# Variance Estimation for Noise Components in Time Series from a Survey

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## 1. Background

Models for economic time series of the form $Y = Trend + Seasonal + Irregular$ typically assume that each term is stochastic with a noise component. A fourth noise component, which we denote $e_t$, enters the picture when the series is observed from a survey. Pfeffermann (1994) presents a method for obtaining variance measures for seasonal adjustment with the X11 method. The variance measures are all approximated from autocovariances of the error terms $e_t$ and the irregular $I_t$. These autocovariances are estimated from their relation to X11 irregular autocovariances, in turn estimated by the method of moments from X11 output. Breidt (1992) offers an alternative method working in the frequency domain. Chen, Wong, Morry, and Fung (2003) develop a second method for spectral estimation of the "combined error" autocovariances and compare their methods to the method of moments. This paper continues the work of comparing these methods.

The next section reviews the method of moments and the spectral methods of Chen et al. Section 3 presents several comparisons based on simulation experiments. Some experiments revisit the simulations in Chen et al and some are based on models suggested by a U.S. labor force series. Section 4 looks at theoretical properties of autocovariances and spectra for a few error models. The final section contains conclusions.

## 2. Methodology

Assume the observed series is $y_t = Y_t + e_t$, where $Y_t = T_t + S_t + I_t$ represents the signal or population process with trend, seasonal, and irregular components and $e_t$ represents sampling error. We denote population and estimated values of the seasonally adjusted series as $A_t = Y_t - S_t = T_t - I_t$ and $\hat{A}_t = Y_t - \hat{S}_t$, respectively. The most basic variance measure considered in Pfeffermann (1994) and in Scott, Sverchkov, and Pfeffermann (2005) is

$$Var\left(\hat{A}_t - A_t\right). \qquad (2.1)$$

Let $e_t = I_t + e_t$ denote the combined error and

$$V_k = Cov\left(e_t, e_{t+k}\right). \qquad (2.2)$$

To estimate (2.1) from the observed series $y_t$, one needs to calculate the autocovariances $V_k$. It is natural to assume the irregular and sampling error are independent, so we have $V_k = n_k + l_k$, $n_k = Cov\left(I_t, I_{t+k}\right)$, and $l_k = Cov\left(e_t, e_{t+k}\right)$.

*Method of Moments* (MM)

Pfeffermann (1994) develops an approximation to $V_k$ in terms of the autocovariances of the X11 estimated irregulars $R_t$. Begin by assuming that the combined errors form a stationary process and that

$$Ew_I\left[T_t\right] = Ew_I\left[S_t\right] = 0, \qquad (2.3)$$

where $w_I$ represents the symmetric X11 irregular filter. Pfeffermann derives the approximate equation

$$R_t = \sum_j w_I\left(j\right) y_{t-j} \approx \sum_j w_I\left(j\right) e_{t-j} \qquad (2.4)$$

where $w_i(j)$ denotes the X11 symmetric irregular filter weight at distance $j$. In practice, this means using data only from the central part of the X11 decomposition, where the filters are time-invariant. Taking autocovariances in (2.4), we obtain the desired relation between autocovariances $U\left(k\right)$ and $V\left(k\right)$ of $R_t$ and $e_t$, respectively, namely

$$U\left(k\right) = \sum_{j=0}^{C} d_{kj} V\left(j\right), \quad k = 0, \ldots, C, \qquad (2.5)$$

assuming

$$V\left(k\right) = 0, \quad k > C,$$

for some cutoff value *C*; in matrix terms,

$$\mathbf{U} = \mathbf{DV}. \qquad (2.6)$$

For the method of moments, which we denote MM, given X11 results, we compute estimates

$$U\left(k\right) = \frac{1}{n} \sum_t R_t R_{t+k} \qquad (2.7)$$

where the time points are restricted to the central portion of the series, and solve the linear system (2.6) to obtain the desired estimates for $V\left(k\right)$.

When there is survey error present with a known structure, we simply subtract the known part $\mathbf{D}l$ and solve

$$\mathbf{U} - \mathbf{D}l = \mathbf{D}n \qquad (2.8)$$

for $n$. In addition to making use of extra information, this method of solution reduces the linear system to a very small size, given that we are willing to model the irregular as a low order MA(q) process, which includes the usual assumption of white noise. The advantages of this solution come with the price that sometimes the covariance matrix corresponding to an estimate $n$ fails to be nonnegative definite. In this case, our usual practice is to assume the irregular is negligible and set $n = 0$.

