

A hand holding a blue credit card, surrounded by numerous colorful circular icons representing various lifestyle and financial concepts like travel, health, technology, and shopping.

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approximately. We also find that a monthly chained, Laspeyres index shows substantial drift.

Background

In August 2002, the CPI program began publishing the C-CPI-U. The C-CPI-U was implemented to address concerns that the official Consumer Price Index for All Urban Consumers (CPI-U) suffered from upper-level substitution bias, given that it is calculated using a Laspeyres (technically, a Lowe) formula. As described by Robert Cage, John Greenlees, and Patrick Jackman in a 2003 conference paper, the C-CPI-U “employs a superlative Törnqvist formula and utilizes expenditure data in adjacent time periods in order to reflect the effect of any substitution that consumers make across item categories in response to changes in relative prices.”^[3]

The Boskin Commission report submitted to the U.S. Senate Finance Committee in December 1996 estimated that the official CPI-U was biased upward by 0.15 percentage points because of consumer substitution *across* item categories (upper-level substitution) and that it was biased upward by 0.25 percentage points because of consumer substitution *within* item categories (lower-level substitution).^[4] Upper-level substitution refers to substitution between these categories. Lower-level substitution refers to substitution within any of 243 item categories at the lowest level of aggregation in the CPI classification scheme.

According to the Boskin Commission, “Substitution bias occurs because a fixed market basket fails to reflect the fact that consumers substitute relatively less for more expensive goods when relative prices change.”^[5] For the CPI-U, U.S. city average, all-items index, an example of upper-level substitution bias is when consumers substitute chicken for steak or beer for wine. An example of lower-level substitution bias is when consumers substitute low-fat milk for whole milk. “Levels” refer to the placement of categories of goods and services within the CPI aggregation structure.

The CPI-U is based on prices collected from monthly surveys—as well as alternative data sources for some item categories—and on expenditures collected from the Consumer Expenditure Surveys administered monthly during 24-month intervals between biennial expenditure-weight updates. The CPI-U thus measures the changes in prices for a market basket of goods and services that remains unchanged between biennial updates (specific items may change because of item replacement and sample rotation), with quantity data captured implicitly via lagged expenditures. Biennial updates also mean that price and quantity data are “chained” at each biennial “rebase” using a Lowe formula. A geometric mean formula is used in the official CPI-U to address lower-level substitution bias.^[6]

In the C-CPI-U, price and quantity data are chained each month using a Törnqvist formula that calculates an index incorporating both monthly price changes *and* monthly expenditure changes across item categories. Because of the amount of time that is necessary to process expenditure data, C-CPI-U data are released first in preliminary form. Three months later, they are released again in their first interim form. Six months later, they are released again in their second interim form. Nine months later, they are released again in their third interim form. In these three quarterly intervals following release in preliminary form, BLS uses a constant elasticity-of-substitution formula for the initial estimates. Twelve months later, exactly a year after the preliminary release, a Törnqvist formula is used to produce the final estimates.^[7]

By addressing potential substitution bias, the C-CPI-U is designed to be a closer approximation to a “true” cost-of-living index (COLI). Given that the implicitly derived quantities that consumers purchase are taken to be the optimal quantities on the basis of their income or wealth, COLIs, originally conceived by A. A. Konüs, are based on the economic theory of consumer demand.[8] Under the standard framework, consumers maximize utility and minimize cost, given a level of wealth or income. More specifically, under “Hicksian” demand, consumers minimize the expenditures necessary to attain a given standard of living—that is, they seek to maximize their utility at the minimal cost.[9] When relative prices change, the relative affordability of different goods and services changes, which affects consumers’ standard of living. The COLI is designed to measure the compensation necessary to afford the original standard of living. Consumer inflation is thus calculated as the ratio of minimum expenditures in two periods necessary to achieve the same standard of living. In the following equation, which illustrates this ratio, ${}_iIX_{(0,t)}$ is the index that represents consumer inflation over the set of i goods from period 0 to t , with the basket of goods, and thus the standard of living, remaining constant; ${}_iP_0$ is the price of good i in period 0, ${}_iQ_0$ is the quantity of good i in period 0, ${}_iP_t$ is the price of good i in period t , and ${}_iQ_t$ is the quantity of good i in period t :

$${}_iIX_{(0,t)} = \frac{\min \sum {}_iP_t \times {}_iQ_t}{\sum {}_iP_0 \times {}_iQ_0}.$$

However, the choice between a chained or fixed-base index presents a tradeoff between representativeness and transitivity. Indexes are “representative” when they accurately represent not only the price trend of a “market basket” of goods and services but also the composition of the “market basket” as it changes over time. As the 2004 *Consumer Price Index Manual* explains,

The main problem with the use of fixed base Laspeyres indices is that the period 0 fixed basket of commodities that is being priced out in period t can often be quite different from the period t basket. Thus, if there are systematic trends in at least some of the prices and quantities in the index basket, the fixed base Laspeyres price index $P_L(p^0, p^t, q^0, q^t)$ can be quite different from the corresponding fixed base Paasche price index, $P_P(p^0, p^t, q^0, q^t)$. This means that both indices are likely to be an inadequate representation of the movement in average prices over the time period under consideration.[10]

Christian G. Ehemann defines an index number formula as transitive if chaining from $t = 1$ to $t = 2$ and then from $t = 2$ to $t = 3$ yields the same index value for the index at $t = 3$ as the direct index from $t = 1$ to $t = 3$: $I(p^0, p^1, q^0, q^1) * I(p^1, p^2, q^1, q^2) * I(p^2, p^3, q^2, q^3) = I(p^0, p^3, q^0, q^3)$. If this condition is not met, the index is nontransitive. The monthly chained Törnqvist formula used for the C-CPI-U is nontransitive and, as a result, is subject to drift.[11] Frequent weight updates may lead to increased drift in nontransitive indexes. The official index—the Consumer Price Index for All Urban Consumers (CPI-U), U.S. city average, all items—is a chained index in the sense that weights are updated biennially, but it also is a fixed-base index when it is calculated between expenditure-weight updates. The C-CPI-U uses monthly chaining with monthly weight updates (when finalized) and, in principle, may be at greater risk of measurable chain drift.

We now clarify some of the terminology associated with price-index methodology. A price index can be either fixed base or chained, and it can be either bilateral or multilateral. (In this article, we use the terms “fixed base” and “direct” interchangeably.) We rely on the *Consumer Price Index Manual* for precise definitions of these terms. According to the *Manual*, the word “bilateral” refers to the assumption that a function P , or a price index number formula, $P(p^0, p^1, q^0, q^1)$, depends only on the data pertaining to the two situations or periods being compared. For example, P is regarded as a function of the two sets of price and quantity vectors, p^0, p^1, q^0, q^1 , “that are to be

aggregated into a single number that summarizes the overall change in the n price ratios $p^1(1)/p^0(1), \dots, p^1(n)/p^0(n)$.”[12] “Multilateral index number theory,” however, “refers to the case where there are more than two situations whose prices and quantities need to be aggregated.”[13]

On the distinction between a fixed-base index and a chained index, the *Manual* explains that the “chain system measures the change in prices going from one period to a subsequent period using a bilateral index number formula involving the prices and quantities pertaining to the two adjacent periods. These one-period rates of change (the links in the chain) are then cumulated to yield the relative levels of prices over the entire period under consideration.” Consider, for example, a bilateral price index P . The index is “chained” if index calculation generates the following sequence:[14]

$$1, P(p^0, p^1, q^0, q^1), P(p^1, p^2, q^1, q^2).$$

Under a fixed-base system, however, the bilateral index number formula P “simply computes the level of prices in period t relative to the base period 0 as $P(p^0, p^t, q^0, q^t)$.”[15] As such, the fixed-base system generates the following sequence of price levels for periods 0, 1, and 2:

$$1, P(p^0, p^1, q^0, q^1), P(p^0, p^2, q^0, q^2).$$

Empirically, chain drift is often defined as the difference between the chained and fixed-base versions of a price index. Chain drift can occur when expenditure-share weight updates lag short-term price oscillations, distorting trend reversion and leading to nontransitivity in a price index. However, several factors can lead to divergence, including the representativity of the market basket of goods and services in the base period. Our analysis also indicates that choice of base month and seasonal patterns may affect the amount of drift. Gregory Kurtzon refers to divergence resulting from consumer substitution as “good” drift. Divergence resulting from nontransitivity, which can be analyzed with the unity test, is, unambiguously, “bad” drift.[16]

Chaining: theory and practice

In addition to the choice of index number formula, chaining an index addresses substitution bias by providing an approach for incorporating changes in quantity over time. According to F. G. Forsythe and R. F. Fowler, Francois Divisia “put forward the new concept of an index of prices based specifically on the assumption that an index of price changes over a period of time 0 to t should depend not only on prices and quantities at 0 and t but also on the movement of prices and quantities throughout the interval 0 to t . In other words, the index should depend on the path and take account of all the data relating to prices and quantities in the interval.”[17] This idea has its earliest roots in the 19th-century work of Julius Lehr and Alfred Marshall, the latter introducing the principle of chaining as a way of making index formulas more representative of ongoing changes in economic activity.[18] As Forsythe and Fowler write, “Marshall was concerned only with the practical problem of allowing for the introduction of new commodities into an index of prices which he thought would be greatly facilitated if the weights were changed every year and the successive yearly indices linked or chained together by simple multiplication.”[19]

In principle, a Divisia index perfectly captures changes in price and quantity as they occur. Because a continuous time index is not feasible, numerous discrete time-index formulas have been developed, such as the Lowe and Törnqvist formulas used for the official CPI-U and the C-CPI-U, respectively. These formulas are weighted versions

of arithmetic and geometric means. The economic theory of consumer demand provides the theoretical connection between a Divisia index and a cost-of-living index.[20]

Forsyth and Fowler noted a tradeoff between maintaining base-period weights and using chained indexes. Chained indexes have the potential for drift but represent recent expenditure patterns. Fixed-base indexes have no drift, but the base-period consumption basket becomes less representative as consumption patterns change. Forsyth and Fowler argued that the benefits of representativity generally outweigh the relatively small drift in a Fisher index.[21]

The C-CPI-U combines a Törnqvist formula with monthly chaining to provide a measure of consumer inflation that reflects monthly purchases in response to monthly changes in price. The C-CPI-U is not only a measure of consumer inflation but a measure of revealed preference. One major problem that can emerge with chaining, however, is a violation of transitivity, which is one of several axiomatic properties that are desirable for indexes to have.[22] An index is transitive if long-term price change calculated with updated quantity data in each incremental period is equal to long-term price change without frequent updating from a reference period to a comparison period.

More precisely, Ehemann defines an index number formula as “transitive if chaining from $t = 1$ to $t = 2$ and then from $t = 2$ to $t = 3$ gives the same index value for the index at $t = 3$ as the direct index from $t = 1$ to $t = 3$ ”:[23]

$$I(p^1, p^2, q^1, q^2) * I(p^2, p^3, q^2, q^3) = I(p^1, p^3, q^1, q^3).$$

A violation of the transitivity axiom could indicate that chain drift is a problem with the index choice. If indexes are transitive, they do not exhibit chain drift. Conversely, indexes that are not transitive exhibit chain drift. Indexes that exhibit chain drift diverge from their “true” long-term trend. As a result, a chained index makes an index more representative of market-basket composition (what consumers are purchasing), but often at the expense of providing an inaccurate measure of long-term inflation. Chaining can thus involve a tradeoff between representativity and transitivity.[24] For any price index, the central question is whether, in any given case, the benefits of representativity are outweighed by the cost of nontransitivity (i.e., chain drift).

Bohdan J. Szulc identified “bounce” behavior in prices resulting from such factors as seasonality or price wars as a major source of drift. “Bounce” is more likely to cause drift if a “peak” or “trough” diverges from the long-term trend.[25] Thus, following Kurtzon, we can imagine the prices of two goods with equal expenditure shares “bouncing” between two periods, with the price of each good either \$1 or \$2 in every period, generating a price relative of 2 or 1/2. In this situation, there is no long-term inflation, but, as Kurtzon states, “this index relative would give an inflation rate of $1/2(2 + 1/2) = 1.25$, or 25 [percent] inflation every period.”[26] Price oscillation may reflect trend reversion as competition prevents sellers from charging prices that deviate from their long-term trends as consumer substitution “puts downward pressure on prices that are comparatively high, and it may also put upward pressure on prices that are unusually low by making them attract high sales.”[27] Lorraine Ivancic, Kevin J. Fox, and W. Erwin Diewert provide a similar explanation of drift:

As a result, it is not necessarily the case that prices and quantities in adjacent periods are more similar than those in periods which are not adjacent when subannual data is used. In particular, when an item goes off sale and prices return to their “regular” price, we would expect that the use of a chained superlative index would simply (more or less exactly) reverse the previous downward movement in the index and take us back to the “regular” price level. *However, in practice this may not happen because when an item comes off sale,*

consumers are likely to purchase less than the “average” quantity of that item for some period of time until their inventories of the item have been depleted. It is only over time that the quantities sold will gradually recover to their pre-sale levels [emphasis added]. If prices do not change over the post-sale period, all reasonable indexes will show no price change over these “regular” price periods. Thus, under these conditions (i.e., where sales are apparent), chained superlative indexes will tend to have a *downward drift* when compared to their fixed base counterparts.[28]

Methodology: test for chain drift

Because chaining has the potential to make a price index more representative of consumption behavior and marketplace activity, it is of considerable interest to determine if the benefits of a more representative index are outweighed by any “drift” that causes the index to diverge from long-term price trends. In this study, we used several tests to determine the extent to which the C-CPI-U exhibits undesirable drift. We first address the second stage of index aggregation in which component indexes—which are constructed from aggregations of price observations pertaining to specific geographic areas and item categories—are combined. We then focus on subaggregate indexes at the expenditure-class level. In the CPI aggregation structure, there are 70 expenditure-class indexes, which are one level of aggregation above the “lower level” 243 item-level indexes, which, combined with 32 index areas, form 7,776 item-area, lower-level, index-area “cells,” the building blocks of CPI index construction. Item-area component indexes use fixed-weight formulas, but, like all bilateral indexes, they are effectively chained at each weight update. Every time the relative weights of price quotes change within a cell, which can occur because of subsampling and partial cell sample rotation, drift can occur.

