Communications

Estimating lost future earnings using the new worklife tables

George C. Alter and William E. Becker

Since the 1982 publication of the Bureau of Labor Statistics updated worklife tables, articles have appeared in the *Monthly Labor Review* and several legal journals regarding the use of such tables in liability proceedings.[1] As stated in these articles expert witnesses in wrongful death and injury litigation are interested primarily in using the increment-decrement worklife tables to find the expected number of years an individual would have been active in the work force had an injury or death not occurred. This expected worklife is then used to calculate the present value of "expected" earnings lost between the date of death or injury and the date of expected final separation from the work force.

It will be shown here that such methods do not yield a mathematically defensible expectation of future earnings, because the sum of earnings over the expected worklife need not equal the sum of expected yearly earnings over life. A model based on the increment-decrement worklife table is developed for calculations of expected earnings in each year of possible life. This model is then modified to obtain the discounted present value of expected future earnings. The final section of this article presents our calculations of expected earnings for representative individuals who die prior to age 85, and compares them with those reported by David Nelson and Kenneth Boudreaux in past issues of the *Review*.[2]

Expected earnings

It is a simple exercise to show that the sum of earnings over expected worklife need not equal the sum of expected yearly earnings over life. For instance, assume that a cohort of 1,000 people are initially active in the work force but, at the end of the first year, 400 become inactive. Similarly, in the second and third years, 300 become inactive at the end of each year. The expected worklife for this hypothetical cohort is 1.9 years. If individual earnings in each subsequent year are projected to be \$25,000, \$30,000, and \$35,000, then, using current techniques, an expert witness would conclude that expected earnings are \$52,000 (= \$25,000 + 0.9 × \$30,000), ignoring discounting and other adjustments. But such a calculation overlooks the interaction of the probability of being active in each year and the earnings which are projected for the year. The true mathematical mean, or expected earnings, is \$53,500 (= 0.4 × \$25,000 + 0.3 × \$55,000 + 0.3 × \$90,000).

For pedagogical ease, the above example assumes that the hypothetical cohort of 1,000 remains alive for all 3 years. It does not allow for both movement into and out of the work force. These complications affect the calculation of expected income. Using all the information now available in the increment-decrement worklife tables, the true mathematical expected earnings of an active individual at age x can be derived in the following manner.

Let q_x represent the probability (or more precisely, the relative frequency) of death in the year following exact age x. Let l_x represent the number of survivors at age x. At each age, survivors can be divided into those who are active in the work force and those who are not. In addition, at each age, a survivor who is active may stay active or leave the work force, while someone who is inactive may stay inactive or move into the work force. Let the four relevant probabilities (or relative frequencies) for work force transition be represented as follows:

${}^{I}p_x^{A}$ = the probability that someone who is inactive at age x will be active at age $x + 1$;

${}^{I}p_x^{I}$ = the probability that someone who is inactive at age x will be inactive at age $x + 1$;

${}^{A}p_x^{I}$ = the probability that someone who is active at age x will be inactive at age $x + 1$; and,

${}^{A}p_x^{A}$ = the probability that someone who is active at age x will be active at age $x + 1$

The above transitional probabilities are conditional on survival from age x to age $x + 1$. Thus:

$${}^{I}p_x^{A} + {}^{I}p_x^{I} = 1, \text{ and } {}^{A}p_x^{I} + {}^{A}p_x^{A} = 1$$

Assuming that the probability of death and the probabilities of transition between work force states are independent, the

George C. Alter is an assistant professor of history and William E. Becker is a professor of economics at Indiana University, Bloomington.

number of inactive survivors at age $x + 1$ (that is, ${}^{I}l_{x+1}$) and the number of active survivors at age $x + 1$ (${}^{A}l_{x+1}$) can now be defined as:

$${}^{I}l_{x+1} = (1 - q_x)\,({}^{I}l_x\,{}^{I}p_x^{I} + {}^{A}l_x\,{}^{A}p_x^{I});\ \text{and}$$

$${}^{A}l_{x+1} = (1 - q_x)\,({}^{I}l_x\,{}^{I}p_x^{A} + {}^{A}l_x\,{}^{A}p_x^{A})$$

where $l_x = {}^{I}l_x + {}^{A}l_x$, and $l_{x+1} = l_x\,(1 - q_x)$.

