## An Introductory Look at the Chained Consumer Price Index

**U.S. Department of Labor • Bureau of Labor Statistics • Washington, DC 20212**

### ESTIMATION METHODOLOGY

1. **Price Index Formula Notation**

   - **Index symbol**: \( IX_L^{[0;t]} \)
   - **Base period of index**: \( 0 \)
   - **Comparison period of index**: \( t \)
   - **Item, area, and population represented**: \( i, a, u \)

2. **The Cost-of-Living Index Concept**

   \[
   IX^C_{[0;1]} = \frac{\min \sum_i P_t \times Q_t}{\sum_i P_0 \times Q_0}
   \]

   - The minimum expenditure \( (P_t \times Q_t) \) required in comparison period \( t \) to attain the same level of satisfaction or utility \( U_0 \) achieved in base period \( 0 \), divided by the actual expenditure \( (P_0 \times Q_0) \) in base period \( 0 \).

3. **Price Index Formulas Commonly Used to Approximate a Cost-of-Living Index**

   **FIRST ORDER APPROXIMATIONS:**

<table>
<thead>
<tr>
<th>Abbreviation of underlying price index formula</th>
<th>Index symbol</th>
<th>Abbreviation of underlying price index formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>LASPEYRES: ( IX_L^{[0;1]} = \sum_i s_i \left( \frac{P_t}{P_0} \right) )</td>
<td>( IX_L^{[0;1]} )</td>
<td>( IX_L^{[0;1]} )</td>
</tr>
<tr>
<td>PAASCHE: ( IX_P^{[0;1]} = \left[ \sum_i s_i \left( \frac{P_t}{P_0} \right) \right]^{-1} )</td>
<td>( IX_P^{[0;1]} )</td>
<td>( IX_P^{[0;1]} )</td>
</tr>
<tr>
<td>GEOMETRIC MEAN: ( IX_G^{[0;1]} = \prod_i \left( \frac{P_t}{P_0} \right)^{s_i} )</td>
<td>( IX_G^{[0;1]} )</td>
<td>( IX_G^{[0;1]} )</td>
</tr>
</tbody>
</table>

   **SECOND ORDER APPROXIMATIONS:**

<table>
<thead>
<tr>
<th>Abbreviation of underlying price index formula</th>
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</tr>
</thead>
<tbody>
<tr>
<td>TORNQVIST: ( IX_T^{[0;1]} = \prod_i \left( \frac{P_t}{P_0} \right)^{\frac{s_i}{2}} )</td>
<td>( IX_T^{[0;1]} )</td>
<td>( IX_T^{[0;1]} )</td>
</tr>
<tr>
<td>FISHER IDEAL: ( IX_F^{[0;1]} = \left( IX_L^{[0;1]} \times IX_P^{[0;1]} \right)^{1/2} )</td>
<td>( IX_F^{[0;1]} )</td>
<td>( IX_F^{[0;1]} )</td>
</tr>
</tbody>
</table>

**KEY:**

- \( P_t \) = Price of item \( i \) in comparison period \( t \)
- \( P_0 \) = Price of item \( i \) in base period \( 0 \)
- \( s_i \) = Expenditure on item \( i \) in comparison period \( t \), divided by expenditures on all items in comparison period \( t \)
- \( s_0 \) = Expenditure on item \( i \) in base period \( 0 \), divided by expenditures on all items in base period \( 0 \)
4. Estimation of Price Change in the Chained Consumer Price Index (C-CPI-U)

**LOWER-LEVEL AGGREGATION:**

\[
i_{i,a} IX^L_{[0;\tau]} = \sum_{k \in i,a} k S_0 \left( \frac{k P_t}{k P_0} \right)
\]

**UPPER-LEVEL AGGREGATION:**

**Long-term Price Change**

\[
i_{i,a} IX^G_{[0;\tau]} = \prod_{n=1}^{\tau} i_{i,a} IX^G_{[n-1;\tau]}
\]

**Month-to-Month Price Change**

\[
i_{i,a} IX^G_{[t-1;\tau]} = \lambda_y \prod_{i,a \in I, A} \left( \frac{i_{i,a} IX^L_{[t;\tau]}^{[0;\tau]}}{i_{i,a} IX^L_{[0;\tau-1]}^{[0;\tau-1]}} \right) i_{i,a} S_{b_2}
\]

**KEY:**

- **k** = unique good or service
- **A** = CPI aggregate area
- **a** = CPI elementary area
- **i** = CPI elementary item
- **0** = elementary index base period
- **t** = month
- **y** = year
- \(P_t\) = price of good (k) in month (t)
- \(P_0\) = price of good (k) in base-period (0)
- \(S_0\) = expenditure for good (k) in base period (0), divided by expenditure for all (k) goods in elementary item (i), area (a) in base period (0)
- **z** = December 1999 index base period
- **b_1** = expenditure reference period of CPI-U index of year (y); \(b_1 = 1999-2000\) for y=2002 and y=2003.
- **b_2** = expenditure reference period of CPI-U index of year (y+1); \(b_2 = 2001-2002\) for y=2003.
- **IX^L** = Laspeyres elementary index
- **IX^G** = Geometric Mean elementary index
- **IX^G_i** = Initial C-CPI-U index
- **IX^G_r** = Interim C-CPI-U index
- **IX^T** = Final C-CPI-U index
- **i_{i,a} S_{b_1}** = expenditure for elementary item (i) in area (a) in expenditure period (b), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b)
- **i_{i,a} S_{b_2}** = expenditure for elementary item (i) in area (a) in expenditure period (b), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b)
- **i_{i,a} S_t** = expenditure for elementary item (i) in area (a) in month (t), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t)
- **i_{i,a} S_{t+1}** = expenditure for elementary item (i) in area (a) in month (t-1), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t-1)
- **\lambda_y** = Adjustment factor used in year (y) to calculate Initial (y) and Interim (y-1) C-CPI-U indexes published in year (y); \(\lambda_y = 1\) for C-CPI-U indexes published in 2002.