Unity test

C. M. Walsh introduced the unity test as one method for detecting drift. As the *Consumer Price Index Manual* explains, this test uses “the bilateral index formula $P(p^0, p^1, q^0, q^1)$ to calculate the change in prices going from period 0 to 1.” It then uses “the same formula evaluated at the data corresponding to periods 1 and 2, $P(p^1, p^2, q^1, q^2)$, to calculate the change in prices going from period 1 to 2” and then uses “ $P(p^{T-1}, p^T, q^{T-1}, q^T)$ to calculate the change in prices going from period $T - 1$ to T .” The unity test then “introduce[s] an artificial period $T + 1$ that has exactly the price and quantity of the initial period 0 and use[s] $P(p^T, p^0, q^T, q^0)$ to calculate the change in prices going from period T to 0.” In the last step, “multiply all of these indices together.”[29]

The unity test thus chains index formulas from point 0 to point T and sets the price and quantity in period $T + 1$ equal to the price and quantity observed in period 0. In the next step, $I(p^0, p^T, q^0, q^T)$ is used to calculate the change in prices from period T to period 0. In the final step, multiply all of the chained indexes together. According to the *Consumer Price Index Manual*, if the result is an index value equal to the index value in period 0, “we end up where we started, [and] the product of all of these indices should ideally be one.”[30] Mathematically, we have the following:

$$\text{Drift}_{\text{Unity},t} = I(p^0, p^1, q^0, q^1) * I(p^1, p^2, q^1, q^2) * \dots * I(p^t, p^0, q^t, q^0).$$

This procedure computes the chained price index until month t and then appends the initial set of prices and quantities (p^0, q^0) to the end of the series. Because the final period is the same as the first, a fully transitive index should show no change. We produced estimates of drift over the entire series and, iteratively, sequentially conducted this test for each period t after the base period, setting the terminal month indexes and weights equal to

the exact values from the starting month iteratively from December 1999 to November 2017. Thus, in the first run of the test, we conducted an iterative unity test with December 1999 as the starting point, and each successive month up to November 2017, with December 2017 as the terminal month. Then, we ran another iterative unity test, with January 2000 as the starting point, and each successive month up to November 2017, with December 2017 as the terminal month. We repeated this process through November 2017. We also produced unity tests on indexes that are based on bounded price relatives. Monthly price changes in these indexes are capped at 95-percent declines and 2,000-percent increases.

Circularity test

The unity test, or what Diewert called the “multiperiod identity test,” is a special case of the circularity test.^[31] In both cases, the aim is to test whether transitivity holds. If transitivity holds, index formulas generate the same measurement of long-term price change. If it does not, fixed-base formulas and chained-index formulas diverge. Or, stated differently, chained indexes drift. Note that the *Consumer Price Index Manual* recommends the circularity and unity tests not as measures for deciding whether to use fixed-base or chained indexes, but as measures of “how ‘good’ a particular index number formula is.”^[32] Drift is also a reason why the *Manual* recommends chaining for series that have smooth trends. As the *Manual* notes, Alterman, Diewert, and Feenstra

“show that if the logarithmic price ratios $\ln(\frac{p_i^t}{p_i^{t-1}})$ trend linearly with time t and the expenditures shares s_i^t also trend linearly with time, then the Törnqvist index P_T will satisfy the circularity test exactly.”^[33]

The circularity test originates in the work of Harald Westergaard and Irving Fisher and helps “to determine if there are index number formulae that give the same answer when either the fixed base or chain system is used.”^[34] If an index formula yields the same calculation of long-term price change regardless of whether a fixed base or chaining is used, it passes the circularity test. That is, an index number formula that passes the circularity test is transitive. No drift will be detected in the amount of price change calculated by the chained index formula. As explained by Ehemann, “The testing of an index number formula for transitivity by determining whether the chained and direct calculation of the index value are equal is known as a circularity test.”^[35] Mathematically, the test is represented as follows:

$$I(p^0, p^1, q^0, q^1) * I(p^1, p^2, q^1, q^2) = I(p^0, p^2, q^0, q^2).^{[36]}$$

Here, we express drift as the ratio of the chained index relative to the fixed-base index relative in period t :

$$\text{Drift}_{\text{Circularity},t} = \frac{P_{\text{Chained}}}{P_{\text{Fixed base}}}.$$

For the Törnqvist price index, $\text{Drift}_{\text{Circularity},t}$ and $\text{Drift}_{\text{Unity},t}$ are equivalent. Multiplying the Törnqvist chained index by an additional term returning to the base period is equivalent to dividing the chained Törnqvist index by the fixed-base Törnqvist index, because $\text{Torn}(p^t, p^0, q^t, q^0) = \text{Torn}(p^0, p^t, q^0, q^t)^{-1}$.

Multilateral index comparisons

Ivancic et al. developed a rolling-window time version of the Gini, Eltetö, Köves, and Szulc (GEKS) formula originally used for interarea price comparisons.^[37] The full GEKS index is transitive. However, the fully transitive GEKS index must be reestimated every month, in every period, and it produces revisions in previous period estimates. The rolling-window GEKS formula produces a current-period estimate without revising prior months.

Although not fully transitive, the rolling-window GEKS formula produces a chained index with attenuated drift. Once the initial GEKS index is estimated, it is updated through a splicing method. We focus on the results of a mean splice. Ivancic et al. originally used a Fisher formula to make the bilateral comparisons in each element of the GEKS index. Here, as recommended by Diewert and Fox, we use a Törnqvist index in place of the Fisher index to produce GEKS-Törnqvist indexes, also referred to as Caves-Christensen-Diewert-Inklaar (CCDI) indexes.^[38] This also allows us to make a more direct comparison with the C-CPI-U because that index is based on a Törnqvist formula:

$$\text{Drift}_{\text{CCDI},t} = \frac{P_{\text{chained},t}}{\text{CCDI}_t}.$$

For these tests, a value equal to 1 indicates no chain drift, a value less than 1 indicates downward drift, and a value greater than 1 indicates upward drift.

Data

The data used for this article consist of monthly item-area CPI indexes and cost weights from December 1999 to December 2017. This period corresponds to the interval between the 1998 and 2018 geographic area sample revisions. In January 2018, the CPI program implemented a geographic area sample revision. As part of this revision, the number of primary sampling units (PSUs) declined from 87 to 75 and the number of index areas for purposes of index construction declined from 38 to 32. Meanwhile, there are now 243 item categories. The present study avoids the complications associated with reconciling the new geographic design with the geographic area sample that prevailed during the 1999–2017 period. Although changes in item structure occurred during this period, the geographic area sample remained stable in 38 index areas and 87 PSUs, avoiding complications that arise from changes in the item-area index aggregation structure as a result of a new geographic area sample. Although we briefly considered conducting an analysis across the geographic revision implemented in January 2018, we only had 1.5 years of data beyond 2018 at the time of this analysis; the change from 211 to 243 lower-level item categories in the CPI aggregation structure would have added complications that are unrelated to chain drift.

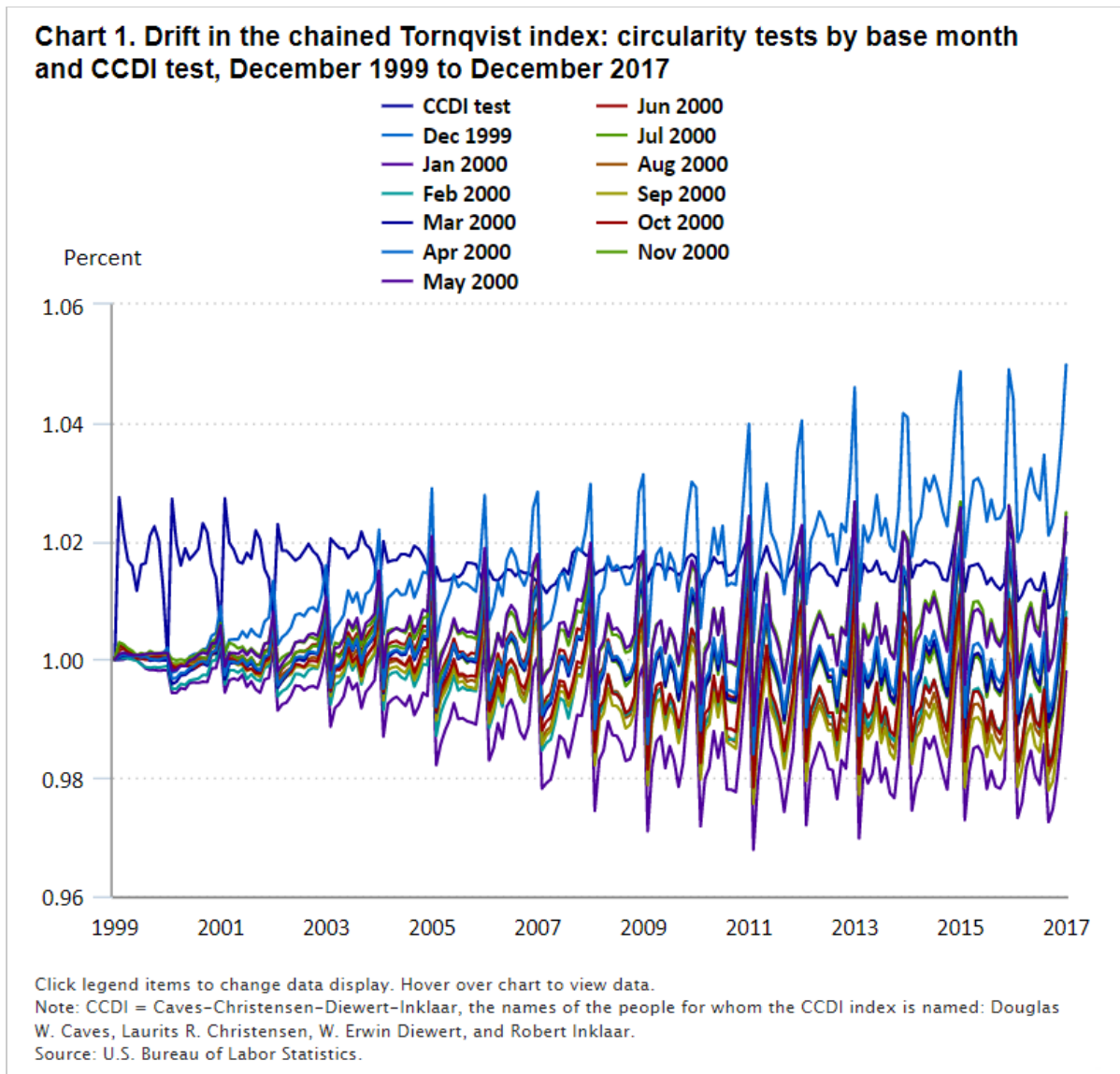
The published C-CPI-U accounts for these structural changes, while the chained Törnqvist and other indexes discussed in this article use a dataset that is based on a harmonized structure, so there are some differences between the published C-CPI-U indexes and the research results presented here. Nevertheless, we view the chained Törnqvist index as analogous to the C-CPI-U; the chained Törnqvist index displays a close relationship to the C-CPI-U. The C-CPI-U shows a 39.5-percent increase from December 1999 to December 2017, while the chained Törnqvist index shows a 39.9-percent increase (an annualized difference of just 0.02 percent).

Results

On the basis of the CCDI test, we find that the all-items monthly chained Törnqvist index displays a small amount of drift: 0.11 percent annually, for a total of 2.1 percent over the 18-year period analyzed in this study. Results from unity and circularity tests show similar levels of drift but vary substantially, depending on the choice of base and end month. Iterative comparisons of different subperiods show that the drift, as measured by the circularity and unity tests, is partly related to seasonality. Multilateral indexes also show small amounts of drift in the monthly chained Törnqvist index. A monthly chained Laspeyres index shows large amounts of upward drift. We show that

drift declines with more infrequent weight updates for both Törnqvist and Laspeyres indexes. At the expenditure-class level, some categories show substantial drift. Unity and circularity tests have nearly equivalent results, with some differences because of rounding. The CCDI test helps mitigate drift in those indexes with large amounts of drift.

Chart 1 summarizes the estimated chain drift in the chained Törnqvist index based on the CCDI and circularity tests.



The CCDI test shows upward chain drift across the entire time span of 1 to 2 percent, depending on the month. The circularity test is quite sensitive to the choice of base month. Of the first 12 months in the series, using December 1999 as a base month implies the most drift, nearly 5.0 percent, while using January 2000 implies the least drift, at 0.2 percent.

Unity test

We relied on a dataset of continuous elementary item-area monthly indexes and weights from December 1999 to December 2017, and we ran iterative calculations of the unity test over the same 1999–2017 period. We systematically varied all base periods, starting with December 1999, and all end periods, ending with December 2017, until we obtained drift estimates based on the unity test for all chronological combinations of base and end months.