As in the published increment-decrement worklife tables, these formulas yield:

$$\text{Expected worklife for persons active at age } x = (1/{}^{A}l_x) \sum_{n=0}^{M} [(1 - q_{x+n})\,({}^{A}l_{x+n}\,{}^{A}p_{x+n}^{A} + 0.5\,{}^{A}l_{x+n}\,{}^{A}p_{x+n}^{I} + 0.5\,{}^{I}l_{x+n}\,{}^{I}p_{x+n}^{A}) + 0.5\,{}^{A}l_{x+n}q_{x+n}]$$

where M is the number of ages remaining after age x until the cohort is extinguished.

The above formula for expected worklife is based on a cohort that dies out over $x + M + 1$ years. At age x, for each of the remaining $M + 1$ years, there are four terms over which yearly summation takes place. The first three terms refer to persons who survive to the next year of age. Among these survivors there are those who are active at the start of the year and stay active for a full year. For this group, $(1 - q_{x+n})({}^{A}l_{x+n}\,{}^{A}p_{x+n}^{A})$ is the total number of active years accumulated between ages $x + n$ and $x + n + 1$. Persons who survive the year, but move from active to inactive or from inactive to active status, are assumed to be active for one-half year. Thus $(1 - q_{x+n})(0.5\,{}^{A}l_{x+n}\,{}^{A}p_{x+n}^{I})$ and $(1 - q_{x+n})(0.5\,{}^{I}l_{x+n}\,{}^{I}p_{x+n}^{A})$ are the total numbers of active years accumulated in year $x + n$ by individuals who live to age $x + n + 1$, and who make midyear work force transitions from either active to inactive or inactive to active status, respectively. Persons who were active at the beginning of the year and die in the interval are also considered active for one-half year. Thus, $0.5\,{}^{A}l_{x+n}q_{x+n}$ is the total number of active years between years $x + n$ and $x + n + 1$ for individuals who are assumed to die at age $x + n + 0.5$.

Unlike the simpler mortality tables, the increment-decrement model poses an added complication in the formulation of expected worklife which has implications for calculating expected earnings: The values for survivors by age and work force status depend upon the age at which one begins the computations and the distribution of persons by work force status at that age. In a mortality table, any arbitrary value for l_0 will yield the same expectation of life for each successive age. In the increment-decrement table, one must set either the active or inactive population to zero at the starting age. For example, the expected working life for persons inactive at age 16 is computed by setting ${}^{I}l_{16} =$ 1,000 and ${}^{A}l_{16} = 0$. The associated ${}^{I}l_x$ and ${}^{A}l_x$ can then be computed from these two initial values. If one starts at age 17, or calculates the table for persons out of the work force at age 16, all of the ${}^{I}l_x$ and ${}^{A}l_x$ values will change.

Expected earnings at age x are calculated by introducing annual earnings. Let total annual earnings in year $x (y_x)$ be paid in two equal biannual payments. The payments to persons changing work force status during a year can be approximated by assuming that a person who becomes inactive or dies is active for the first half of the year, and that a person who is inactive and becomes active has earnings in the last half of the year. Under these conditions:

$$\text{Expected earnings for active person at age } x = (1/{}^{A}l_x) \sum_{n=0}^{M} [(1 - q_{x+n})\,({}^{A}l_{x+n}\,{}^{A}p_{x+n}^{A}y_{x+n} + 0.5\,{}^{A}l_{x+n}\,{}^{A}p_{x+n}^{I}y_{x+n} + 0.5\,{}^{I}l_{x+n}\,{}^{I}p^{A}_{x+n}y_{x+n}) + 0.5\,{}^{A}l_{x+n}q_{x+n}y_{x+n}]$$

As does the formula for expected worklife, this formula for expected earnings describes four groups who work (or, more precisely, are active) for different portions of the year between ages $x + n$ and $x + n + 1$. Years of work, however, are now evaluated in terms of total dollars earned by each of the four groups.

The above formula for calculating expected earnings involves an assumption either that time has no value, or that productivity and inflation gains are exactly offset by the market rate of interest. While some expert witnesses still advocate the use of such a "total offset method,"[3] courts today will accept the discounting of future earnings to reflect the net time value of money.[4] The expected earnings equation can be modified to accommodate discounting by defining either a continuous compounding rate (r) or its annual discount rate equivalent (d), that is, $1 + d)^{-n} = e^{nr}$. (For instance, if the annual rate of discount is 11 percent, its continuous compounding equivalent is 10.44 percent.) The present value of expected earnings for an active person at age x, in continuous discounting form is:

$$(1/{}^{A}l_x) \sum_{n=0}^{M} \{(1 - q_{x+n})\,[0.5\,{}^{A}l_{x+n}\,{}^{A}p^{A}{}_{x+n}y_{x+n}(e^{(n+.5)r} + e^{(n+1)r}) + 0.5\,{}^{A}l_{x+n}\,{}^{A}p^{I}{}_{x+n}y_{x+n}e^{(n+.5)r} + 0.5\,{}^{I}l_{x+n}\,{}^{I}p^{A}{}_{x+n}y_{x+n}e^{(n+1)r}] + 0.5\,{}^{A}l_{x+n}q_{x+n}y_{x+n}e^{(n+1)r}\}$$

Corresponding expressions for the expected worklife, expected earnings, and present value of expected earnings for persons inactive at age x can be derived in a similar way.