*Spectral Method* (SP)

For the spectral method, Chen also begins with the approximation (2.4). As before, we use the observed $R_t$ to make inferences about the combined errors $e_t$. In the center of the series, where a time-invariant symmetric linear filter is applied, the spectra of $e_t$ and $R_t$ satisfy

$$f_R(w) = |W(w)|^2 \; f_e(w) \qquad (2.9)$$

where $W$ is the gain or transfer function of X11's symmetric irregular filter

$$W_I(w) = \sum_{j=-m}^{m} w_I(j)\cos(jw), \qquad (2.10)$$

Writing $J_e^*$ as an estimator of the spectrum of $e_t$ and $J_R$ for the periodogram of $R_t$, we have

$$J_e{*}(w) = |W(w)|^{-2} \; J_R(w). \qquad (2.11)$$

Now we use the fact that for a stationary process the autocovariance and the spectrum form a Fourier pair. For applications considered here, we mostly assume the stationary process is or can be approximated by an MA(q) process, so the spectrum takes the form

$$f_e(w) = \frac{1}{2p}\left[g_e(0) + 2\sum_{k=1}^{q} g_e(k)\cos(kw)\right]. \quad (2.12)$$

Given that we can obtain observations of $f_e(w)$ from (2.11), we may view (2.12) as a regression model, in which the regression coefficients provide the desired autocovariances of $e_t$:

$$J_e{*}(w) = \frac{1}{2p}\left[g_e(0) + 2\sum_{k=1}^{q} g_e(k)\cos(kw)\right] + \text{error} \qquad (2.13)$$

Chen addresses several technical issues, including places where periodogram smoothing is needed. For estimating the regression coefficients, "observations" $J_e{*}(w)$ are obtained for a set of Fourier frequencies, less points around the seasonal frequencies where the estimates are least stable.

Let us now assume sampling error information is available. The spectra add: $f_e = f_I + f_e$. Converting to a sample equation and assuming $f_e$ is known, we have a simple estimate

$$J_I{*} = J_e{*} - f_e.$$

While by construction $J_e{*}(w) \geq 0$, in practice there is no guarantee that this holds for $J_I{*}$. A modified estimator is

$$J_I{**}(w) = \max(J_e{*}(w) - f_e(w), 0).$$

Chen's preferred method is to compute $J_I{*}$ or $J_I{**}$ and then approximate it by a best-fitting linear spectrum (constrained to be nonnegative). Given this spectrum estimate $\hat{f}_I$, we have

$$\hat{n}(k) = 2\int_0^p \hat{f}_I(w)\cos(kw)\,dw, \quad k = 0,1.$$

The justification for modeling a linear spectrum is this: an MA(1) model should suffice for modeling $I_t$, in which case

$$f_I(w) = \frac{1}{2p} g_I(0) + \frac{1}{p} g_I(1)\cos(w), \quad 0 \leq w \leq p.$$

Since the cosine function is monotone on $[0, p]$, a linear approximation should be reasonable.

Summing up, when sampling error information is available, Chen's method (1) yields estimates for V consistent with an MA(1) model for the irregular and (2) always produces nonnegative definite estimates of the covariance matrix for $I_t$. We will denote this method as LSP.

## 3. Simulation comparisons between moments (MM) and spectral (SP, LSP) methods

For the default options of a 3x5 seasonal filter and a 13-point Henderson trend filter, the X11 symmetric irregular filter has length 169. A symmetric filter of length $2m+1$ can be applied to only $N-2m$ central values from an input time series. Chen has carried out simulations with series having 222 monthly observations, or 18½ years, so that the full filter applies to only the central $222-2\times 84 = 54$ points. In order to use more data, Chen approximates the irregular filter with a symmetric filter of length 79, allowing use of $222-2\times 39 = 144$ time points. We follow Chen by forming alternative filters of length 79 and 121. A very basic property of the irregular filter is that its weights add to 0. For our filter approximations , we reallocate the tail weights by two methods: distributing the net amount equally among the remaining weights and distributing them proportionally, each remaining weight receiving a weight proportional to its own value. In our empirical work, we present results using the latter, but differences between these two choices is small. As mentioned in analyzing Tables 3.3 and 3.4, however, results can be quite sensitive to even small departures from $\sum_k w_k = 0$. There is a trade-off between using a more accurate filter and obtaining more central values for computing key statistics.