Charts 2–5 show the results of this iterative, sequentially run unity test when we used bounded Laspeyres and bounded Törnqvist formulas. Charts 2 and 3 show the per annum percent change in the monthly chained index value from unity (the beginning month in which the index equals 100) to the ending month in each iteration, up to the terminal month of December 2017. Chart 2 displays the results when the Laspeyres formula is used.

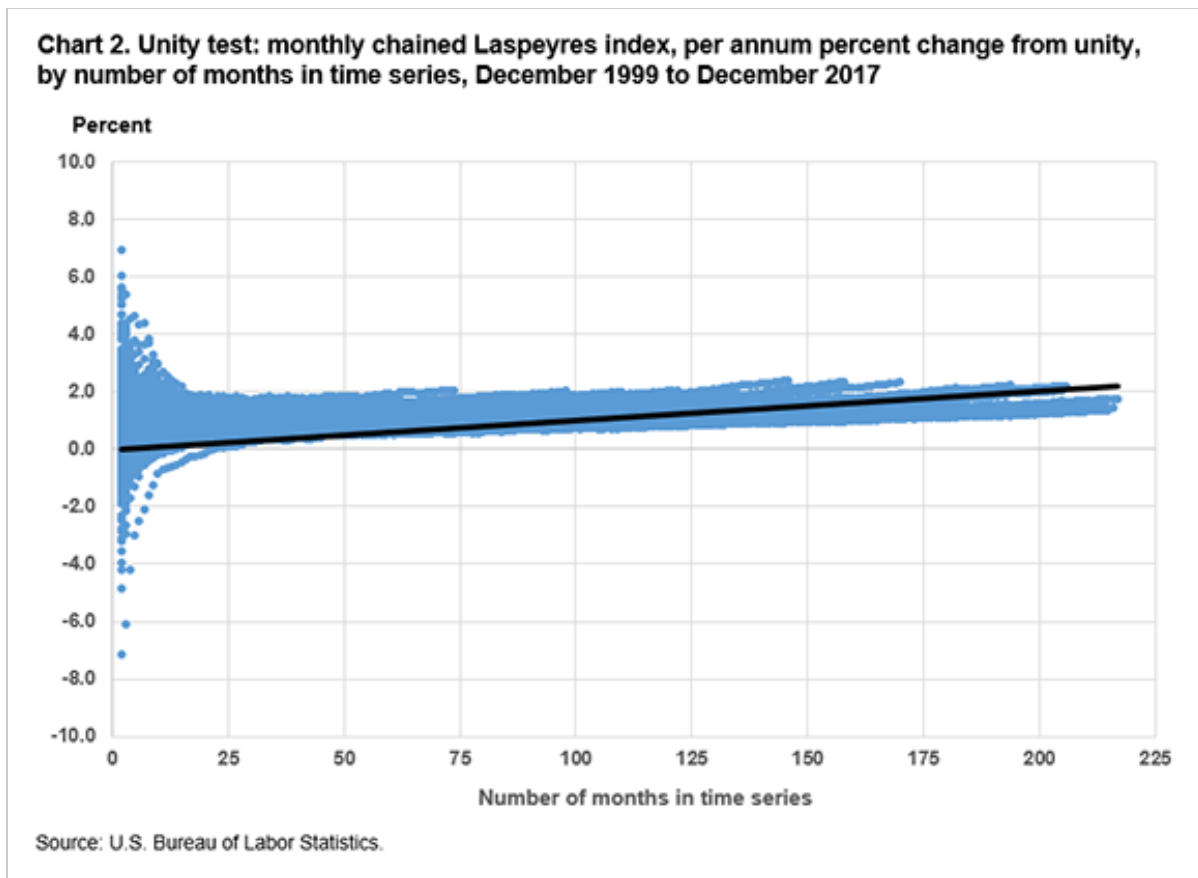
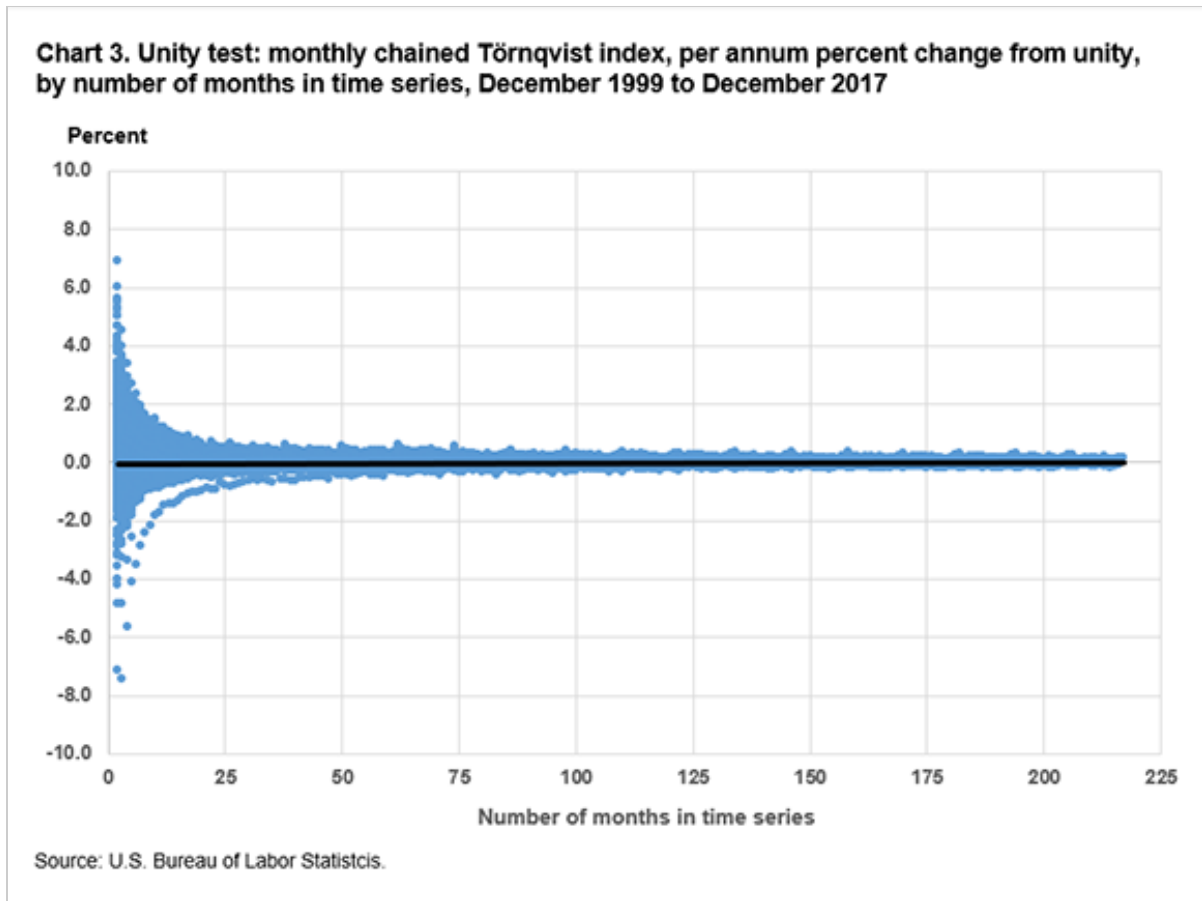


Chart 3 displays the results when the Törnqvist formula is used.



Charts 4 and 5 show the cumulative percent change in the monthly chained index value from unity (the beginning month in which the index equals 100) to the ending month in each iteration, up to the terminal month of December 2017. Chart 4 displays the results when the Laspeyres formula is used.

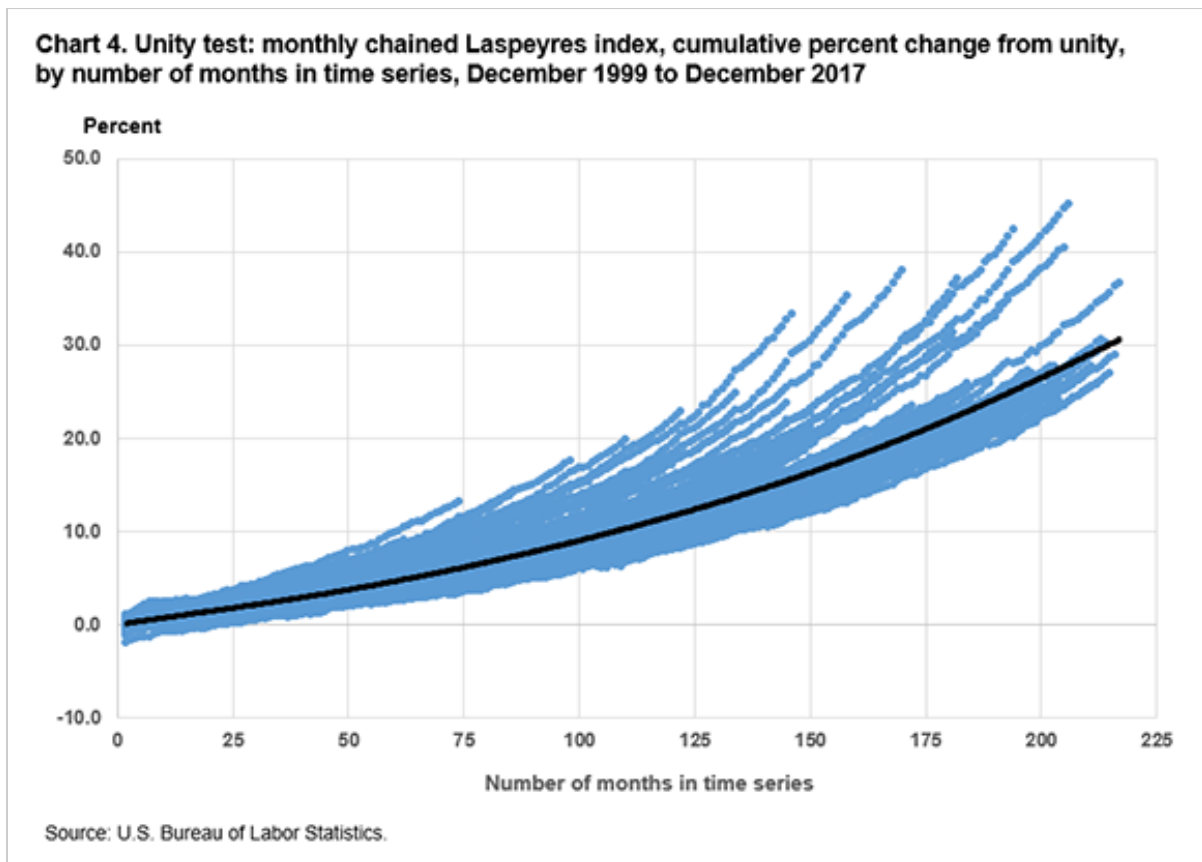
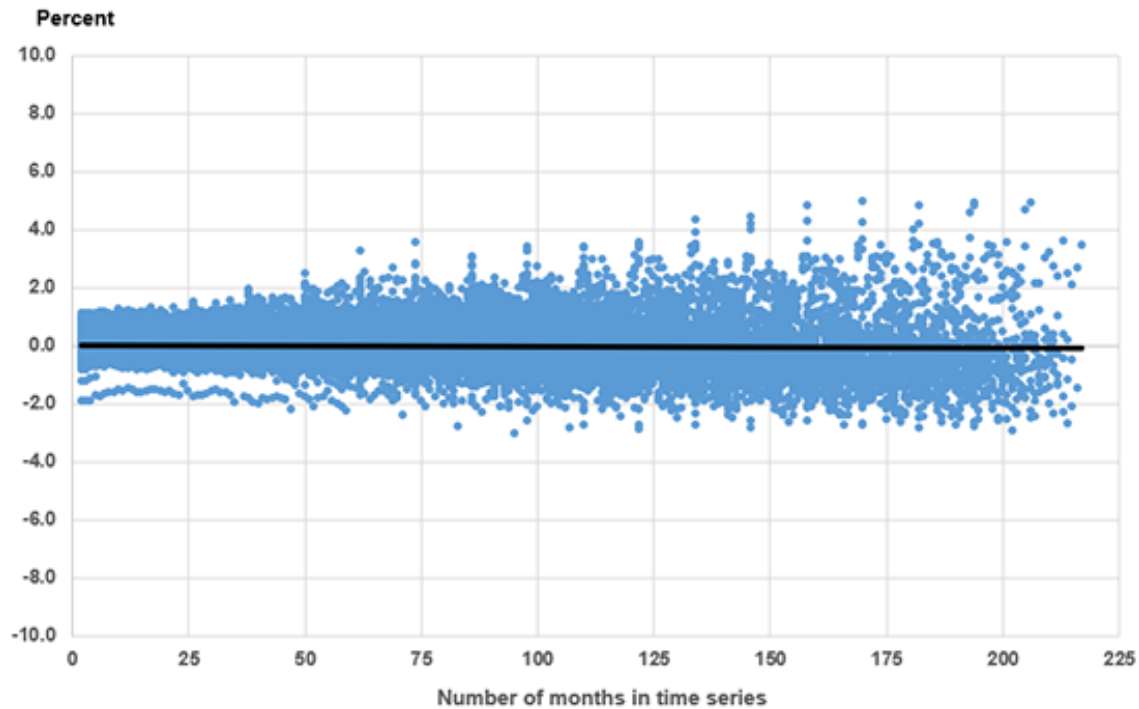


Chart 5 displays the results when the Törnqvist formula is used.