Calculation procedures

In her comment on Boudreaux's and Nelson's methods for adjusting the worklife tables to estimate lost earnings, Shirley Smith notes that "frequently, economists want to look past the lifetime-worklife expectancy figure to study the timing of the potential earnings stream." Here we argue that in the calculation of expected lost earnings it is not sufficient to know the "median number of years until final separation," as defined by Nelson; to adjust this figure by assuming that activity is evenly spread over the entire period until retirement, as suggested by Boudreaux; or to know any other single number that represents the possible length of time that a person will be active. A true mathematical expectation of lost earnings requires knowledge of the timing of probable activity and of the potential (nominal or discounted) earnings during the period of probable activity.

Because the timing of probable activity is sensitive to both the initial work force status and the age of an individual, our development of the true mathematical expected earnings, unlike the approaches of Nelson and Boudreaux, emphasizes an active or inactive starting point. To assess the consequences of this distinction, consider the example provided by Boudreaux. A man age 30 with annual earnings of $25,000 (using a current market interest rate of 11 percent and an annual earnings increase of 4.5 percent) has a present value of "expected" earnings of $332,913, by the worklife table estimate of 29.2 years of remaining worklife for the entire population. Using Nelson's 31.5 years to final separation criterion, the present value of "expected" earnings is $341,857. Boudreaux's 7.3-percentage reduction criterion drops this estimate to $316,901. However, our calculations show that the true mathematical present value of expected earnings for an active man at age 30 is $319,397, and for an inactive man at age 30 it is $273,535.

In some cases, one might wish to ignore initial work force status. A weighted average of our active and inactive estimates can be obtained by using the proportions of men active and inactive at the initial age. In the above example of a man at age 30, this average present value of expected earnings is $316,502, which compares favorably to Boudreaux's estimate of $316,901. Given the ease of using Boudreaux's adjustment method, one might question the practical value of using our more complicated true mathematical expectation method.

Unfortunately, Boudreaux's approximation is close to the true mathematical expectation only for younger men. His assumption that inactivity is spread evenly over the entire period until retirement is inappropriate at older ages, when the proportion that are inactive rises rapidly. For a younger person, changes in expected earnings caused by increasing probabilities of inactivity later in life are mitigated by high

Table 1. Probabilities of work force transitions for men, by age and work force status