### 3.1 Experiments derived from a U.S. labor force series

Important applications of seasonal adjustment at BLS include household and establishment employment and unemployment surveys. Both major surveys have sampling error information. Simulations will be based loosely on Adult Female Unemployment from the household survey, the U.S. Current Population Survey conducted by the Census Bureau. Sampling error information suggests an MA(2) model with parameters $q_1 = .30$ and $q_2 = .18$, reflecting positive correlation across months which dies out quickly. Based on data for the span 1993-2002 and taking account of the sampling error model, William Bell's REGCMPNT program (Bell, 2003) estimates an ARIMA model for the signal part of the model. Standard decomposition of this model through TRAMO/SEATS-style signal extraction leads to the following models for trend, seasonal, and irregular:

$$(1-B)^2 T_t = Z_t^T + .03 Z_{t-1}^T - .97 Z_{t-2}^T ,$$

$$(1+B+\cdots+B^{11}) S_t = \sum_{j=0}^{11} q_j Z_{t-j}^S ,$$

$$I_t = Z_t^I ,$$

where $Z_t^T$, $Z_t^S$, and $Z_t^I$ are white noise processes with fixed variances of $s_T^2$, $s_S^2$ and $s_I^2$, respectively. The survey error follows an $MA(2)$ process with covariance vector

$$l = l_0 [1,.315,.160].$$

Simulations are executed with three values of $l_0$, 55.0, 28.1, and 10.1, i.e., large, moderately large, and about equal in comparison to the irregular variance $n_0 = 8.5$. The following are results obtained averaging 1000 replications of series of length 240 using the approximate filter of length 121. The matrix D in (2.5) and the transfer function $W$ in (2.10) come from the symmetric X11 irregular filter. All that we need for estimating **V** is a set of X11 irregular estimates. Thus, for each simulated series, we apply the X11 irregular filter to the 120 central points, and use

$$R_t = \hat{I}_t = w_I [y_t] ,$$

to estimate V.

Table 3.1 contains the mean and standard deviation of the $N = 1000$ estimates for comparison to the theoretical values derived from the models. Estimates of the irregular variance and first order autocovariance are given for the moments method (MM) for MA(q) models, $q = 0$ to 2, and for the linear spectral method (LSP) which assumes an MA(1). All are significant overestimates. This is especially true for the LSP method and for MM with $q = 2$ or $q = 3$, where some of the estimates are roughly double to triple the true value. With MM, even for the correct value $q = 0$, there is approximately a 50% overestimate. Not surprisingly, all the standard deviations decrease markedly as $l_0$ decreases and the estimates decrease as well. The overestimation would appear to be largely due to noise from the trend and seasonal components not being filtered out.

To check this, we carry out simulations with exactly the same models as above, except for changing $s_T^2$ and $s_S^2$ from 3.4 and 1.7 respectively to 0.5. In addition to the WN model for $I_t$, we obtain results for MA(1) and MA(2) models with parameters $(q_1, q_2) = (.4,0)$ and $(q_1, q_2) = (.36,.20)$. For the MA models, the disturbances are selected so that the irregular variance $n_0$ remains 8.5. Table 3.2 contains results for the intermediate sampling error case, $l_0 = 28.1$, again with results for MM ($q = 0$ to 2) and LSP (q=1). Each column contains estimates for the three simulations in which the $I_t$ model varies. As before, the standard deviation of each estimate is shown in parentheses.

As expected, the overestimation is greatly reduced. For MM, all the estimates and standard deviations increase as $q$ increases. All the estimates for $n_0$ are overestimates. The $q = 0$ estimate is closest for the WN and MA(1) cases and $q = 1$ comes closest for the MA(2) case. LSP overestimates $n_0$ for all three models, but by a decreasing amount as $I_t$ goes from WN to MA(2). The overestimation with LSP and with the correct q values with MM are at least in part due to noise coming from the trend and seasonal components. The decrease in estimates when the true irregular is MA(1) or MA(2) can be due to irregular noise being partially filtered out by X11's irregular filter (see the end of Sec. 4 below for further comment).