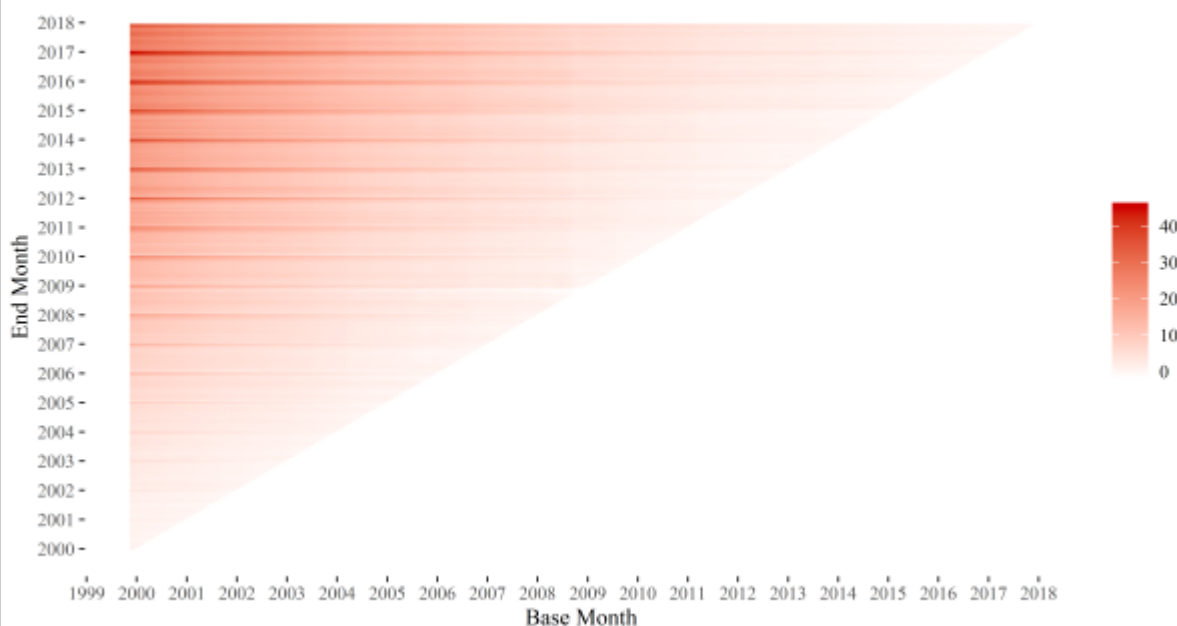
Chart 5. Unity Test: monthly chained Törnqvist index, cumulative percent change from unity, by number of months in time series, December 1999 to December 2017



Source: U.S. Bureau of Labor Statistics.

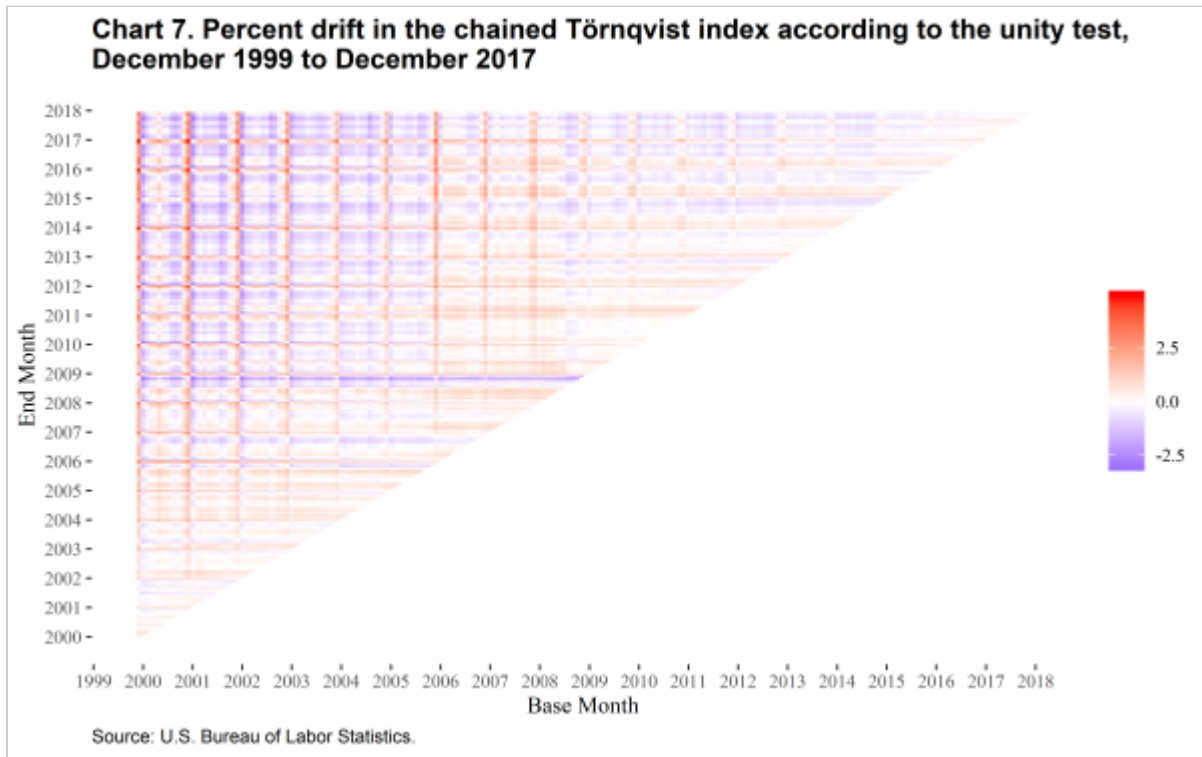
Chart 6 shows the accumulated amount of drift in the Laspeyres index between each pair of base and end months in the form of a heatmap.

Chart 6. Percent drift in the chained Laspeyres index according to the unity test, December 1999 to December 2017



Source: U.S. Bureau of Labor Statistics.

Chart 7 shows the accumulated amount of drift in the chained Törnqvist index.



The chained Laspeyres index shows substantial upward drift in almost all periods, while the chained Törnqvist index shows a seasonal pattern with strong upward drift when December is the base month and slight downward drift for most of the rest of the year. Several months also stand out as having stronger effects on drift, especially when December 1999 is the base month and the end months are in late 2008.

In these charts, we observe chain drift when the Laspeyres formula is used (charts 2 and 4) but not when the Törnqvist formula is used (charts 3 and 5). Table 1 shows summary statistics for the per annum percent changes from unity for the Laspeyres and Törnqvist formulas.

Table 1. Summary statistics for the per annum percent changes from unity for the Laspeyres and Törnqvist formulas, 1999–2017

Statistic	Laspeyres	Törnqvist
Mean	1.00	0.05
Median	1.00	0.00
Standard deviation	0.43	0.37
Source: U.S. Bureau of Labor Statistics.		

Table 2 shows the percentage of months in the time series in which we observe upward or downward annual drift (on a cumulative basis).

Table 2. Percentage of months in which upward or downward per annum drift was observed, 1999–2017

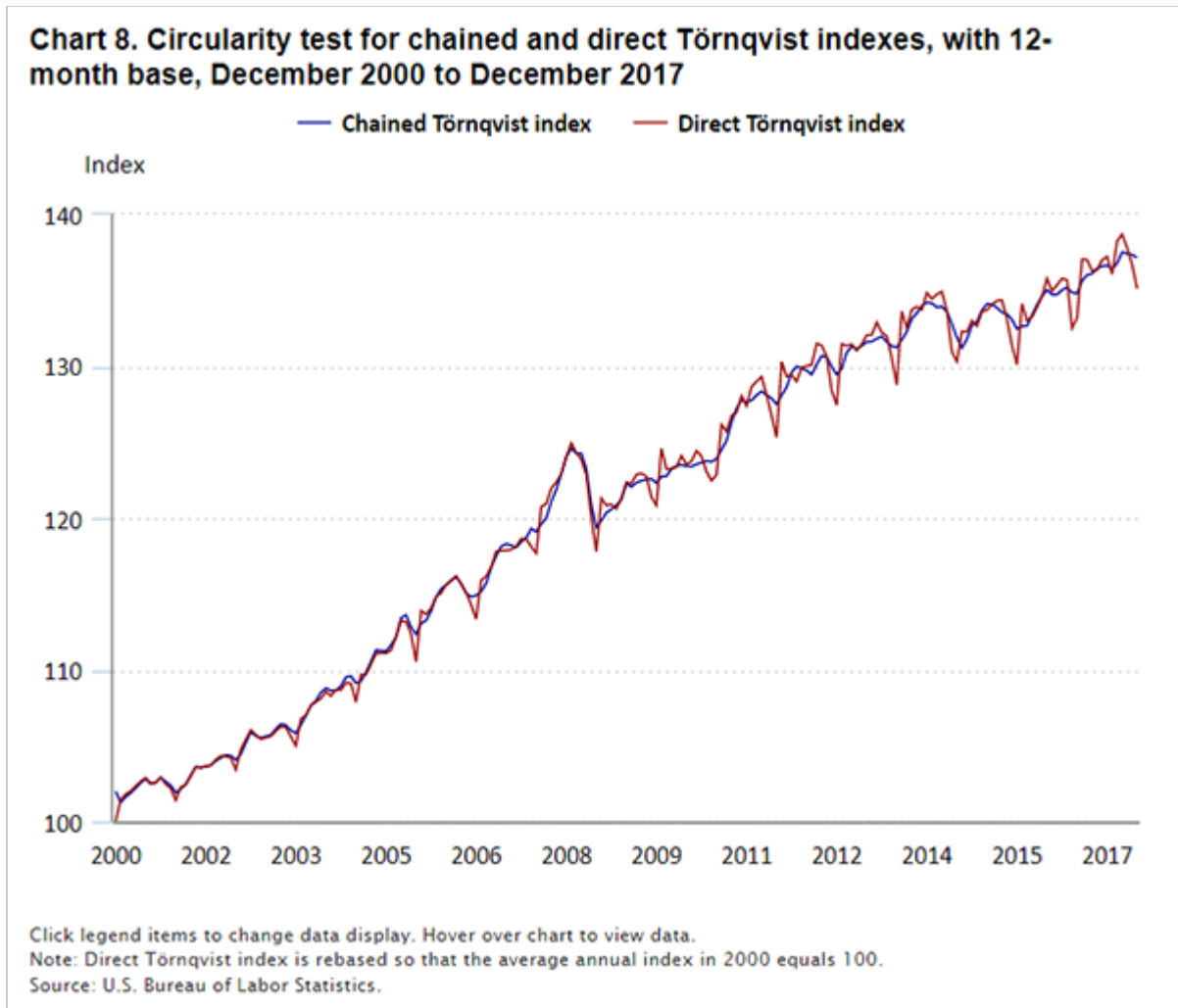
Drift	Laspeyres	Törnqvist
Upward	99.0	50.4
Downward	1.0	49.6
Source: U.S. Bureau of Labor Statistics.		

From the preceding charts and tables, we conclude that chain drift in a monthly chained Törnqvist index at the U.S. city average, all-items level, is minimal, compared with the drift in a monthly chained Laspeyres index at the same level, which is sizeable.

Circularity test

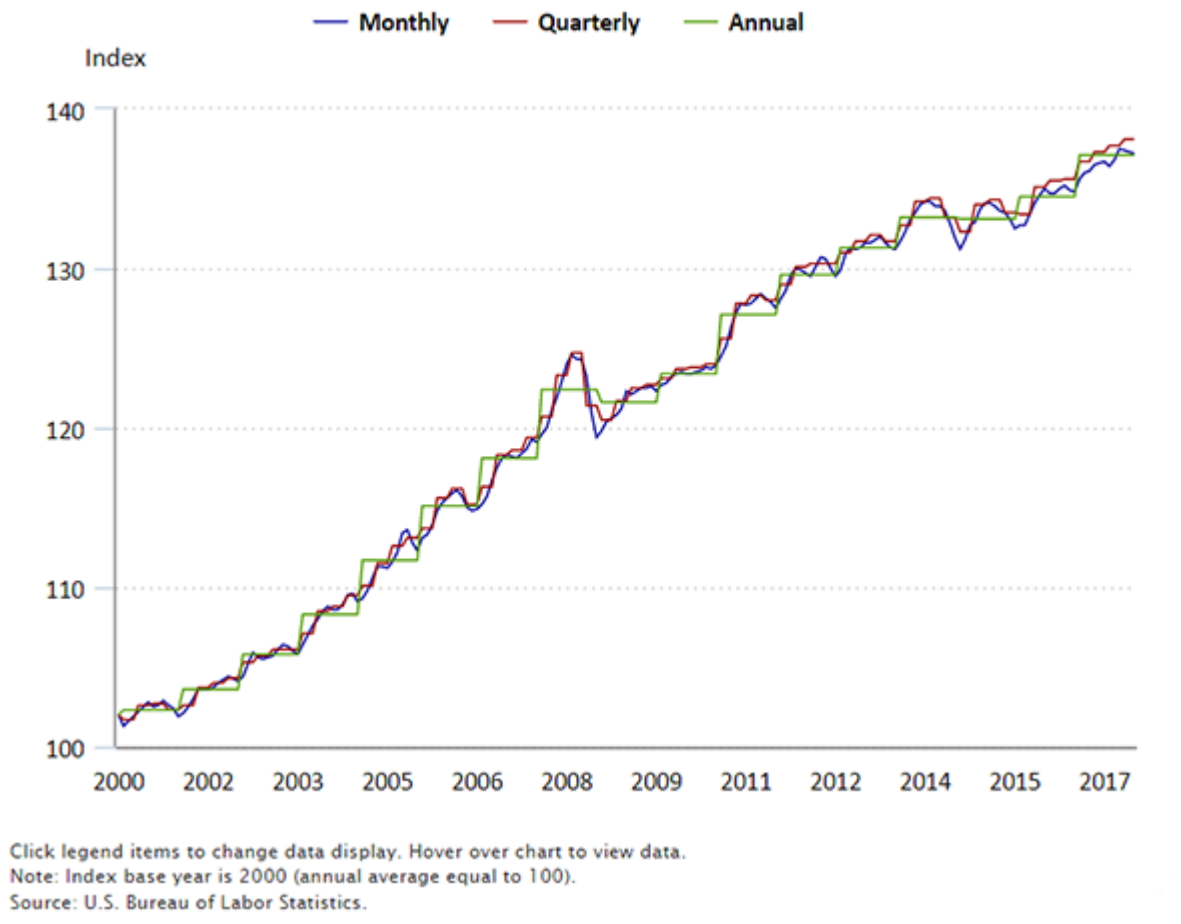
We conducted a circularity test by comparing chained and direct versions of the CPI-U, U.S. city average, all-items index, using both a Laspeyres formula and a Törnqvist formula. The relevant comparisons are between the chained and direct Laspeyres index and between the chained and direct Törnqvist index. In both cases, we observed visible drift. The CPI-U, U.S. city average, all-items index calculated using a Laspeyres formula with monthly chaining exhibits upward drift relative to the same index directly calculated using a Laspeyres formula (without chaining). When the CPI-U, U.S. city average, all-items index is calculated using a Törnqvist formula with monthly chaining, it also exhibits upward drift relative to the same index directly calculated using a Törnqvist formula. The Törnqvist finding conflicts with a previous empirical analysis by Klick that showed lower drift for the chained CPI-U, U.S. city average, all-items index using a Törnqvist formula.[\[39\]](#)