Age	Rate of —		Status in next year for survivors —				Age	Rate of —		Status in next year for survivors —			
	Survival	Death	Active at age *x*		Inactive at age *x*			Survival	Death	Active at age *x*		Inactive at age *x*	
			Active	Inactive	Inactive	Active				Active	Inactive	Inactive	Active
16	0.99870	0.00130	0.73633	0.26367	0.70348	0.29652	51	0.99090	0.00910	0.97211	0.02789	0.84637	0.15363
17	0.99848	0.00152	0.83598	0.16402	0.73269	0.26731	52	0.99005	0.00995	0.97115	0.02885	0.86455	0.13545
18	0.99832	0.00168	0.82814	0.17186	0.68197	0.31803	53	0.98919	0.01081	0.96918	0.03082	0.88187	0.11813
19	0.99821	0.00179	0.82234	0.17766	0.63228	0.36772	54	0.98829	0.01171	0.96582	0.03418	0.89427	0.10573
20	0.99810	0.00190	0.86112	0.13888	0.60466	0.39534	55	0.98737	0.01263	0.96144	0.03856	0.89962	0.10038
							56	0.98634	0.01366	0.95790	0.04210	0.90767	0.09233
21	0.99800	0.00200	0.88646	0.11354	0.59445	0.40555	57	0.98509	0.01491	0.94989	0.05011	0.91160	0.08840
22	0.99793	0.00207	0.90865	0.09135	0.59370	0.40630	58	0.98353	0.01647	0.93407	0.06593	0.91543	0.08457
23	0.99792	0.00208	0.92901	0.07099	0.58156	0.41844	59	0.98174	0.01826	0.91500	0.08500	0.92765	0.07235
24	0.99795	0.00205	0.94483	0.05517	0.57096	0.42904	60	0.97974	0.02026	0.88540	0.11460	0.93765	0.06235
25	0.99799	0.00201	0.95668	0.04332	0.56366	0.43634							
26	0.99803	0.00197	0.96503	0.03497	0.56330	0.43670	61	0.97769	0.02231	0.85444	0.14556	0.94056	0.05944
27	0.99807	0.00193	0.97052	0.02948	0.56318	0.43682	62	0.97571	0.02429	0.82607	0.17393	0.94039	0.05961
28	0.99810	0.00190	0.97424	0.02576	0.56642	0.43358	63	0.97389	0.02611	0.79895	0.20105	0.94124	0.05876
29	0.99812	0.00188	0.97614	0.02386	0.58214	0.41786	64	0.97217	0.02783	0.76808	0.23192	0.94353	0.05647
30	0.99814	0.00186	0.97908	0.02092	0.60012	0.39988	65	0.97042	0.02958	0.73537	0.26463	0.94273	0.05727
							66	0.96846	0.03154	0.71640	0.28360	0.94702	0.05298
31	0.99814	0.00186	0.98082	0.01918	0.61932	0.38068	67	0.96612	0.03388	0.70816	0.29184	0.95150	0.04850
32	0.99811	0.00189	0.98212	0.01788	0.65411	0.34589	68	0.96325	0.03675	0.69670	0.30330	0.95379	0.04621
33	0.99803	0.00197	0.98295	0.01705	0.67299	0.32701	69	0.95987	0.04013	0.69525	0.30475	0.95789	0.04211
34	0.99792	0.00208	0.98414	0.01586	0.68539	0.31461	70	0.95623	0.04377	0.68951	0.31049	0.96207	0.03793
35	0.99778	0.00222	0.98545	0.01455	0.70813	0.29187							
36	0.99761	0.00239	0.98600	0.01400	0.73233	0.26767	71	0.95239	0.04761	0.68370	0.31630	0.96371	0.03629
37	0.99743	0.00257	0.98645	0.01355	0.75924	0.24076	72	0.94816	0.05184	0.67571	0.32429	0.96540	0.03460
38	0.99723	0.00277	0.98710	0.01290	0.75448	0.24552	73	0.94351	0.05649	0.66528	0.33472	0.96817	0.03183
39	0.99700	0.00300	0.98629	0.01371	0.75752	0.24248	74	0.93844	0.06156	0.66368	0.33632	0.97240	0.02760
40	0.99675	0.00325	0.98477	0.01523	0.75835	0.24165	75	0.93297	0.06703	0.64235	0.35765	0.96161	0.03839
							76	0.92714	0.07286	0.46071	0.53929	0.56857	0.43143
41	0.99645	0.00355	0.98388	0.01612	0.75415	0.24585	77	0.92100	0.07900	0.00000	1.00000	1.00000	0.00000
42	0.99612	0.00388	0.98391	0.01609	0.75912	0.24088	78	0.91461	0.08539	0.00000	1.00000	1.00000	0.00000
43	0.99575	0.00425	0.98295	0.01705	0.76601	0.23399	79	0.90805	0.09195	0.00000	1.00000	1.00000	0.00000
44	0.99533	0.00467	0.98170	0.01830	0.76927	0.23073	80	0.90148	0.09852	0.00000	1.00000	1.00000	0.00000
45	0.99488	0.00512	0.98111	0.01889	0.77840	0.22160							
46	0.99438	0.00562	0.98059	0.01941	0.78560	0.21440	81	0.89513	0.10487	0.00000	1.00000	1.00000	0.00000
47	0.99382	0.00618	0.97837	0.02163	0.81025	0.18975	82	0.88943	0.11057	0.00000	1.00000	1.00000	0.00000
48	0.99319	0.00681	0.97601	0.02399	0.82041	0.17959	83	0.88503	0.11497	0.00000	1.00000	1.00000	0.00000
49	0.99249	0.00751	0.97529	0.02471	0.83038	0.16962	84	0.88298	0.11702	0.00000	1.00000	1.00000	0.00000
50	0.99172	0.00828	0.97388	0.02612	0.83728	0.16272	85	0.00000	1.00000	0.00000	1.00000	1.00000	0.00000

discounting of expected earnings in distant years. For older people, the mitigating effect of discounting is not present. Thus, for a man age 45 with the same earnings stream used above, Nelson's and Boudreaux's methods of estimating the present value of potential earnings yield $256,044 and $242,217, respectively. Our mathematical expectations are $236,626 for an active man, $155,310 for an inactive man, and $231,325 for the weighted average of active and inactive persons.

