Introduction

Questions:
- What is the marginal propensity to consume (MPC) out of permanent income and income shocks? How does it differ across gender, race, education and age groups?

Why is it important:
- Implications for predicting household responses to tax reforms and other redistributive policies

What has been done:
- In the literature, a common measure of permanent income is the average annual income over some time period

What is new:
- Use a large household dataset which includes measures of both total consumption expenditure and income
- Take into serious consideration the changing individual characteristics when calculating permanent income
This Project

- Provide a regression framework for analyzing the effects of individual attributes on total family after-tax income
  - the Consumer Expenditure Survey
- Construct transition path of these attributes over the life cycle
  - the Panel Study of Income Dynamics
- Use the estimated regression coefficients and the projected demographic profiles to predict family permanent income
- Calculate the MPC by regressing consumption expenditure on permanent income and income shocks
Preview of Findings

- On average, households spend \( 22.37 \) cents out of each dollar of income shocks.
- \$1\ increase in permanent income drives up current consumption expenditure by \( 2.06 \) cents.
- The MPC out of income shocks is not statistically different for people at different permanent income levels.
Related Literature

- Predictions about the **level** of consumption in relation to the **level** of the permanent component of income

- Predictions about **changes** in consumption in response to predictable or unpredictable, transitory or permanent, income **changes**
The life cycle and the permanent income models have constituted the main analytical tools to the study of consumption behaviour, both at the micro and at the aggregate level.

Total consumption is much smoother than disposable income, and the consumption profile is flat after accounting for changes in family size and composition.

- use the following equivalence scale: assign weight 1 to the male adult, 0.8 to the female adult, and 0.4 to a kid.
Consumer Expenditure Survey

- Data on expenditures, income, assets, and demographic characteristics of consumers in the United States
- The Quarterly Interview Survey is designed to collect data on large and recurring expenditures
- Use the subsample of households that have completed all four quarterly interviews
  - Their annual consumption expenditure chronologically corresponds to the reported income, which is over the last twelve months
- Aggregate expenditures from quarterly to annual frequency using data between 1990 and 2016
- Convert all values into real 2016 dollars based on chained CPI

Imputation of Chained CPI Before 1999
Regression Framework

Apply weighted least squares, minimizing the sum of squared residuals weighted by the inverse probabilities of selection:

\[
\ln y_i = \alpha_0 + \alpha_1 age_i + \alpha_2 age_i^2 + \beta_0 x_i + \beta_1 x_i age_i + \beta_2 x_i age_i^2 + \gamma z_i + \eta_i
\]

- \( y_i \) is total family after-tax income in 2016 dollars
- \( x \) includes marital status, number of children, gender, employment status, employment status* \( I \) (employer pension contributions), employment status* \( I \) (self-employment), full-time equivalent of weeks worked, educational attainment and race
- \( z_i \) is a set of birth-year dummies to control for cohort effects
- \( \eta_i \) is an income shock
- Use robust estimator of variance, which gives heteroskedasticity-consistent standard error
**Top-coding**

- The CE protects the respondents’ identity by changing sensitive data with topcoding
  - Topcoding refers to the replacement of data if the value of the original data exceeds prescribed critical values
- Each observation that falls outside the critical value is replaced with a topcoded value that represents the mean of the subset of all outlying observations
  - Do not correct the top-coding in household expenditures, income and assets
  - Bosworth, et al. (1991): the top coding has only a minimal effect on the surveys conducted after 1981
Income Imputation

- Starting with the publication of the 2004 data, the CE includes income data that have been produced using multiple imputation
  - fill in blanks due to nonresponse, i.e., the respondent does not know or refuses to provide a value for a source of income
  - The process preserves the mean of each source of income, and also yields variance estimates that take into account the uncertainty built into the data from the fact that some observations are imputed, rather than reported.

