Abstract
The Great Recession (Dec07-Jun09) and its aftermath have raised major challenges to the practice of seasonal adjustment by statistical agencies around the world. Recent studies have questioned the adequacy of the standard approach to handling recession related disturbances and have offered different interpretations and solutions. A problem with evaluating these alternatives is that we can never know the true recession effects. A useful way to obtain objective information is to directly test the robustness of commonly used seasonal adjustment filters to pre-specified test series designed to reflect plausible recession effects. This study focuses on sensitivity testing of X-11 filters using artificial test series based on the behavior during the Great Recession of U.S. total non-farm payroll employment from the Current Employment Statistics (CES) program. The results show that the standard methodology is reasonably robust to recession effects.

Key Words: Ramps, Henderson Filter, X-11

1. Introduction
The Great Recession (Dec07-Jun09) and its aftermath have raised major challenges to the practice of seasonal adjustment by statistical agencies around the world. Since the 1990’s ARIMA mode-based methods of intervention analysis and automatic outlier detection (Findley, et al., 1998) have become standard tools for handling recession effects and other data irregularities in mainstream seasonal adjustment software (X-12, SEATS, X-13A-S). It is now standard procedure to first run automatic outlier detection prior to seasonal adjustment where the most commonly tested outliers are additive outliers, temporary change, and permanent level shifts (LS). The automated methods are important to statistical agencies not only because they makes it more manageable to adjust a large number of series but also provide objective rules for identifying, estimating and adjusting for outliers.

Recent studies have questioned the adequacy of the standard approach to handling recession related disturbances and have offered different interpretations and solutions. A problem with evaluating these alternatives is that we can never know the true recession effects. Like seasonal effects, they are inherently unobservable. All that we have are estimates and it is not always clear how to compare them.

A useful way to obtain objective information is to directly test the robustness of commonly used seasonal adjustment filters to pre-specified test series designed to reflect plausible recession effects. This study focuses on sensitivity testing of X-11 filters (as implemented in the X-12 or X-13A-S software) using artificial test series based on the behavior during the Great Recession of seasonally adjusted (SA) U.S. total non-farm payroll employment from the Current Employment Statistics (CES) program.
2. Alternatives to Conventional Practice

Studies in both the U.S. and Europe have questioned the adequacy of the standard approach during the Great Recession. Alternatives have been proposed within and outside government statistical agencies. As motivation for this paper, we review selected studies based on either subjective or data driven adjustments.

2.1 Subjective Intervention

It has been a long standing reoccurring argument that recessions distort seasonally adjusted estimates (Dagum and Morry, 1985). The basic intuition is that official seasonal adjustment procedures confuse the downturn due to recessions with a change in seasonality and then propagate recession effects into the post-recession period as seasonal effects. This argument resurfaced among Wall Street analysts frustrated by their inability to forecast CES payroll employment following the Great Recession (Wieting, 2012).

Jonathan Wright also pursued this issue in his paper “Unseasonal Seasonals” (2013) where he argued that large declines in CES, especially following the collapse of Lehman Brothers in September 2008, were partially absorbed into seasonal factor estimates causing the winter effect to become more negative. Furthermore, he argued that in the following years this distortion in the seasonal factors biased up SA estimates in the winter and down in the rest of the year.

2.2 Data Driven Intervention

A number of statistical agencies have experimented with ramps to model recession shifts (Table 1). While standard software allows the fitting of ramps there is no automated method for doing so. Lytras and Bell (2013) propose analysts intervention using a systematic approach and broaden ramps to include quadratic behavior. We make use of a quadratic ramp in our testing.

The last three studies in Table 1 used X-11 filters to adjust their series. They found that recession pre-adjustments had little effect on the final seasonal adjustments. As discussed later, the results of our sensitivity testing provides an explanation for these empirical findings.

Table 1: Studies of Recession Related Modeling with Ramps

<table>
<thead>
<tr>
<th>Agency</th>
<th>Authors</th>
<th>Number of Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of Spain</td>
<td>Maravall &amp; Cañete (2012)</td>
<td>1</td>
</tr>
<tr>
<td>Bureau of the Census</td>
<td>Lytras &amp; Bell (2013)</td>
<td>23</td>
</tr>
</tbody>
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3. Robustness of X-11 Filters

Major disturbances in a series during a recession period do not necessarily imply inadequacies in the X-11 filters, even if model fits are improved by outlier identification. For this reason it is necessary to consider the robustness properties of the filters.
X-11 filters are cascade filters resulting from convolution of various filters derived from selected Henderson trend and seasonal filters. The properties of these filters were discussed in detail by Dagum et al. (1996). The symmetric Henderson filters are designed to reproduce polynomials up to third order; thus they can account for characteristics essential to fitting time series such as changes in growth rates and multiple turning points. The concurrent version of the Henderson filter can only track a linear trend-cycle within the span of the filter (usually 7 months), but still allows for non-linear patterns over a longer range of the series. Moreover, the concurrent filter becomes more adaptable with ARIMA forecasting.