OUR METHOD OF CALCULATION requires two modifications of the increment-decrement worklife tables published by BLS.[5] First, the probabilities of transition into and out of the work force at each age must be converted to probabilities that are conditional on survival. Second, conditional probabilities of transition between active and inactive work force status must be added at age 76 to close the table. The relevant probabilities of transition are provided in table 1. A computer program for calculating the present value of expected earnings based on these transitional probabilities is available from the authors. □

FOOTNOTES

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[1] See Shirley J. Smith, "New worklife estimates reflect changing profile of labor force," *Monthly Labor Review,* March 1982, pp. 15–20; Shirley J. Smith, "Using the appropriate worklife estimate in court proceedings," *Monthly Labor Review,* October 1983, pp. 31–32; David M. Nelson, "The use of worklife tables in estimates of lost earning capacity," *Monthly Labor Review,* April 1982, pp. 30–31; Kenneth J. Boudreaux, "A further adjustment needed to estimate lost earning capacity," *Monthly Labor Review,* October 1983, pp. 30–31; Gerald P. Martin, "New Worklife Expectancy Study Favors the Defense," *For the Defense,* March 1983, pp. 3–4; and Melvin Borland and Robert Palsinelli, "Equalizing Wage Differences, Worklife Expectancy Tables and Wrongful Death Litigation," *Trial Lawyer's Guide,* Summer 1983, pp. 213–19.

[2] See Nelson, "The use of worklife tables"; and Boudreaux, "A further adjustment needed."

[3] See Michael T. Brady, "Inflation, Productivity, and the Total Offset Method of Calculating Damages for Lost Future Earnings," *The University of Chicago Law Review,* Fall 1982, pp. 93–122.

[4] Edwin B. Wainscott, "Computation of Lost Future Earnings in Personal Injury and Wrongful Death Action," *Indiana Law Review,* Summer 1978, pp. 648–91.

[5] Shirley J. Smith, *Tables of Working Life: The Increment-Decrement Model,* Bulletin 2135 (Bureau of Labor Statistics, November 1982), pp. 1–65.

Estimating lost future earnings using the new worklife tables: a comment

SHIRLEY J. SMITH

George C. Alter and William E. Becker provide yet another valuable contribution to the ongoing dialog on estimates of lost earnings due to wrongful injury or death. The authors have written a computer program replicating the BLS worklife model, expanding it to manipulate earnings projections by age, and allowing selection of a discount rate to estimate the present value of those lost future earnings.

Shirley J. Smith is a demographic statistician in the Division of Data Development and Users' Services, Bureau of Labor Statistics.

I have no reservations about the worklife component of their model, which is nearly identical to our own. They do use a different closure procedure (for persons age 75 and over) than was employed in the BLS 1977 estimates. Our closure procedure has now been modified for better internal consistency. Alter and Becker also redefine transition rates, making them conditional on survival. Mortality is factored into their model somewhat differently than it is in the BLS procedure. However this is a difference of form rather than substance, the results of the two techniques being virtually identical.

The authors' primary purpose in replicating the BLS model is to draw out some of its unpublished findings having to do with the age-by-age timing of forgone labor force involvement for persons of a known labor force status at the time of injury. Readers involved in liability claims have expressed considerable interest in this type of data. As I noted in an earlier issue of the *Review,*[1] it is possible to derive population-based estimates of worklife during each age from the published tables. Alter and Becker reassert the need for estimates specific to the labor force status of the claimant.

The BLS model produces such estimates, but we have not found it feasible to publish them as part of the Bureau's worklife bulletin. (Status-specific estimates by sex, for 60 initial ages, would add at least 120 pages of tables to an already lengthy publication.) Nevertheless, we have taken note of the demand for such estimates.

Our next worklife publication is slated to include tables not only by sex, but also by race and education. This expansion of the output from 2 to 12 reference groups will require a cutback in the number of data items published for each group. We hope to be able to retain the estimates most useful for analysis of lost earnings. In addition, we hope to be able to provide on request some of the unpublished findings of the model, such as initial-status-specific worklife expectancies within each age, in some form certifiable for use in court.

The Alter and Becker model estimates lost earnings under the assumption of biannual payments over the claimant's natural lifetime. Doing so entails the use of very detailed worklife data (specifically, estimates of labor force entries and exits at each subsequent age, for a cohort of a given initial age and labor force status). We may also attempt to provide counts of these flows in the unpublished tables, to facilitate this type of computation. □

FOOTNOTE

[1] Shirley J. Smith, "Using the appropriate worklife estimate in court proceedings," *Monthly Labor Review,* October 1983, pp. 31–32.