- Use reported values if available. Otherwise, use imputed values and apply the method of repeated-imputation inference
Repeated-Imputation Inference

The proper estimation uses all five imputations for income by estimating the regression model once with each imputation

- The point estimates, $\bar{b} = \frac{\sum_{i=1}^{m} b_i}{m}$, where $m$ equals the number of imputations
- The variance of the point estimate, $T_m = \bar{U}_m + (1 + m^{-1})B_m$
  - $\bar{U}_m = \frac{\sum_{i=1}^{m} U_i}{m}$ and $U_i$ is the variance of the estimated coefficient for imputation $i$
  - $B_m = \frac{\sum_{i=1}^{m} (b_i - \bar{b})^2}{m-1}$, which takes into account the uncertainty involving the point estimate
Panel Study of Income Dynamics

- Originally designed to study the dynamics of income and poverty
  - an over-sample of 1,872 low income families from the Survey of Economic Opportunity (the “SEO sample”)
  - a nationally representative sample of 2,930 families designed by the Survey Research Center at the University of Michigan (the “SRC sample”)

- “Sample persons” include all persons living in the PSID families in 1968 plus anyone subsequently born to or adopted by a sample person
  - All sample members are followed even when leaving to establish separate family units (FUs)

- The most common example of “non-sample persons” are those who after 1968 marry sample persons.
  - Information on non-sample persons is collected while they are living in the same family unit as a sample person
Transition Matrix of Attributes

- Use the PSID to construct transition matrix of all attributes for each gender, race, age and education group:

\[ P_{ij}^g = \Pr(X_{t+1}^g = j | X_t^g = i) = \frac{n_{ij}^g}{\sum_j n_{ij}^g} \]

- \( \sum_j P_{ij}^g = 1 \) and \( P_{ij}^g \geq 0 \). \( n_{ij}^g \) is the weighted sum of the number of observations where individuals in group \( g \) transition from state \( i \) at age \( t \) to state \( j \) at age \( t + 1 \) in attribute \( X \).

- Impose the restrictions that \( Educ_t \leq Educ_{t+1} \leq Educ_t + 1 \). In addition, if one’s education increases from high school graduates to some college in year \( t \), then there can be no more increase in the next 2 years.
Assumptions I

- Include an observation if and only if the individual has a positive sample weight in both last year and the current year, and there is no missing information about their age, gender, race and education as well as the variable in question.

- If there is not enough information to calculate the transition probability of a variable for a given group at a given age, then assume that the next state is the same as the initial state with probability 1.

- Transition probabilities after age 65 are the same as those at age 65 due to a lack of data.
Assumptions II

- Split the PSID sample into three time periods: 1990-1997, 1997-2005 and 2005-2015 to allow transition matrix to vary over time
  - Suppose an individual was 20 in 1996, then apply the 1990-1997 transition matrix to infer the probabilities of their attributes at age 21, and apply the 1997-2005 transition matrix for predictions at age 22

- In order to classify an individual as a self-employed or wage worker who claims to be both in the PSID, compare their labor and asset income from business with their wages and salaries
  - If the former is larger, identify the individual as self-employed
  - If this information is unavailable, assume an individual is a wage worker
One-Year Transition Matrix

PSID was conducted annually until 1996, and biennially since 1997

- Impute 1-year transition matrix by calculating the principal square root of 2-year transition matrix
  - The unique square root for which every eigenvalue has nonnegative real part
- Underlying assumption: transition probabilities are the same from $t$ to $t + 1$ and from $t + 1$ to $t + 2$ between 1997 and 2015
- If $[V, D] = eig(A)$, then the squareroots have the general form $Y = V \cdot S/V$, where $D = S \cdot S$, and the unique principal square root is chosen

Square Root of a Matrix
Transition Path of Attributes

- Assume sequence of all attributes follows a Markov chain, then the conditional probability is:

\[
\Pr(X_{i,t+1} = x_{i,t+1} | X_{i,0} = x_{i,0}, X_{i,1} = x_{i,1}, \ldots, X_{i,t} = x_{i,t})
\]

\[
= \Pr(X_{i,t+1} = x_{i,t+1} | X_{i,t} = x_{i,t})
\]

\[
= P^g_{x_{i,t}x_{i,t+1}}
\]

- The probability of each realization of the random variable \(X_{i,t}\) for individual \(i \in g\) is:

\[
\Pr(X_{i,t+k} = x_{i,t+k}) = \sum_{\{x_{i,t+k}\}} \cdots \left[ \sum_{\{x_{i,2}\}} \left( \sum_{\{x_{i,1}\}} P^g_{x_{i,0}x_{i,1}} P^g_{x_{i,1}x_{i,2}} P^g_{x_{i,2}x_{i,3}} \right) \right] \cdots P^g_{x_{i,t+k-1}x_{i,1+k}}
\]
Permanent Income