Table 3.1. Estimates of $n_k$ when $I_t$ □ $WN(0,8.5)$ $s_T^2 = 3.4$ $s_S^2 = 1.7$

| $l_0$ | $k$ | $n_k$ | $q=0$ | $q=1$ | $q=2$ | LSP $q=1$ |
|---|---|---|---|---|---|---|
| 55.0 | 0 | 8.5 | 12.9 (8.3) | 18.3 (11.7) | 22.5 (15.7) | 23.1 (14.0) |
| | 1 | 0 | 0 | 6.6 (8.5) | 10.8 (12.1) | 5.4 (8.0) |
| 28.1 | 0 | 8.5 | 12.8 (5.4) | 16.4 (7.5) | 20.3 (10.1) | 20.8 (9.4) |
| | 1 | 0 | 0 | 5.2 (5.7) | 8.7 (8.2) | 5.4 (5.9) |
| 10.1 | 0 | 8.5 | 12.7 (3.2) | 15.9 (4.7) | 19.2 (6.6) | 19.6 (6.0) |
| | 1 | 0 | 0 | 4.7 (3.6) | 7.7 (5.5) | 5.7 (4.0) |

Table 3.2. Estimates of $n_k$ when $I_t$ □ $WN$, $MA(1)$, $MA(2)$ $s_T^2 = 0.5$ $s_S^2 = 0.5$

| $l_0$ =28.1 | $k$ | $n_k$ | $q=0$ | $q=1$ | $q=2$ | LSP $q=1$ |
|---|---|---|---|---|---|---|
| WN | 0 | 8.5 | 9.2 (4.9) | 10.8 (6.4) | 12.2 (8.3) | 13.1 (7.2) |
| | 1 | 0 | 0 | 2.0 (4.5) | 3.7 (5.8) | 1.5 (4.5) |
| MA (1) | 0 | 8.5 | 7.2 (4.6) | 10.5 (6.6) | 12.0 (8.6) | 12.9 (7.8) |
| | 1 | 2.9 | 0 | 4.4 (4.7) | 6.0 (6.4) | 3.6 (4.2) |
| | 2 | 0 | 0 | 0 | 1.7 (3.9) | 0 |
| MA (2) | 0 | 8.5 | 6.5 (4.3) | 9.1 (6.2) | 12.0 (8.4) | 11.8 (7.0) |
| | 1 | 3.1 | 0 | 3.4 (4.4) | 6.1 (6.4) | 2.9 (4.1) |
| | 2 | 1.4 | 0 | 0 | 2.8 (4.0) | 0 |

3.2. Experiments from Chen et al (2003)

Next we revisit several comparisons done in Chen et al (2003). Again we simulate $N$ replicates of a time series model of the form $y_t = T_t + I_t + e_t$, this time with length 222, compute autocovariance estimates using both methods for each replication, and compare the mean estimates to the theoretical values.

The simulations are carried out comparing the moment method with the linear spectral approach or the spectral approach, depending on whether sampling error (SE) information is known or not.

Consider the observed series from three models M1, M2, and M3, for the combined error

$$(M1) \quad e_t = x_t + 0.8x_{t-1} + 0.64x_{t-2}$$

$$(M2) \quad e_t = x_t - 0.75x_{t-1} + 0.125x_{t-2},$$

where $x_t$ is white noise with mean 0 and variance $s_t^2 = 25$, or

$$(M3) \quad e_t = x_t,$$

where $x_t$ is white noise with mean 0 and variance $s_t^2 = 36$.

For some simulations a trend component is added to the combined error. The trend component $T_t$ is governed by an equation of the form

$$T_t = T_{t-1} + z_{t-1} + h_{1,t}$$
$$z_t = z_{t-1} + h_{2,t} \qquad (3.1)$$

where $h_{j,t}$ $(j = 1,2)$ is white noise with mean 0 and variance $s_j^2$. The noise standard deviations for the trend process are simulated using model (3.1) with one of two different parameter pairs from the set

$$\{(s_1, s_2) \mid (0,0), (0.8,0.6)\} \qquad (3.2)$$

First we present results using 1,000 replicates for series with a trend component $T_t$ and with one of the two models M1 or M2 for $e_t$ when there is no knowledge of the sampling error, cases appearing in Table 3.5.1 of Chen et al. (2003). Tables 3.3 and 3.4 show our results for the two most extreme trend models for the moments method with $q = 0$ to 2 and the spectral method ( $q = 1$ ).

Table 3.3 shows that for the combined error model M1, when the correct value $q = 2$ is chosen, MM gives estimates with very little bias for both trend models. The SP estimates with $q = 2$ are underestimates, closer than MM for $V_1$, but less close for $V_0$ and $V_2$. SP also has greater variability. MM gives severe underestimation with $q = 0$ or 1.