The preceding analysis suggests that the existence and extent of chain drift may depend on the choice of base period. We investigated this issue by changing the base period to 2000 and conducting additional circularity tests. We set calendar year 2000 as the base period in the direct Törnqvist calculations. Chart 8 shows the results when we compared the direct Törnqvist index with the monthly chained version, the former rebased so that the average annual index in 2000 was equal to 100, and we observed no significant drift.



Törnqvist indexes with quarterly and annual chaining behave similarly to a chained index with a 12-month base. (See chart 9.) This implies that most of the drift we see in the monthly chained Törnqvist index is due to the base period rather than the effects of chaining in intermediate periods.

Chart 9. Chained Törnqvist indexes, by frequency, December 2000 to December 2017

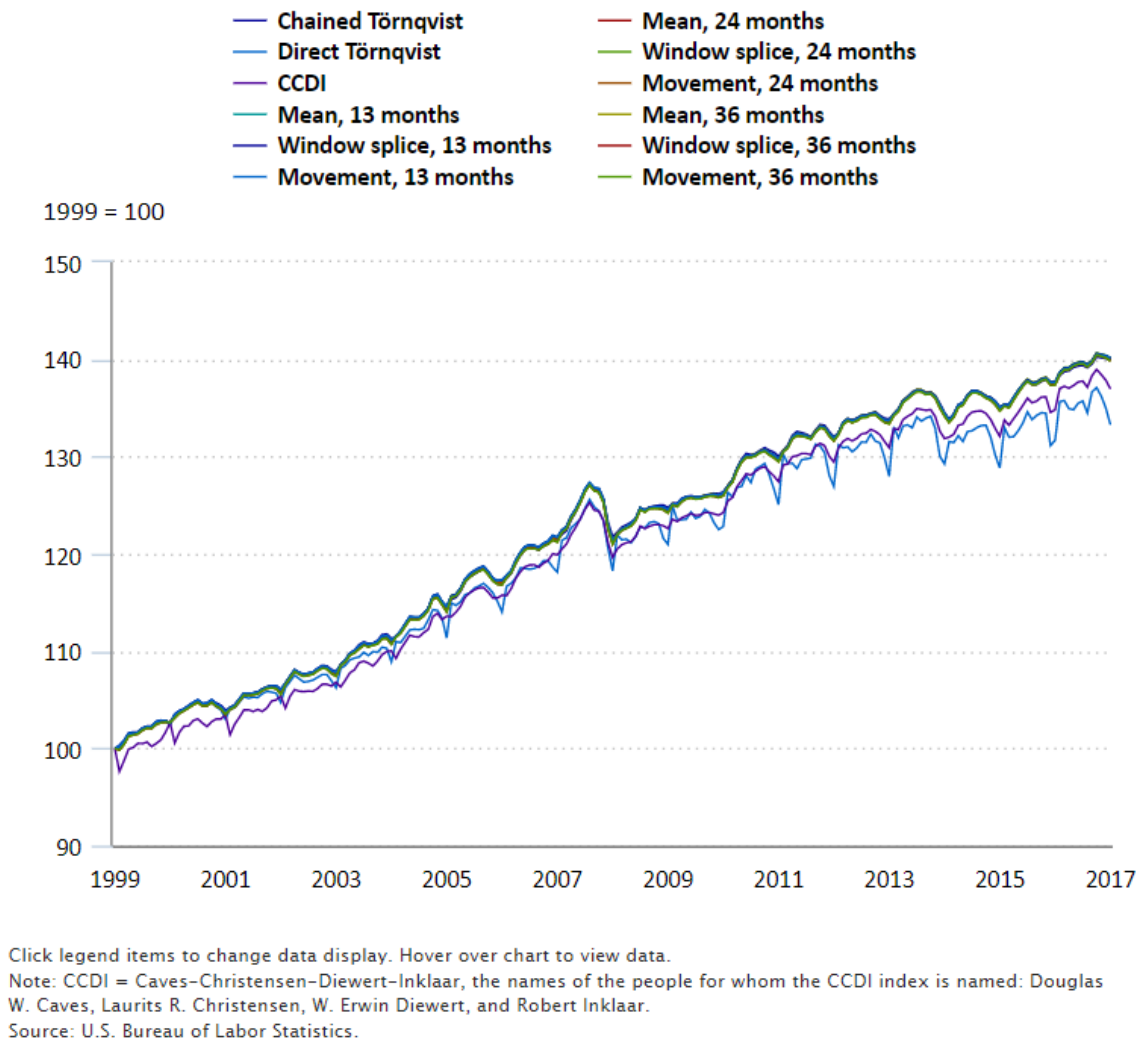


Multilateral index comparisons

The CCDI index provides an alternative measure of price change. In general, a multilateral chained index is more representative than a direct index while maintaining transitivity and thereby avoiding chain drift. The CCDI index also avoids the base-period sensitivity issue. We used the IndexNumR package in R to test various versions of the CCDI index by varying the method of splicing and changing window length.[\[40\]](#)

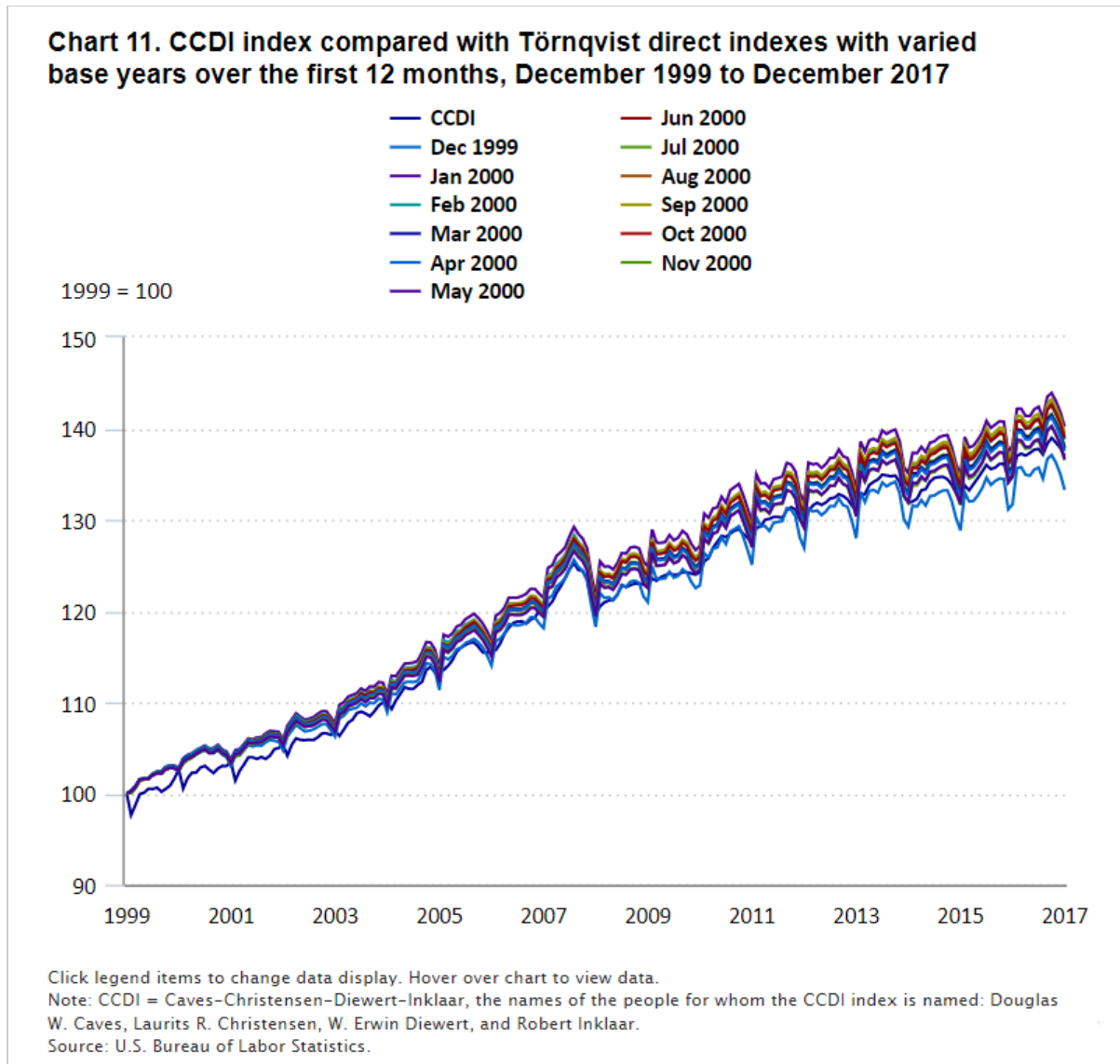
Chart 10 displays the chained and direct Törnqvist indexes discussed previously compared with a full-period CCDI index and the results of applying various extension methods to extend a CCDI index.

Chart 10. Törnqvist index method compared with various CCDI methods, December 1999 to December 2017



Previous research has explored the relative merits of various extension methods and then the length of a rolling window needed to mitigate drift from multilateral indexes. We find that three common extension methods (mean, movement, and window splices) and three window lengths (13, 24, and 36 months) all produce indexes similar to the monthly chained Törnqvist index. Although the direct Törnqvist and CCDI indexes increase at a slightly lower rate, the remaining indexes lie on top of each other. The results show that methodological choices for extension methods have little bearing for the index at the top level of aggregation.^[41]

The CCDI analysis produces similar results except that the direct Törnqvist index rises slightly more slowly than the monthly chained Törnqvist index. This difference might be driven more by sensitivity to the base period (December 1999) than by chain drift. In chart 11, we compare these indexes with a Törnqvist direct index, iterating over the first 12 months and taking each in turn as a fixed base. The index that uses December 1999 as the base period stands out as rising more slowly than indexes that use other months as the base period. The rolling CCDI index also generally falls within the range of the other months.