Discontinuous change is more difficult to approximate, hence the emphasis on detecting level shifts in the standard approach (Findley, et al., 1998).

4. Sensitivity Testing of X-11 Filters

We consider 3 types of recession effects each of which take the form of a special type of ramp as shown in Table 2. In each case, we assume no change in the true seasonal pattern.

Table 2: Three Types of Ramps

<table>
<thead>
<tr>
<th>Type</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear ramp (LR)</td>
<td>constant rate of decline</td>
</tr>
<tr>
<td>Quadratic ramp (IQR, QRamp)</td>
<td>increasing rate of decline (IQR) or with an inflection point (QRamp)</td>
</tr>
<tr>
<td>Level shift (LS)</td>
<td>one period ramp</td>
</tr>
</tbody>
</table>

Given,

- \( t_0, t_1 = \) start, end point
- \( \lambda = \) total change in level, and
- \( w_i = \) proportion of total adjustment at \( t \)

we compute three alternative paths to a new lower recession level.

\[
LR_i = \lambda w_i \\
IQR_i = \lambda w_i^2 \\
LS_i = \lambda w_i, \ t_1 = t_0 + 1
\]

To motivate the selection of the magnitude (\( \lambda \)) and length (\( t_0, t_1 \)) of the decline we consider the behavior of the CES SA estimates during the Great Recession as shown in Figure 1. From January 2008 to February 2010 a total of 8.7 million jobs were lost which in relative terms was a 6.3% drop. Within this period (Feb08-Mar09) there was an accelerated decline of 4.3% with a loss of 5.9 million jobs. During this period Lehman Brothers went bankrupt and a world-wide financial panic followed.
Figure 1: U.S. Total Non-Agricultural Seasonally Adjusted CES

Jonathan Wright in his previously cited paper (2013) used fictitious linear ramps applied at the detailed industry level to replace real CES data from July 2008 to July 2009. The CES series with the linear ramps were seasonally adjusted and compared with the official estimates to provide a measure of bias. Wright criticized the Kropf and Hudson (2012) study for using ramps with longer periods. For our first set of test series we also focus on ramps that are no more than 12 months in length. We set the size of the total decline for each of the three alternatives to 5% and the length of decline to 12 months for LRamp and IQRamp. The length of decline for LS, of course, must always be one month. Later we double the length and magnitude of the decline.

The test series are read into the X-13 software with the extreme value adjustment of the input series turned off. Also, no forecasting is done. We selected these options to test the robustness of the original X-11 filters with no pre-processing or model assistance.

For filter selection we use the commonly selected X-11 options which consists of the Henderson 13-term and 3x5 seasonal filters. The effective half length of the symmetric X-11 cascade filters is three years. The total length of the one-sided concurrent cascade filters is also about 3 years.

Since the test series are not seasonal, the true seasonal component is equal to zero and the true SA series is equal to the observed series. Large non-zero estimated values for the seasonal component indicate that the recession disturbances, as represented by the ramps, are not well estimated; as a result the SA estimates will have a bias equal in magnitude to its difference from the observed series.

4.1 Test Series with a decline of 5% over one year or less
For each of the three types of recession effects, a three part piece-wise test signal is constructed with the first part being a pre-recession level set at 100, the last part a permanent lower level of 95 and the recession effect in the middle. For the level shift the...
entire adjustment occurs over the one month period January to February 2008 and for the linear and quadratic ramps the decline is spread over the period January 2008 to January 2009.

The level shift is by definition a discontinuous change but there are also discontinuities of smaller magnitudes in the test signals with linear and quadratic ramps. Within the adjustment period change is continuous but at either end there is a discontinuity as shown in Figure 2.

**Figure 2:** Rate of Change in Test Series Compared to CES SA Change

For the linear ramp the rate of change goes from zero to -0.42 at the start and back to zero at the end of the adjustment period. For the quadratic ramp the discontinuity is about twice as large but only occurs at the end of the adjustment period. Compared to the rate of change in the CES (dashed black line in Figure 2) these discontinuities are much larger in magnitude than what occurred in the CES. These test series thus represent a more severe test of the adaptability of the X-11 filters.

### 4.1.1 Response to linear ramp

The response of the symmetric X-11 filters to the LRamp is shown in Figure 3. The test series is represented by the open round markers, the output of the symmetric Henderson filter by the blue line, and the symmetric seasonal cascade filter by the red line. The vertical axis on the left-hand side measure the seasonal component; the right hand vertical axis measures the levels for the series and H13 filter output. The Henderson filter clearly reproduces the linear ramp and the estimated seasonal factors (SF) are all zero indicating no distortions to the seasonally adjusted series.
Figure 3: Response of Henderson Trend Filter (H13) and Cascade Seasonal Filter (SF) to Linear Ramp (LRamp)

4.1.2 Response to increasing quadratic ramp
Figure 4 shows the response of the X-11 filters to the quadratic ramp with an increasingly negative slope. The decline in the series is closely followed, but some over-smoothing occurs as the accelerating decline switches to zero change in January 2009. This discontinuity generates very small dips in the SF’s for January as well as slight blips in the SA series.