Table 3.4 gives the results for the two methods when the true combined error model is given by model M2. Both methods come very close with the correct value $q = 2$. MM is a bit closer to the true values in all cases, but here SP has less variability in its estimates. MM underestimates with $q = 0$ but its estimates are close for $q = 1$, due to M2's small covariance at lag 2 – it is *practically* an MA (1) model.

The results given in the two above tables differ substantially from those reported in the Chen et al. (2003) because of the use of the unbiased filter in our simulations. Both these tables show substantial changes in mean estimates with the moments method as q changes. While only $q = 2$ values are shown for the spectral method, our work shows that it changes much less with the choice of q.

Next in our analysis we look at the case when there is known sampling error present. The observed series are based on trend and irregular models indicated in Table 3.5 with an AR(1) sampling error defined in Chen et al. (2003) by

$$e_t = 0.5 e_{t-1} + z_t ,$$

where $z_t$ is a white noise process with variance 36.

This gives the survey error an overall variance of 48 similar in magnitude to the variance of the irregular term. Table 3.5 gives the results for MM ($q = 1$, 2) and LSP ($q = 1$). Compared to LSP, MM is less biased with $q = 1$ and more biased with $q = 2$ for both trend cases. Again, we note that M2 is fairly close to an MA(1) model.

Table 3.3 Results for Combined Error Model M1

| $s_1$ $s_2$ | $k$ | $V_k$ | $q = 0$ | $q = 1$ | $q = 2$ | SP $q = 2$ |
|---|---|---|---|---|---|---|
| 0 0 | 0 | 51.2 | 22.5 (6.1) | 34.2 (11.1) | 51.3 (17.9) | 48.3 (20.5) |
| | 1 | 28.3 | 0 | 17.4 (8.1) | 32.8 (14.6) | 29.6 (16.6) |
| | 2 | 16.0 | 0 | 0 | 16.0 (8.4) | 12.8 (9.0) |
| .8 .6 | 0 | 51.2 | 22.9 (6.2) | 34.8 (11.2) | 52.3 (18.1) | 49.4 (20.8) |
| | 1 | 28.3 | 0 | 17.7 (8.2) | 33.4 (14.8) | 30.3 (16.9) |
| | 2 | 16.0 | 0 | 0 | 16.3 (8.5) | 13.1 (9.1) |

Table 3.4 Results for Combined Model M2

| $s_1$ $s_2$ | $k$ | $V_k$ | $q = 0$ | $q = 1$ | $q = 2$ | SP $q = 2$ |
|---|---|---|---|---|---|---|
| 0 0 | 0 | 39.5 | 52.5 (13.2) | 36.3 (8.5) | 39.5 (13.9) | 40.0 (10.7) |
| | 1 | -21.1 | 0 | -24.2 (12.3) | -21.3 (10.4) | -19.7 (8.0) |
| | 2 | 3.1 | 0 | 0 | 3.0 (12.4) | 2.6 (8.4) |
| .8 .6 | 0 | 39.5 | 53.0 (13.3) | 37.0 (8.6) | 40.5 (14.0) | 41.0 (10.8) |
| | 1 | -21.1 | 0 | -23.8 (12.3) | -20.6 (10.3) | -19.0 (8.2) |
| | 2 | 3.1 | 0 | 0 | 3.3 (12.5) | 2.9 (8.4) |

Table 3.5 Results with Irregular Model M2 and Known AR(1) Survey Error

| Model | Actual | MM $q = 1$ | MM $q = 2$ | LSP $q = 1$ |
|---|---|---|---|---|
| Trend: $s_1$ =0 $s_2$ =0<br>Irregular: M2<br>Survey error: AR(1) | $n_0$ = 39.5 | 40.9 (12.1) | 46.2 (16.5) | 44.834 (12.62) |
| | $n_1$ = -21.1 | -19.1 (11.0) | -16.0 (12.3) | -15.00 (7.25) |
| | $n_2$ = 3.1 | 0 0 | 7.3 (12.0) | 0 0 |
| Trend: $s_1$ =0.8 $s_2$ =0.6<br>Irregular: M2<br>Survey error: AR(1) | $n_0$ = 39.5 | 41.23 (12.33 ) | 47.13 (16.29) | 45.201 (12.728) |
| | $n_1$ = -21.1 | -19.04 (11.17) | -15.67 (12.19) | -14.97 (7.35) |
| | $n_2$ = 3.1 | 0 0 | 7.5 (11.9) | 0 0 |