- Calculate $PI_i = A_{i,t} + \sum_{k=0}^{T-t} \left( \frac{1}{1+r} \right)^k E_t [Y_{i,t+k}]$, where

  $$E_t [Y_{i,t+k}] = \delta_{i,t+k} \sum_l \Pr( Y_{i,t+k} = y_{i,l} ) \times y_{i,l}$$

  $$\Pr( Y_{i,t+k} = y_{i,l} ) \times y_{i,l} = \Pr( X_{i,t+k} = x_{i,t+k} )$$

  $$[\hat{\alpha}_0 + \hat{\alpha}_1 \text{age}_{i,t+k} + \hat{\alpha}_2 \text{age}^2_{i,t+k} + \hat{\beta}_0 x_{i,t+k} + \hat{\beta}_1 x_{i,t+k} \text{age}_{i,t+k} + \hat{\beta}_2 x_{i,t+k} \text{age}^2_{i,t+k} + \hat{\gamma} z_i]$$

- $A_{i,t}$ is a CU’s current net worth
- Assume that $r = 0.04$; $t$ corresponds to the age of the individual when interviewed; $T = 94$, the oldest age in the CE
- Mortality rates, $\delta_{i,t}$, is a function of education levels and marital status along with other demographic variables (gender, race, age)
- Use the maximum of the head’s and the spouse’s predicted permanent income as the family permanent income
Average Propensity to Consume

Compute the APC by dividing consumption expenditure by permanent income

- more variation across households as they age
- increase monotonically with age, rising from 2% at age 20 to over 10% at age 90
- adjustment using the equivalence scale increases its curvature
- similar results after talking into account the economies of scale of joint living
- not a lot of variation across race nor education groups
### Table 1: Summary Statistics for the APC

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<tr>
<td>APC per adult equivalent</td>
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</tr>
<tr>
<td>APC joint living</td>
<td>.0248291</td>
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Marginal Propensity to Consume

- Compute the MPC out of income shocks, $\theta_1$, and out of permanent income, $\theta_2$:

$$c_i = \theta_0 + \theta_1 \hat{\eta}_i + \theta_2 P\hat{l}_i$$

$$+ \alpha_1 \text{age}_i + \alpha_2 \text{age}^2_i + \beta_0 x_i + \beta_1 x_i \text{age}_i + \beta_2 x_i \text{age}^2_i + \gamma z_i + \epsilon_i$$

- Measure income shocks as:

$$\hat{\eta}_i = \ln y_i - (\hat{\alpha}_0 + \hat{\alpha}_1 \text{age}_i + \hat{\alpha}_2 \text{age}^2_i + \hat{\beta}_0 x_i + \hat{\beta}_1 x_i \text{age}_i + \hat{\beta}_2 x_i \text{age}^2_i + \hat{\gamma} z_i)$$

### Table 2: Marginal Propensity to Consume

|          | Coef.       | Bootstrapped Std. Err. | t    | P>|t| |
|----------|-------------|------------------------|------|-----|
| $c_i$    | 0.2237425   | 0.0048216              | 46.40| 0.000|
| $\hat{\eta}_i$ | 0.0205654 | 0.0010507              | 19.57| 0.000|
| $P\hat{l}_i$ |            |                        |      |     |

- Households spend 22.4 cents out of each dollar of income shocks
- If permanent income goes up by $1, then consumption expenditure goes up by 2.1 cents
Life-Cycle Profiles of MPC

- Before 55, younger households spend more out of each dollar of income shocks; after 55, older households do.
- Before 55, older households spend more out of each dollar increase in permanent income; after 55, younger households do.
Life-Cycle Profiles of MPC

- To test this hypothesis, examine MPC calculated based on consumption expenditure per adult equivalent.