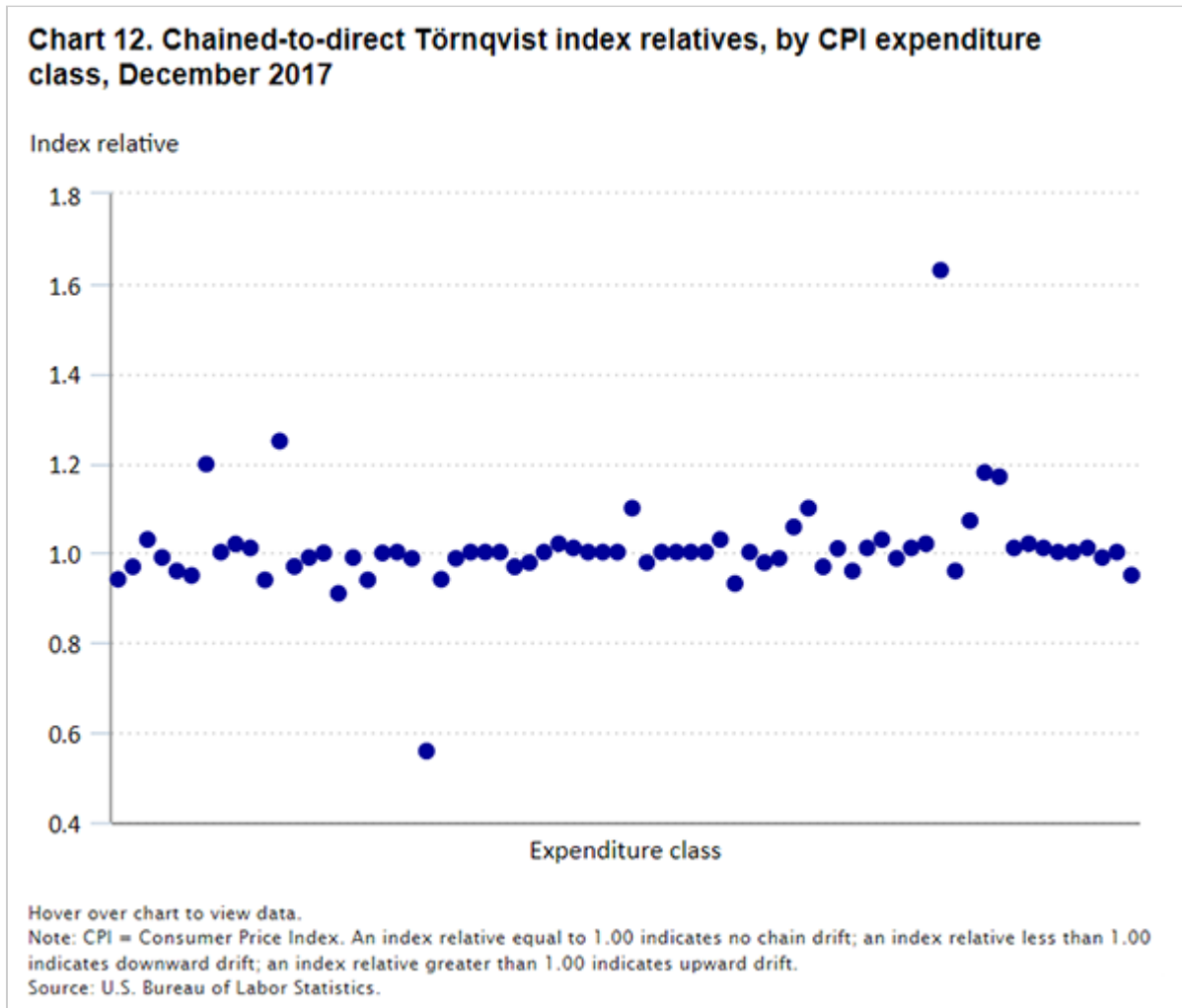
Circularity test at the expenditure-class level

Although chain drift at the highest aggregate level appears to be minor and mostly explained by base-period sensitivity and seasonality, we find that certain subaggregate indexes display relatively large amounts of drift. Overall, our analysis shows that a minority of the 70 expenditure-class CPIs satisfy the circularity test exactly, but most expenditure-class indexes satisfy the circularity test approximately.

During the period from December 1999 to December 2017, the CPI aggregation structure consisted of 8 major groups, 70 expenditure classes, 211 item categories, and various intermediate-level indexes, such as the index for all items less food and energy. We conducted a circularity test for each of the 70 expenditure classes. Using December 1999 as the base month, we used a Törnqvist formula to calculate index values for each month from December 1999 to December 2017. First, we performed the calculation with monthly chaining. Second, we performed the calculation directly with December 1999 as the base month. To conduct the circularity test, we

calculated a relative that divides the December 2017 indexes calculated with a chained Törnqvist formula by the December 2017 indexes calculated with a direct Törnqvist formula.

If the relative between the chained and direct Törnqvist indexes is equal to 1.00, the monthly indexes are the same—that is, the chained Törnqvist indexes are perfectly transitive and our circularity tests indicate no chain drift. If the relative is less than 1.00, then there is downward drift; if the relative is greater than 1.00, then there is upward drift. Our results show that 27 percent (or 19 of 70) of the expenditure-class indexes had a relative of approximately 1.00. Chart 12 shows the results of the circularity tests.



The results of our circularity tests at the expenditure-class level were essentially equivalent to those of our unity tests. The CCDI tests implied levels of drift similar to those of the other tests. (Appendix table A-1 shows the test results for each of the 70 expenditure classes, according to the unity, circularity, and CCDI tests.)

According to the *Consumer Price Index Manual*, “it is not useful to ask that the price index P satisfy the circularity test *exactly*.” It is, however, “of some interest to find index number formulae that satisfy the circularity test to some degree of approximation, since the use of such an index number formula will lead to measures of aggregate price change that are more or less the same no matter whether we use the chain or fixed base systems.”^[42] As a result, we calculated the percentage of expenditure-class indexes whose chained-to-direct relatives fell within a band of 0.95 and 1.05, as well as within a band of 0.90 and 1.10. We found that 74 percent of the expenditure-

class indexes had relatives between 0.95 and 1.05, while 89 percent had relatives between 0.90 and 1.10. Tables 3 and 4 summarize these results.

Table 3. Percentage of expenditure-class indexes that fall within the 0.95-to-1.05 relative bands, 1999–2017

Characteristic	Percent
Relative between 0.95 and 1.05	74.0
Relative outside the 0.95-to-1.05 band	24.0
Source: U.S. Bureau of Labor Statistics.	

Table 4. Percentage of expenditure-class indexes that fall within the 0.90-to-1.10 relative bands, 1999–2017

Characteristic	Percent
Relative between 0.90 and 1.10	89.0
Relative outside the 0.90-to-1.10 band	11.0
Source: U.S. Bureau of Labor Statistics.	

The results indicate that 11 percent of the expenditure-class indexes exhibited chain drift greater than 10 percent over the 18-year period of the study. Extreme outliers are shown in table 5.

Table 5. Expenditure-class indexes that exhibited chain drift greater than 10 percent, 1999–2017

Expenditure class	Chained-to-direct relative in December 2017
Video and audio	1.633
Information technology, hardware, and services	1.249
Jewelry and watches	1.203
Photography	1.183
Other recreational goods	1.171
Fresh fruits	0.556
Note: Other recreational goods include toys; sewing machines, fabric, and supplies; music instruments and accessories; and unsampled recreation commodities.	
Source: U.S. Bureau of Labor Statistics.	

The practical implication of these relatives is that chained and direct Törnqvist calculations of long-term price change show significant divergence. In the most extreme case, for fresh fruits, the chained Törnqvist formula shows long-term *deflation* of 24 percent, while the direct Törnqvist formula shows long-term *inflation* of 37 percent. Table 6 summarizes the extent of divergence for the categories shown in table 5, with the rate of inflation (or deflation) calculated from December 1999 to December 2017.

Table 6. Extent of divergence for expenditure-class categories that exhibited drift of greater than 10 percent

Expenditure class	Chained Törnqvist formula		Direct Törnqvist formula	
	Percent divergence	Inflation or deflation?	Percent divergence	Inflation or deflation?
Information technology, hardware, and services	79.0	Deflation	83.0	Deflation
Other recreational goods	61.0	Deflation	67.0	Deflation
Photography	29.0	Deflation	40.0	Deflation
Fresh fruits	24.0	Deflation	37.0	Inflation
Jewelry and watches	21.0	Inflation	1.0	Inflation
Video and audio	19.0	Deflation	51.0	Deflation

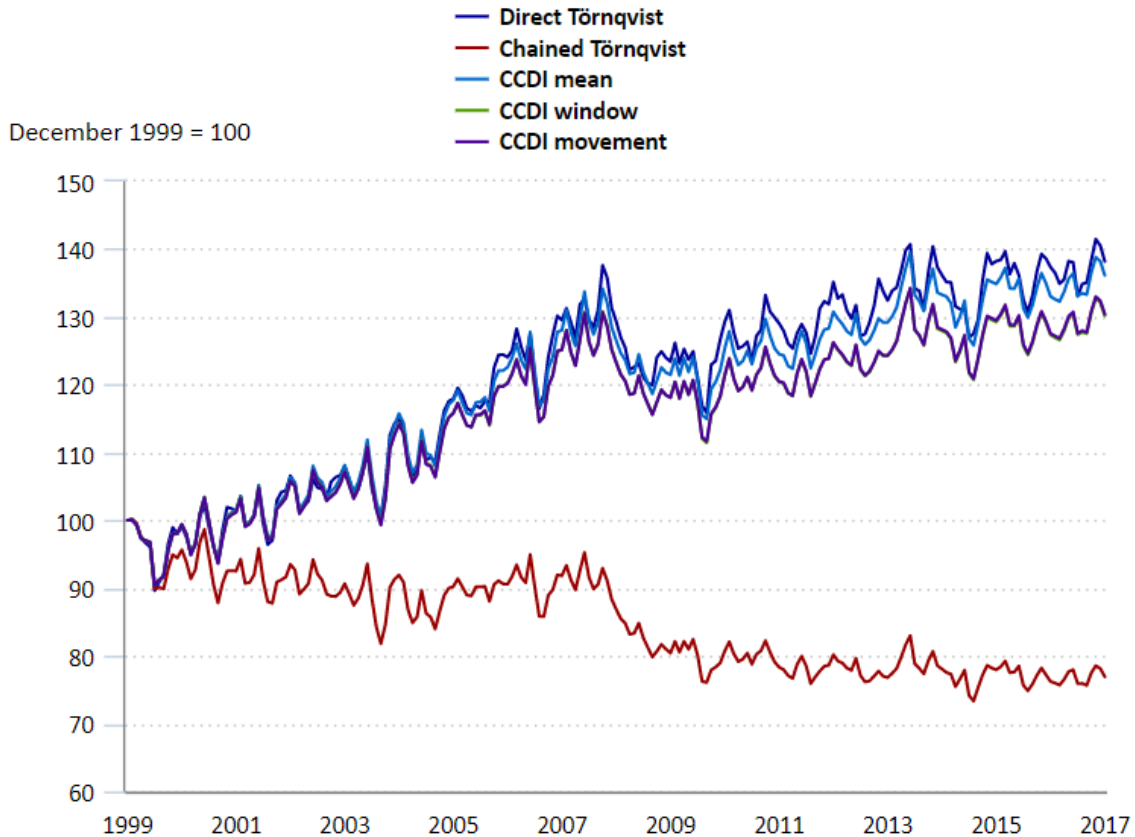
Note: Other recreational goods include toys; sewing machines, fabric, and supplies; music instruments and accessories; and unsampled recreation commodities.

Source: U.S. Bureau of Labor Statistics.

According to the analysis in this section, most expenditure-class indexes satisfy the circularity test to a reasonable degree. However, outliers exist, which raises doubts about using these indexes as measures of inflation for items in those index categories. The most glaring outlier was the index for fresh fruits, which shows a 24-percent *deflation* rate with a chained Törnqvist index but a 37-percent *inflation* rate with a direct Törnqvist index. As we suggested earlier in this article, drift often depends on the timing of price change in relation to the weight of an index. As the *Consumer Price Index Manual* states, “the more prices and quantities are subject to large fluctuations (rather than smooth trends), the less the correspondence” between a fixed-base and a chained index.^[43]

To investigate the issue further, we conducted a multilateral analysis on the fresh fruits aggregate index. The multilateral indexes helped address chain drift in this index. Chart 13 shows the difference between the chained and direct Törnqvist indexes, and the rolling multilateral indexes are close to the direct index. We explored three “splicing” options to update the CCDI index on a monthly basis. Of these, the mean splice was closest to the direct index, while the movement and window splices were almost identical to each other.

Chart 13. Fresh fruits: direct and chained Törnqvist indexes, with CCDI mean, window, and movement indexes, December 1999 to December 2017



Click legend items to change data display. Hover over chart to view data.

Note: CCDI = Caves-Christensen-Diewert-Inklaar, the names of the people for whom the CCDI index is named: Douglas W. Caves, Laurits R. Christensen, W. Erwin Diewert, and Robert Inklaar.

Source: U.S. Bureau of Labor Statistics.

Table 7 shows the correlation matrix between the different CCDI indexes produced at the expenditure-class level.

Table 7. Correlation matrix between the different CCDI indexes produced at the expenditure-class level, 1999–2017

Item	Mean, 13 months	Window splice, 13 months	Movement, 13 months	Full
Mean, 13 months	1.0000	0.9999	0.9999	0.9988
Window splice, 13 months	[1]	1.0000	1.0000	0.9986
Movement, 13 months	[1]	[1]	1.0000	0.9987
Full	[1]	[1]	[1]	1.0000

[1] Not applicable.

Note: CCDI = Caves-Christensen-Diewert-Inklaar, the names of the people for whom the CCDI index is named: Douglas W. Caves, Laurits R. Christensen, W. Erwin Diewert, and Robert Inklaar.

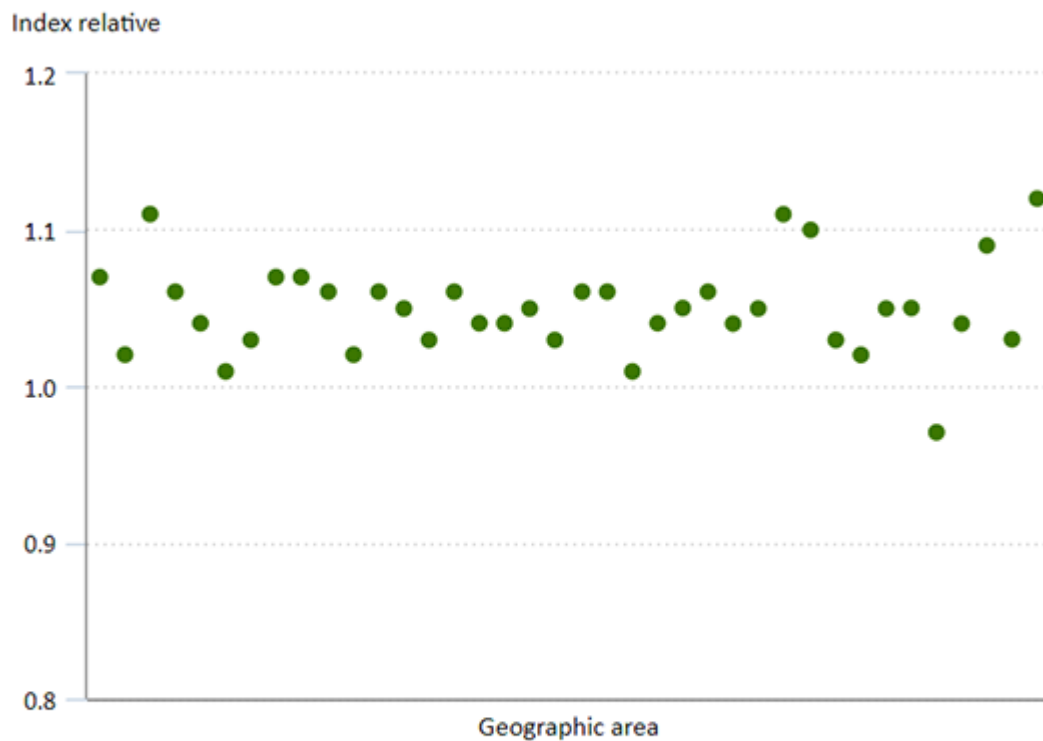
Source: U.S. Bureau of Labor Statistics.