4. Theoretical characteristics of the X11 irregular

Key to both methods are properties of the X11 irregular. Properties of the X11 filters have been carefully studied; a recent paper is Findley and Martin (2003). We briefly exhibit theoretical properties for autocovariances and spectra for simple error processes along the lines of processes described in earlier sections. We write $\{e_t\}$ for the error processes and $\{R_t\}$ for the processes stemming from application of the X11 irregular filter. It is not clear how well these properties carry over to sample quantities, since, for instance, autocovariance estimates have sizable variances and the periodogram must be smoothed, just to provide consistent estimates of spectrum values.

Figure 1 shows autocorrelation functions for unfiltered and filtered error models, white noise (WN) and two MA(1) processes, $q_1 = \pm 0.3$. With WN, as is well known, the irregular filter induces sizable negative autocorrelations at lags 1 and 12, about -1/3 at lag 1. For the MA(1) process with positive autocorrelation ($q_1 = +0.3$), the lag 1 autocorrelation moves toward 0, but the lag 2 autocorrelation has a large negative value near -0.4. For $q_1 = -0.3$, the negative lag 1 autocorrelation is accentuated, moving close to -1/2. For an MA process with $q_{12} > 0$ and $q_k = 0, k \neq 12$, (not shown) the negative autocorrelation at lag 12 is dampened. Figure 1 also shows the spectra for these processes. For WN, the spectrum of $R_t$ is roughly horizontal except for suppression at all the low (trend) frequencies and in a neighborhood of the seasonal frequencies. For both MA(1) processes, the spectrum $f_R$ is zero or nearly so up to the first seasonal frequency and in a neighborhood of the other seasonal freqencies. The nonzero parts of the spectrum mimic the shape of the spectrum $f_e$ of $e_t$, especially beyond p/3. Since much of the power for the positive MA(1) is in the lower frequencies, $f_R$ is, overall, comparatively low.

The shape of either the theoretical autocovariance function or spectrum provides information about $e_t$. These same theoretical functions can be easily calculated for more complicated models for $e_t$. Further work could be conducted to evaluate whether consideration of *sample* quantities from such error models can help in identifying the error structure, e.g., selection of $q$. Note that the decline in estimates of $n_0$ as the model for $I_t$ changes from MA(0) to MA(2) (with positive autocorrelations) fits with suppression of some of the error process by the irregular filter.

5. Conclusions

Our experience with the spectral method is quite limited, so these conclusions are tentative. Both methods performed well in the Chen simulations. Especially in the first simulation based on the Adult Female Unemployment series, strong overestimation occurred.

The spectral method often behaves more stably than the moments method. However, using the correct of q, the moments method can have much smaller bias and lower MSE. A further advantage with the moments method is its modification to use all the estimated irregulars, not just the central values. This modification, presented in Pfeffermann and Scott (1997), is important, since series are often seasonally adjusted with 10 years or less. Extension of the spectral method to the case of non-stationary irregular has not been done.

In some of our simulations for Adult Female Unemployment, sampling error, irregular, and differenced trend components all follow moving average models. Asking a single filter to sort out the trend part from the noise is unrealistic. The linear spectrum strategy of insuring valid covariance estimates and seeking nothing more complicated than an MA(1) irregular has much appeal.

Section 4 illustrates that it is easy to derive properties of the X11 irregular when the inputs are simple error processes (or sums of simple processes). For the moments method, one could consider smoothing a sample estimate of the irregular autocovariance function by using a theoretical function from a model suggested by the sample autocorrelations or spectrum. Whether their sample properties are strong enough to support this approach has not yet been evaluated.

The analysis in this paper has emphasized a difficult case. However, based on experience reported in Scott, Sverchkov, and Pfeffermann (2005) typical applications of the X11 method yield acceptable results from the moments method and, likely, from the spectral method as well.