- The shape of MPC out of income shocks does not change much.
- MPC out of permanent income monotonically increases with age after change in family size is taken into account.
MPC of Income Shocks at Different PI Levels

Table 3: MPC of Income Shocks at Different PI Levels

|                  | Coef.       | Bootstrapped Std. Err. | t   | P>|t| |
|------------------|-------------|------------------------|-----|-----|
| $\hat{c}_i$      |             |                        |     |     |
| $\hat{\eta}_i \times PI_i$ | 1.24e-08   | 1.20e-08               | -1.03 | 0.303 |
| $\hat{\eta}_i$  | 0.3871338   | 0.0615386              | 6.29 | 0.000 |
| $\hat{\eta}_i \times age$ | -0.0054707 | 0.0023142              | -2.36 | 0.018 |
| $\hat{\eta}_i \times age^2$ | 0.0000491  | 0.0000243              | 2.02 | 0.043 |
| $PI_i$           | 0.0178924   | 0.0055083              | 3.25 | 0.001 |
| $PI_i \times age$ | 0.0003511  | 0.0002047              | 1.71 | 0.086 |
| $PI_i \times age^2$ | -4.81e-06  | 1.88e-06               | -2.56 | 0.010 |

MPC out of income shocks is not statistically different for people at different permanent income levels

Zheli He (Penn Wharton Budget Model)  Marginal Propensity to Consume  July 2018
Appendix: Imputation of Chained CPI

- Use the series of chained CPI of all items in U.S. city average for all urban consumers from the Bureau of Labor Statistics, which is available from 1999 to 2016.

- To impute chained CPI for the missing years:
  - First, compute the difference in average annual growth rate of chained and unchained CPI between 1999 and 2016.
  - Second, use that difference to adjust the annual growth rate of unchained CPI between 1990 and 1999.

- Use the same deflator for income and expenditures to keep them consistent.
### Table A1: Mean and Variance of the Estimated Coefficients

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### Appendix: Regression Results

#### Table A1: Mean and Variance of the Estimated Coefficients

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## Appendix: Regression Results

### Table A1: Mean and Variance of the Estimated Coefficients

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Appendix: Overview of Multiple Imputation

- The method used to derive the multiple imputations is regression-based
  - A regression is run to provide coefficients for use in estimating values for missing data points
  - The coefficients are then “shocked” by adding random noise to each, and missing values are estimated using the shocked coefficients
  - To each of these estimated values, additional random noise is added, to ensure that consumer units (or members) with identical characteristics (e.g., urban service worker aged 25 to 34) will not receive identical estimates for their income
  - The resulting values are used to fill in invalid blanks where they occur, while reported values are retained
  - This process is then repeated four times
- For the small number of cases in which the respondent does not report receipt of any source of income, receipt of each source is imputed using logistic regression.
  - The income value is treated as a missing data point, and is imputed using the method described above
Appendix: Square Root of a Matrix

- More generally, an \( n \times n \) matrix with \( n \) distinct nonzero eigenvalues has \( 2n \) square roots
  - \( A = VDVi \), where \( V \) is the matrix whose columns are eigenvectors of \( A \); \( D \) is the diagonal matrix whose diagonal elements are the corresponding \( n \) eigenvalues \( \lambda_i \)
  - The square roots of \( A \) are given by \( A = VD^{\frac{1}{2}}V \), where \( D^{\frac{1}{2}} \) is any square root matrix of \( D \), which, for distinct eigenvalues, must be diagonal with diagonal elements equal to square roots of the diagonal elements of \( D \)
  - Since there are two possible choices for a square root of each diagonal element of \( D \), there are \( 2n \) choices for the matrix \( D^{\frac{1}{2}} \)

- \( M \) is called positive semidefinite (or sometimes nonnegative definite) if \( x^* Mx \geq 0 \) for all \( x \) in \( \mathbb{R}^n \)
  - A positive-semidefinite matrix has precisely one positive-semidefinite square root, which can be called its principal square root
Appendix: Life-Cycle Profiles of Permanent Income

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.
Appendix: Life-Cycle Profiles of Permanent Income

Permanent Income by Race

Permanent Income by Education

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.
Appendix: Life-Cycle Profiles of APC

Average Propensity to Consume

Raw predicted values at the top and bottom 5% are trimmed.

Predicted based on individual characteristics and adjusted for mortality rate, 4% discount rate assumed.
Appendix: Life-Cycle Profiles of APC

Average Propensity to Consume: Adult Equivalent

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.

Average Propensity to Consume: Joint Living

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.
Appendix: Life-Cycle Profiles of APC

APC by Race

APC by Education

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.

Predicted based on individual characteristics and adjusted for mortality rate; 4% discount rate assumed.