All of the variations of the CCDI indexes produced similar results. The mean splice was closest to the full-period CCDI index, while the movement and window splice had a slightly closer relationship to each other.

Geographic analysis

In this section, we discuss the results of the circularity tests and the CCDI tests that we conducted on the CPI-U, all-items index, by geographic area. During the period from December 1999 to December 2017, the CPI program produced indexes for 4 census regions, 3 population size classes, and 27 metropolitan areas, in addition to the top-level index for the United States as a whole. Charts 14 and 15 show the results of the circularity tests and the CCDI tests, respectively.

Chart 14. Circularity tests for the Consumer Price Index, by geographic area, December 2017

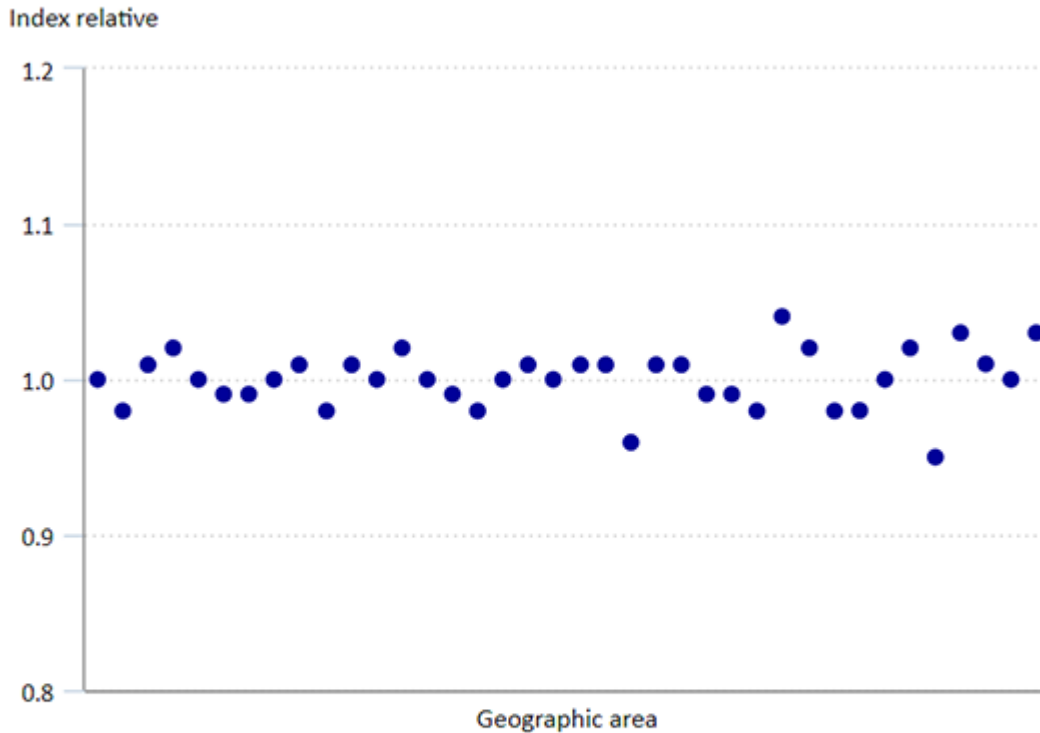


Hover over chart to view data.

Note: An index relative equal to 1.00 indicates no chain drift; an index relative less than 1.00 indicates downward drift; an index relative greater than 1.00 indicates upward drift. In the 1998 geographic sample, the New York-Northern New Jersey-Long Island, NY-NJ-CT-PA, area index was a consolidation of three A-size sampling units (geographic areas), while the Los Angeles-Riverside-Orange County, CA, index was a consolidation of two A-size sampling units (geographic areas). The remaining data point comes from the division of the Washington-Baltimore, DC-MD-VA-WV, index into Washington-Arlington-Alexandria, DC-VA-MD-WV, and Baltimore-Columbia-Towson, MD.

Source: U.S. Bureau of Labor Statistics.

Chart 15. CCDI tests (mean 13) for the Consumer Price Index, by geographic area, December 2017



Hover over chart to view data.

CCDI = Caves-Christensen-Diewert-Inklaar, the names of the people for whom the CCDI index is named: Douglas W. Caves, Laurits R. Christensen, W. Erwin Diewert, and Robert Inklaar. An index relative equal to 1.00 indicates no chain drift; an index relative less than 1.00 indicates downward drift; an index relative greater than 1.00 indicates upward drift. In the 1998 geographic sample, the New York-Northern New Jersey-Long Island, NY-NJ-CT-PA, area index was a consolidation of three A-size sampling units (geographic areas), while the Los Angeles-Riverside-Orange County, CA, index was a consolidation of two A-size sampling units (geographic areas). The remaining data point comes from the division of the Washington-Baltimore, DC-MD-VA-WV, index into Washington-Arlington-Alexandria, DC-VA-MD-WV, and Baltimore-Columbia-Towson, MD.

Source: U.S. Bureau of Labor Statistics.

Most area indexes exhibit cumulative drift of less than 10 percent over the 18-year period. In the circularity tests, only Baltimore, Portland-Salem, San Diego, and the regional population size class consisting of small (size D) Western cities exhibit upward cumulative drift of 10 percent or more (with Washington, DC, showing about 9.5 percent). The index for St. Louis exhibits downward cumulative drift of about 3 percent in the circularity tests. (See chart 14.). On the other hand, the CCDI tests show less drift, with an average of about 1 percent over the period. (See chart 15.) Arguably, the CCDI method keeps more of the “good” drift of representing consumption pattern changes while still eliminating the “bad” drift resulting from nontransitivity.

We can expect to see some level of drift for geographic areas because their sample sizes are smaller. Drift results from short-term price oscillations that distort long-term trends because expenditure-share weights are often inversely proportional to price levels and thus underweight price declines and overweight price increases. Because price oscillations at the microdata level are more likely to affect elementary indexes in areas with smaller samples, chain drift is also more likely.

The circularity tests indicate upward drift at the area level. Other conditions may lead to chain drift in addition to pendular quantities in response to sales. Ludwig von Auer showed that delayed quantity responses to price change, which he calls “sticky quantities,” lead to upward drift.^[44] Recent research on explanations for chain drift have focused on lower-level indexes and consumer behavior. It is unclear whether we are seeing drift at the area level because small sample sizes allow microlevel effects to impact area-level aggregates or if drift is the result of other factors at the aggregate level, such as stochastic variation, seasonality, or the process of weight construction. Table 8 shows the results of the CCDI and circularity tests, by geographic area.

Table 8. CCDI and circularity test results, by Consumer Price Index area, 1999–2017

Area	CCDI test rank	CCDI test (mean 13)	Circularity test
Portland-Salem, OR-WA	1	1.0395	1.1084
West-size class D	2	1.0331	1.1188
Tampa-St. Petersburg-Clearwater, FL	3	1.0251	1.0375
South-size class D	4	1.0190	1.0473
Kansas City, MO-KS	5	1.0162	1.0507
San Diego, CA	6	1.0153	1.1002
Boston-Brockton-Nashua, MA-NH-ME-CT	7	1.0150	1.0562
Honolulu, HI	8	1.0142	1.0180
Midwest-size class D	9	1.0139	1.0504
Minneapolis-St. Paul, MN-WI	10	1.0116	1.0632
Washington, DC-MD-VA-WV	11	1.0115	1.0948
Baltimore, MD	12	1.0096	1.1127
Northeast Urban-size class B/C	13	1.0077	1.0491
Denver-Boulder-Greeley, CO	14	1.0070	1.0666
New York-Connecticut Suburbs	15	1.0066	1.0445
New Jersey Suburbs	16	1.0059	1.0578
South-size class B/C	17	0.9999	1.0521
Los Angeles Suburbs, CA	18	0.9986	1.0280
Anchorage, AK	19	0.9985	1.0745
Midwest-size class B/C	20	0.9982	1.0428
Milwaukee-Racine, WI	21	0.9964	1.0253
Houston-Galveston-Brazoria, TX	22	0.9956	1.0630
Dallas-Fort Worth, TX	23	0.9955	1.0693
West-size class B/C	24	0.9954	1.0301
Chicago-Gary-Kenosha, IL-IN-WI	25	0.9951	1.0415
Phoenix-Mesa, AZ	26	0.9940	1.0424
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	27	0.9914	1.0579
Los Angeles-Orange, CA	28	0.9884	1.0647
Cincinnati-Hamilton, OH-KY-IN	29	0.9873	1.0083
Cleveland-Akron, OH	30	0.9852	1.0290
Miami-Fort Lauderdale, FL	31	0.9849	1.0398
Detroit-Ann Arbor-Flint, MI	32	0.9817	1.0552
Pittsburgh, PA	33	0.9806	1.0480
San Francisco-Oakland-San Jose, CA	34	0.9798	1.0343
Seattle-Tacoma-Bremerton, WA	35	0.9762	1.0222
Atlanta, GA	36	0.9754	1.0241

See footnotes at end of table.

Table 8. CCDI and circularity test results, by Consumer Price Index area, 1999–2017

Area	CCDI test rank	CCDI test (mean 13)	Circularity test
New York, NY	37	0.9615	1.0107
St. Louis, MO-IL	38	0.9503	0.9693

Note: CCDI = Caves-Christensen-Diewert-Inklaar, the names of the people for whom the CCDI index is named: Douglas W. Caves, Laurits R. Christensen, W. Erwin Diewert, and Robert Inklaar. In the 1998 geographic sample, the New York-Northern New Jersey-Long Island, NY-NJ-CT-PA, area index was a consolidation of three A-size sampling units (geographic areas), while the Los Angeles-Riverside-Orange County, CA, index was a consolidation of two A-size sampling units (geographic areas). The remaining data point comes from the division of the Washington-Baltimore, DC-MD-VA-WV, index into Washington-Arlington-Alexandria, DC-VA-MD-WV, and Baltimore-Columbia-Towson, MD.

Source: U.S. Bureau of Labor Statistics.

Conclusion

The Chained Consumer Price Index for All Urban Consumers (C-CPI-U) does not exhibit much drift. To the extent that we see drift, much of it appears to be connected to seasonality and the use of December 1999 as the base year. Moreover, the chained Törnqvist formula used for the C-CPI-U is less susceptible to drift than the chained Laspeyres formula would be for the less frequently updated Consumer Price Index for All Urban Consumers (CPI-U). The advantages from better representing consumer substitution and a timelier market basket appear to outweigh the disadvantages of drift. Although the effects of these advantages and disadvantages should be studied further, there is an implication that chain drift should not be a major concern, and thus that the chained Törnqvist formula is a better approximation of a cost-of-living index than the direct Laspeyres formula currently used for the CPI-U.

Our study demonstrates that some subaggregate indexes do show a certain amount of drift. We can, fortunately, suggest that advances in multilateral indexes promise to provide methods that take advantage of timely weighting while avoiding the problem of drift. We hope that further work at BLS and elsewhere will continue in order to determine the optimal implementation of certain aspects of multilateral index methods, particularly splicing and changing window length. Our results show little difference among the splicing methods. Once consensus is reached on these issues, estimation of multilateral indexes could be used to address drift issues at the lower level.

Appendix: Estimated drift, by expenditure class

Table A-1. Estimated drift, by expenditure class, according to the unity, circularity, and CCDI tests, 1999–2017

Expenditure class	Unity test	Circularity test	CCDI test
Men's apparel	0.941652	0.941828	0.937353
Boys' apparel	0.965863	0.967153	0.982250
Women's apparel	1.029028	1.031830	1.037496
Girls' apparel	0.994046	0.990251	0.985837
Footwear	0.957624	0.959558	0.959986
Infants' and toddlers' apparel	0.955572	0.950252	0.938945

See footnotes at end of table.