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*The views expressed are those of the authors and do not represent official positions of BLS.*

Figure 1a: Autocorrelation Function and Spectrum of the Filtered and Unfiltered White Noise Process

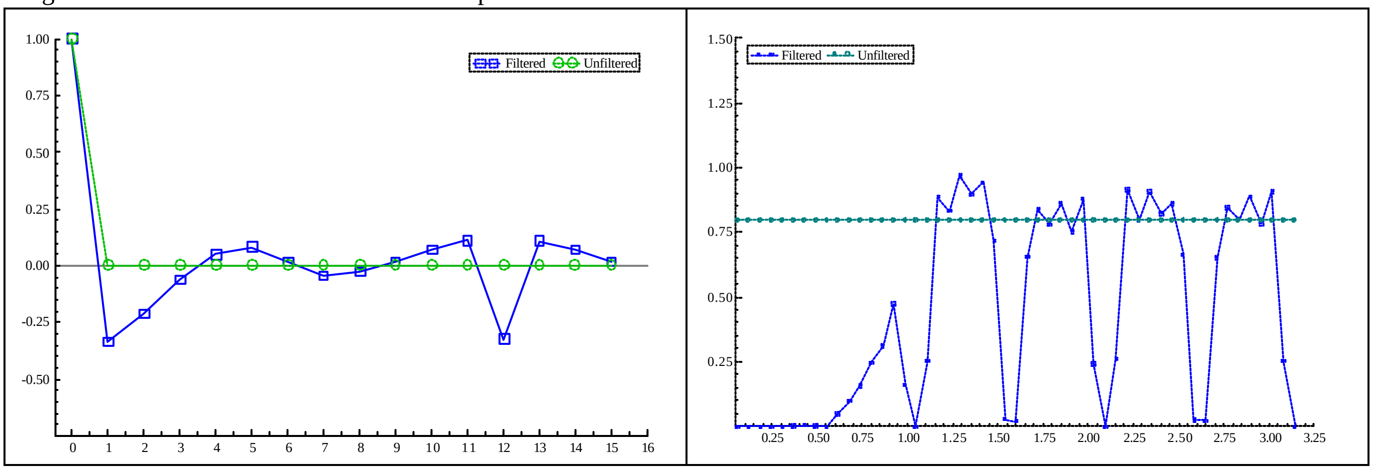


Figure 1b: Autocorrelation Function and Spectrum of the Filtered and Unfiltered MA(1) Process with $q_1 = 0.3$

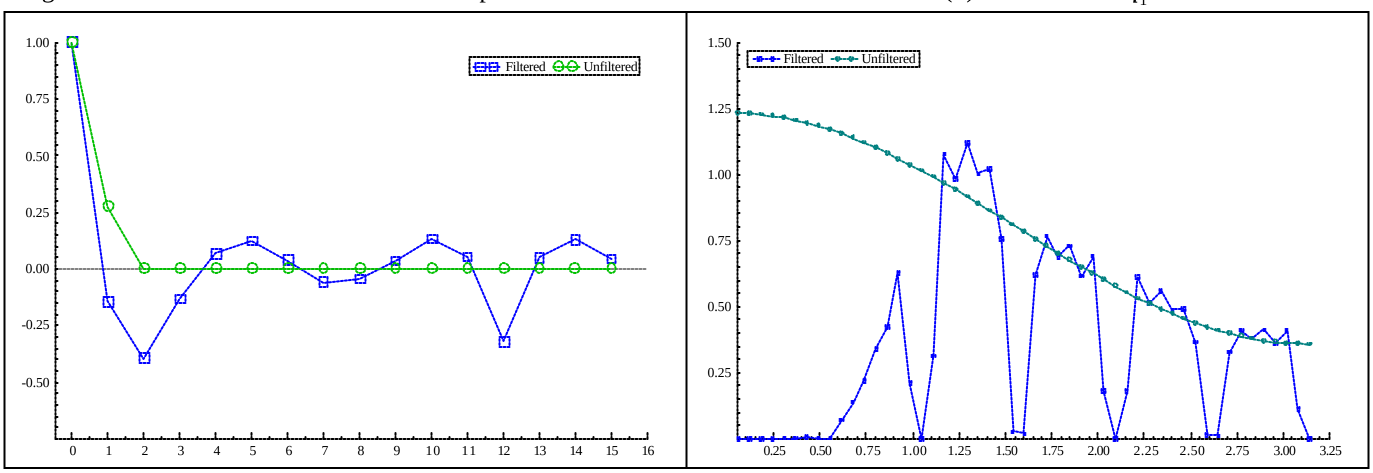


Figure 1c: Autocorrelation Function and Spectrum of the Filtered and Unfiltered MA(1) Process with $q_1 = -0.3$