Figure 4: Response of Cascade Seasonal Adjustment Filter (SA) and Cascade Seasonal Filter (SF) to Increasing Quadratic Ramp (IQRamp)

4.1.3 Response to level shift
Figure 5 shows clear over-smoothing of the series with a level shift by the Henderson filter which misses 40% of the drop from January to February 2008 (dark marker).
Figure 5: Response of Henderson Trend Filter (H13) to Level Shift (LS)

Part of the drop in the level of the series is absorbed into the seasonal component (Figure 6). For 2008 the January seasonal factor shows a peak followed by a February trough. The seasonally adjusted series shows the complement of the SF pattern. The properties of the symmetric filters propagates forward and backward the recession distortions by three years (half length of the filters). Following the downward LS we see in the SA series the pattern of a winter recovery followed by a spring relapse.

Figure 6: Response of Cascade Seasonal Filter (SF) and Cascade SA Filter (TRD) to Level Shift (LS)

Figure 7 presents the percent errors in the SA estimates of month-to-month change for the three types of recession effects. The errors are defined as 100(ΔSA-Δseries)/SA. The dashed black line shows the percent change in the CES with the 2008-2009 recession effects replaced by pre-recession extrapolations. This provides a standard for judging how large the errors are relative to the normal change in the CES. Only a large discontinuity in the series caused by the LS generates much distortion in the SA estimates.
4.1.4 Response to extending the length of the seasonal filter
One possibility for making X-11 filters more robust to discontinuous recession effects is to extend the length of the seasonal filter from its most commonly selected 3x5 filter to the longer 3x9 filter. Figure 8 illustrates the effect of the longer seasonal filter on errors in month-to-month change in the SA estimates due to the level shift. The weights of the longer filter are lower for recent observations and higher for past observations. As a result, the 3x9 filter substantially reduces the magnitude of the LS for the year in which it occurred, 2008, and for the two years preceding and following that year, but magnifies the recession distortion for the third year and extends the distortion for two more years.

Figure 8: Percent Error in X-11 SA Estimates of Month-to-Month Change due to a LS for a 3x5 and 3x9 Seasonal Filter

Imposing longer filter lengths solely to reduce recession effects involves trading off one type of distortion for another. There is, however, a more general argument for using longer seasonal filters for X-11 based on research that shows that the seasonal filters selected automatically tend to be shorter than the length implied by the model based method (Depoutot and Planas. 1998, Chu, Tiao, and Bell, 2012).
4.1.5 Response of Real Time Concurrent filters

In real time the two-sided symmetric filters must be replaced with one-sided concurrent filters. The forecasting option is turned off, which reduces the flexibility of the filter and provides a more stringent test. The effect is shown in Figure 9. Compared to the symmetric filters the concurrent filter uses only current and past observations which results in no distortions in the pre-recession period. The weights, however, tend to be higher on the more recent observations and this results in magnifying the recession distortions in the current and succeeding years. Even so, the distortions are not much larger for the quadratic ramp and remain close to zero for the linear ramp.

![Figure 9: Percent Error in X-11 Concurrent SA Estimates of Month-to-Month Change for Three Types of Recession Effects](image)

4.2 Test Series with Ten Percent Decline over Two Years

As a final demonstration we create a new test series by doubling the drop in the level and the length of the decline by adding an inflection point to the quadratic ramp and extending.

![Figure 10: Quadratic Ramp with an Inflexion Point](image)
its length by one year, where the rate of decline decreases and reaches zero at the end of the recession. This test series tracks the actual change in the CES over its entire period of decline but has a more severe drop of 10% compared to 6.3% for the CES. This is illustrated in Figure 10. The estimated seasonal factors shown in Figure 11 are zero and this results in the SA series exactly reproducing the quadratic recession effect.

![Figure 11: Effect of a Quadratic Ramp with an Inflection Point (solid blue marker) on Seasonal Adjustment](image)

5. Conclusions and implications for Statistical Agencies

The results of our sensitivity tests show that the standard methodology is reasonably robust to recession effects. These results also suggest why the empirical studies cited found that modeling recession effects made little difference to the seasonal adjustments. X-11 filters respond accurately to changes in levels, no matter how large, as long as they are fairly continuous. Major discontinuities resulting from level shifts are especially disruptive, but automated outlier detection can be expected to identify and remove their distortionary effects from the estimation of the seasonal factors.

In the real world the presence of noise and the uncertainty of the future complicates the analysis of recession effects. Special analyst intervention is required to fit ramps and in this environment the risk of spurious identification increases. For government statistical agencies, which are accountable to the public for the methods they use and the results they publish, it makes sense to use intervention analysis cautiously to maintain transparency with their public.

References


Wieting, Steven C., “Broken Seasonal Factors or Changing Seasonal Patterns?”, 2012 Citibank Economic Commentary.