Table A-1. Estimated drift, by expenditure class, according to the unity, circularity, and CCDI tests, 1999–2017

Expenditure class	Unity test	Circularity test	CCDI test
Jewelry and watches	1.204530	1.202724	1.150962
Educational books and supplies	0.988330	0.999926	0.996042
Tuition, other school fees, and childcare	1.017108	1.017554	1.029939
Postage and delivery services	1.010163	1.010069	0.999716
Telephone services	0.920595	0.939250	0.964706
Information technology, hardware, and services	1.255826	1.249000	1.119879
Cereals and cereal products	0.963709	0.966526	0.976675
Bakery products	0.990638	0.990164	0.987380
Beef and veal	1.003275	1.003969	0.998663
Pork	0.910979	0.913383	0.907360
Other meats	0.993535	0.993299	0.994247
Poultry	0.939363	0.937153	0.933386
Fish and seafood	1.005897	1.004070	1.004816
Eggs	0.996227	0.996692	0.994645
Dairy and related products	0.985726	0.986577	0.985187
Fresh fruits	0.556596	0.555873	0.561130
Fresh vegetables	0.939375	0.941835	0.942837
Processes fruits and vegetables	0.986802	0.987155	0.988594
Juices and alcoholic drinks	0.998312	0.998188	0.999799
Beverage materials including coffee and tea	0.994295	0.997309	0.995180
Sugar and sweets	1.004349	1.001851	1.002016
Fats and oils	0.971247	0.974486	0.983493
Other foods	0.984199	0.984025	0.987586
Food away from home	1.001481	1.001670	1.002081
Alcoholic beverages at home	1.020370	1.021433	1.010587
Alcoholic beverages away from home	1.009530	1.010137	1.009256
Tobacco and smoking products	1.002171	1.001898	1.001276
Personal care products	0.998877	0.997457	0.998657
Personal care services	0.996371	0.995939	0.995902
Miscellaneous personal services	1.102182	1.100590	1.110237
Miscellaneous personal goods	0.979213	0.980455	0.975419
Rent of primary residence	1.002715	1.002400	1.001123
Lodging away from home	0.994458	1.002775	1.001720
Owners' equivalent rent of residences	1.002457	1.002322	1.002382
Tenants' and household insurance	0.997705	0.998390	0.996370
Fuel oil and other fuels	1.030241	1.030476	1.034032
Energy services	0.929984	0.930839	0.928660
Water and sewer and trash collection services	1.003097	1.002795	1.004454
Window and floor coverings and other linens	0.979303	0.976859	0.995162
Furniture and bedding	0.994004	0.992963	0.990361
Appliances	1.054744	1.057353	1.029233
Other household equipment and furnishings	1.104847	1.102994	1.055055
Tools, hardware, outdoor equipment and supplies	0.966439	0.968313	0.973173
Housekeeping supplies	1.010245	1.011556	1.002490
Household operations	0.957911	0.956760	0.974772
Professional services	1.008573	1.008217	1.008779

See footnotes at end of table.

Table A-1. Estimated drift, by expenditure class, according to the unity, circularity, and CCDI tests, 1999–2017

Expenditure class	Unity test	Circularity test	CCDI test
Hospital and related services	1.029366	1.028234	1.015461
Health insurance	0.985036	0.985283	0.982734
Medicinal drugs	1.013699	1.012711	1.000114
Medical equipment and supplies	1.019574	1.018104	1.017068
Video and audio	1.632082	1.632832	1.260736
Pets, pet products and services	0.963472	0.961186	0.982019
Sporting goods	1.069327	1.068525	1.039060
Photography	1.182544	1.183394	1.094163
Other recreational goods	1.170802	1.171422	1.077137
Other recreational services	1.007153	1.008762	0.991975
Recreational reading materials	1.024860	1.024699	1.009488
New and used motor vehicles	1.004985	1.005244	1.005059
Motor fuel	1.002221	1.002224	1.002762
Motor vehicle parts and equipment	1.000997	1.002042	1.000716
Motor vehicle maintenance and repair	1.003613	1.005058	1.002593
Motor vehicle insurance	0.990936	0.990766	0.997521
Motor vehicle fees	0.997528	0.997395	1.004337
Public transportation	0.955231	0.954407	0.958536

Note: CCDI = Caves-Christensen-Diewert-Inklaar, the names of the people for whom the CCDI index is named: Douglas W. Caves, Laurits R. Christensen, W. Erwin Diewert, and Robert Inklaar.

Source: U.S. Bureau of Labor Statistics.

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NOTES

¹ The broadest and most comprehensive Consumer Price Index (CPI), or what is sometimes called the “official CPI,” is the Consumer Price Index for All Urban Consumers (CPI-U), U.S. city average, all items (1982–84 = 100). See “Consumer Price Index Frequently Asked Questions,” question 16, “Which index is the ‘official CPI’ reported in the media?,” https://www.bls.gov/cpi/questions-and-answers.htm#Question_16.

² See Joshua Klick, “Measurement of chain drift in the Chained CPI-U,” Office of Survey Methods and Research Statistical survey paper (U.S. Bureau of Labor Statistics, November 2017), <https://www.bls.gov/osmr/research-papers/2017/pdf/st170100.pdf>.

³ Robert Cage, John Greenlees, and Patrick Jackman, “Introducing the Chained Consumer Price Index” (paper presented at the Seventh Meeting of the International Working Group on Price Indices, Paris, France, May 2003), p. ii, <https://www.bls.gov/cpi/additional-resources/chained-cpi-introduction.pdf>.

⁴ Michael J. Boskin, Ellen R. Dulberger, Robert J. Gordon, Zvi Griliches, and Dale Jorgenson, *Final Report of the Advisory Commission to Study the Consumer Price Index* (U.S. Government Printing Office, December 1996), table 3, p. 44, <https://www.bls.gov/publications/otherpublications/final-report-of-the-advisory-commission-to-study-the-consumer-price-index>.

www.finance.senate.gov/imo/media/doc/Prt104-72.pdf. The report is commonly referred to as “The Boskin Commission Report,” named for Michael J. Boskin, the chairman of the Advisory Commission to Study the Consumer Price Index.

⁵ Ibid., p. 1.

⁶ For more on using a geometric mean formula to calculate the Consumer Price Index, see Kenneth V. Dalton, John S. Greenlees, and Kenneth J. Stewart, “Incorporating a geometric mean formula into the CPI,” *Monthly Labor Review*, October 1998, <https://www.bls.gov/opub/mlr/1998/10/art1full.pdf>.

⁷ For more information on this process, see Joshua Klick, “Improving initial estimates of the Chained Consumer Price Index,” *Monthly Labor Review*, February 2018, <https://www.bls.gov/opub/mlr/2018/article/improving-initial-estimates-of-the-chained-consumer-price-index.htm>.

⁸ Peter Hill, ed., *Consumer Price Index Manual: Theory and Practice* (Geneva: International Labour Office, 2004), sec. 17.9, p. 314, https://www.ilo.org/wcmsp5/groups/public/---dgreports/---stat/documents/presentation/wcms_331153.pdf. See also A. A. Konüs, “The problem of the true index of the cost of living,” *Econometrica*, vol. 7, no. 1, January 1939, pp. 10–29, <https://www.jstor.org/stable/pdf/1906997.pdf>.

⁹ The Hicksian demand function (or compensated demand function) is named after noted British Economist John Hicks.

¹⁰ Hill, ed., *Consumer Price Index Manual*, sec. 15.81, p. 281.

¹¹ Christian G. Ehemann, “Chain drift in leading superlative indexes,” BEA Working Paper 2005-09 (U.S. Bureau of Economic Analysis, December 6, 2005), p. 3, <https://www.bea.gov/system/files/papers/WP2005-9.pdf>.

¹² Hill, ed., *Consumer Price Index Manual*, sec. 16.30, pp. 292–93.

¹³ Ibid., sec. 16.30, p. 292, fn. 14.

¹⁴ Ibid., sec. 15.77, pp. 264–65.

¹⁵ Ibid., sec. 15.78, p. 280.

¹⁶ See Gregory Kurtzon, “How much does formula vs. chaining matter for a cost-of-living index? The CPI-U vs. the C-CPI-U,” BLS Working Paper 498 (U.S. Bureau of Labor Statistics, September 2018), p. 12, <https://www.bls.gov/osmr/research-papers/2017/pdf/ec170060.pdf>.

¹⁷ F. G. Forsyth and R. F. Fowler, “The theory and practice of chain price index numbers,” *Journal of the Royal Statistical Society: Series A (General)*, vol. 144, no. 2, Spring 1981, pp. 224–46, <https://doi.org/10.2307/2981921>; quotation, p. 224.

¹⁸ J. Lehr, *Beiträge zur Statistik der Preise* (Frankfurt: J.D. Sauerlander, 1885) and A. Marshall, “Remedies for fluctuations of general prices,” *Contemporary Review* 51, March 1887, pp. 355–75, as cited in Hill, ed., *Consumer Price Index Manual*, sec. 15.77, p. 280, fn. 58.

¹⁹ Forsyth and Fowler, “Theory and practice of chain price index numbers,” p. 224.

²⁰ See “Appendix 15.4: The relationship between the Divisia and economic approaches,” in Hill, ed., *Consumer Price Index Manual*, pp. 287–88.

²¹ Forsyth and Fowler, “Theory and practice of chain price index numbers,” pp. 237–39.

²² Other properties include identity, monotonicity, proportionality in current prices, invariance to proportional changes in quantities, invariance to changes in units, time reversal, and the mean value test. See “Chapter 16: The axiomatic and stochastic approaches to index number theory,” in Hill, ed., *Consumer Price Index Manual*, pp. 289–310.

²³ Ehemann, “Chain drift in leading superlative indexes,” p. 3.

²⁴ Forsyth and Fowler, “Theory and practice of chain price index numbers,” pp. 237–39.

²⁵ Bohdan J. Szulc, “Linking price index numbers,” in W. E. Diewert and C. Montmarquette, eds., *Price Level Measurement: Proceeding of a Conference Sponsored by Statistics Canada* (Ottawa: Minister of Supply and Services Canada, 1983), p. 548. See also, Hill, ed., *Consumer Price Index Manual*, sec. 15.84, p. 281, fn. 61.

²⁶ Kurtzon, “How much does formula vs. chaining matter for a cost-of-living index?” p. 12.

²⁷ Forsyth and Fowler, “Theory and practice of chain price index numbers,” pp. 224–46; and Marshall B. Reinsdorf and Brent R. Moulton, “The construction of basic components of cost-of-living indexes,” in Timothy F. Bresnahan and Robert J. Gordon, ed., *The Economics of New Goods* (Chicago: University of Chicago Press, 1996), p. 405.

²⁸ Lorraine Ivancic, Kevin J. Fox, and W. Erwin Diewert, “Scanner data, time aggregation and the construction of price indexes” (paper presented at the Eleventh Meeting of the Ottawa Group, International Working Group on Price Indices, Neuchatel, Switzerland, May 2009), p. 4, <https://www.ottawagroup.org/Ottawa/ottawagroup.nsf/51c9a3d36edfd0dfca256acb00118404/a49bc2a164b232c4ca2576a100773522?OpenDocument>.

²⁹ Correa Moylan Walsh, *The Measurement of General Exchange Value* (New York: Macmillan, 1901), as cited in *Consumer Price Index Manual*, sec. 15.94, p. 283.

³⁰ Hill, ed., *Consumer Price Index Manual*, sec. 15.94, p. 283.

³¹ W. E. Diewert, “The early history of price index research,” as cited in *Consumer Price Index Manual*, sec. 15.94, p. 283.

³² Hill, ed., *Consumer Price Index Manual*, sec. 15.96, pp. 283–84.

³³ *Ibid.*, sec. 15.93, p. 283.

³⁴ *Ibid.*, sec. 15.88, p. 282. As the *Manual* states (fn. 68), “The test name is attributable to Fisher” (in 1922) and “the concept originated from Westergaard” (in 1890). See the *Manual* for specific source information for Fisher and Westergaard.

³⁵ Ehemann, “Chain drift in leading superlative indexes,” p. 3.

³⁶ Hill, ed., *Consumer Price Index Manual*, sec. 15.88, p. 282.

³⁷ Ivancic et al., “Scanner data, time aggregation and the construction of price indexes,” pp. 33–34.

³⁸ W. Erwin Diewert and Kevin J. Fox, “Substitution bias in multilateral methods for CPI construction using scanner data,” UNSW Business School Research Paper No. 2018-13 (University of New South Wales, October 2018), pp. 12–14, 39, <http://dx.doi.org/10.2139/ssrn.3276457>. See also Douglas W. Caves, Laurits R. Christensen, and W. Erwin Diewert, “The economic theory of index numbers and the measurement of input, output, and productivity,” *Econometrica*, vol. 50, no. 6, February 1982, pp. 1393–1414.

³⁹ Klick, “Measurement of chain drift in the Chained CPI-U”; see graph 4.

⁴⁰ For more information on the “IndexNumR” software package, see Graham White, “IndexNumR: A package for index number calculation,” September 26, 2021, <https://cran.r-project.org/web/packages/IndexNumR/vignettes/indexnumr.html>.

⁴¹ For a helpful resource on rolling-window indexes and extension methods, see W. Erwin Diewert and Kevin J. Fox, “Substitution bias in multilateral methods for CPI construction using scanner data” (presentation to Statistics Norway, Oslo, Norway, May 14, 2019), http://research.economics.unsw.edu.au/kfox/assets/diewertfox_scanner_statsnorway_14may2019.pdf.

⁴² Hill, ed., *Consumer Price Index Manual*, sec. 15.92, pp. 282–83.

⁴³ *Ibid.*, p. 283.

⁴⁴ Ludwid von Auer, “The nature of chain drift” (presentation at the Sixteenth Meeting of the Ottawa Group, International Working Group on Price Indices, Rio de Janeiro, Brazil, May 2019), p. 19, [https://www.ottawagroup.org/Ottawa/ottawagroup.nsf/home/Meeting+16/\\$FILE/The%20Nature%20of%20Chain%20Drift%20pres.pdf](https://www.ottawagroup.org/Ottawa/ottawagroup.nsf/home/Meeting+16/$FILE/The%20Nature%20of%20Chain%20Drift%20pres.pdf).

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