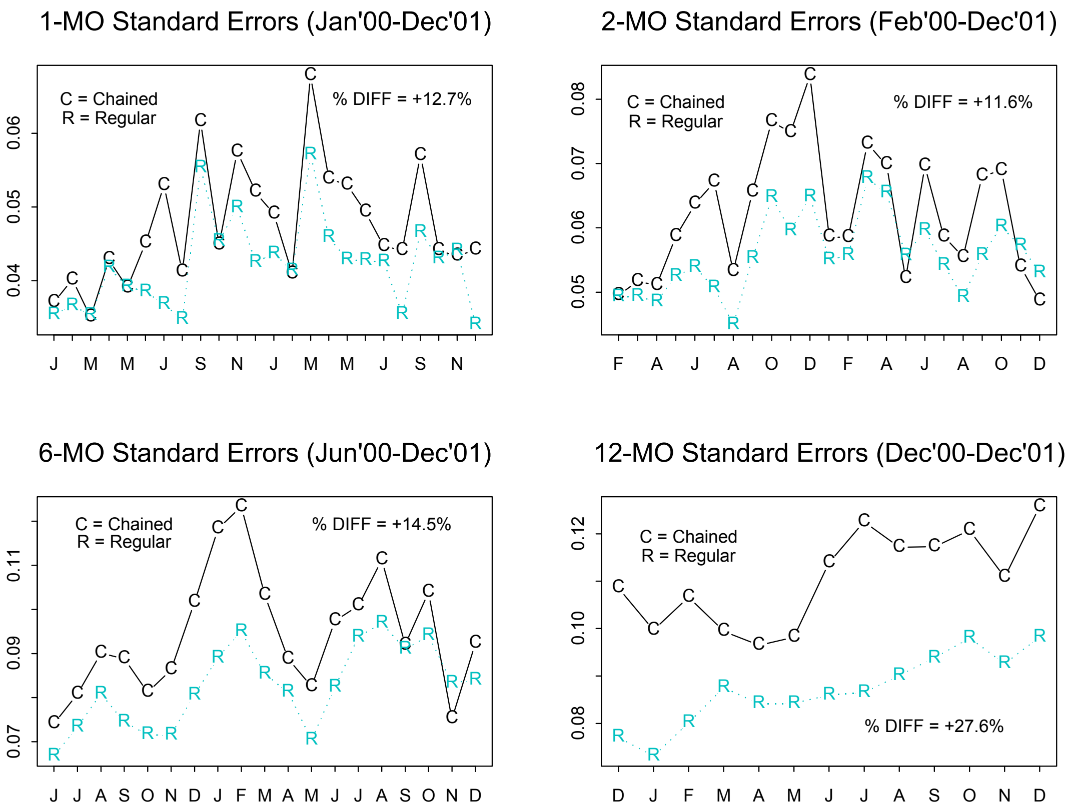
$$\text{where } CW = IX * AGGWT$$

For the C-CPI-U, Equation (1) applies, and

$$PC_{C\text{-}CPI\text{-}U} = (PREL - 1) * 100$$

The same IX's are used in both formulas, but the aggregation weights (AGGWTs) in the regular CPI-U formula are a fixed set of weights, which are anywhere from two to four years old. The monthly Tornqvist weights are from the exact month that is specified. Both sets of weights do come from the same collection of CE (Consumer Expenditure) data, but the two sets of weights are processed differently and of course represent different time periods. The differences between BLS' regular CPI standard errors and the standard errors for the new Tornqvist index are graphed below.

FIGs 5-8. CHAINED vs. REGULAR CPI ALL-US–ALL-ITEMS



One summary statistic, similar to the one in the earlier Price Change graphs, is displayed here: the percent difference (% DIFF) between the respective means of the 1-,2-, 6- and 12-month standard errors. The new Tornqvist standard errors do run higher on average than the regular CPI standard errors, but the differences are not appreciable. At the 1-, 2- and 6-month levels the differences are roughly 13% (comparing Chained to Regular results). The 12-month Tornqvist standard errors are running about 28% higher than their regular CPI standard error counterparts. The chaining effect in the Tornqvist, the geometric formula itself, and the more volatile weights, all contribute to this increased variability. The “superlative” weights are monthly expenditure data values which are smoothed, first over all the ITEMs in the grouping and then back across 12 months of time within the ITEM, which effectively means that the “superlative” weights are carrying a year’s worth of CE data compared to the two to three years’ worth of CE data which the regular (AGGWT) weights reflect. These slightly higher standard errors for the Tornqvist were both expected and yet well within acceptable levels (at least at the All-US–All-Items level).

4. Conclusions

In applying these new standard errors to our Tornqvist results, we are principally interested in noting whether the Tornqvist results are significantly different from the regular CPI results. We can establish confidence bounds at an σ = .05 level by simply multiplying our standard errors by two and then seeing if the regular CPI-U results fall within that 2-sigma bound. For the 24 1-month Tornqvist price changes, only five are significantly different (i.e., higher). For the 23 2-month Tornqvist price changes, we find that only six are significantly different. However, when we move up to the 6- and 12-month price changes, the significance results reverse. *All* of the 13 12-month Tornqvist price changes are clearly significantly different than their CPI-U counterparts, and all but four of the 19 6-month Tornqvist price changes are significantly different than regular CPI. As the time unit is lengthened the standard errors do increase (in a fairly straightforwardly linear fashion, as was mentioned previously), but the differences in the price changes themselves, proportionately, have widened even further. And since it is the 12-month price changes (our *yearly* inflation numbers) that we are most interested in, these clearly significant differences need to be noted.

5. Acknowledgments

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