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The Measurement of Productive Capital Stock, Capital Wealth, and Capital Services

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Abstract

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In order to construct measures of multifactor productivty, the Bureau of Labor Statistics has investigated a number of issues. This paper discusses several related to the vintage aggregation of capital: a primary step in the measurement of capital input.

Unless a capital good's efficiency declines geometrically, its price as it ages will follow a different schedule than its efficiency. The price schedule can be calculated if we assume an efficiency schedule, a discount rate, and the vintage aggregation conditions. Capital services are proportional to a "productive stock" constructed from a perpetual inventory calculation using the age/efficiency schedule. The wealth represented by all assets is consistently estimated by doing a similar calculation using the corresponding age/price schedule. Attempts in the literature to establish that age/efficiency schedules are geometric by studying used asset prices fall short of doing so, because very different age/efficiency schedules can generate very similar age/price profiles. Arguments which rationalize the use of the geometric assumption even when efficiency does not decay geometrically lack merit. All things considered, there is little evidence regarding which age/efficiency pattern is correct, but mounting evidence that the vintage aggregation conditions are badly violated.

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I. Introduction

In capital theory, a careful distinction is made between declines in the efficiency of an asset and the depreciation of that asset. The efficiency of a used asset relative to a new one is defined as the marginal rate of technical substitution of the old asset for the new one. Depreciation is the change in the real price of an asset. That price reflects changes in the discounted stream of future services which can be expected from the asset. These two concepts of decline are strictly related in theory.

This relationship is well illustrated by a light bulb. If the bulb burns steadily for 1000 hours before the filament breaks, it loses no efficiency until its final hour. Yet, since inflation and interest are minor factors in the 6 odd weeks of its life, its value can be expected to decline in a straight line pattern. That is, after 500 hours, it should be worth one half what a new bulb costs (assuming changing and selling of bulbs is costless). In this case efficiency follows a one hoss shay pattern and price a straight line pattern. The efficiency function of an asset might take any conceivable form. Economists usually define initial efficiency as 100% and assume efficiency declines monotonically, approaching zero eventually. Besides the patterns we've mentioned, geometric decay functions, and a whole family of hyperbolic functions meet these criteria. Others are possible as well.

Economists frequently assume that the relative efficiency of an asset declines geometrically over time. This assumption has several advantages.

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Under geometric efficiency decline, depreciation is also geometric and proceeds at the same constant rate. Therefore, no distinction need be made in the measurement process. Furthermore, because of the constancy of this rate, no system of vintage accounting needs to be maintained. Finally, under geometric efficiency decline, depreciation is independent of the real own rate of interest. These properties are enjoyed only by the geometric form. In general efficiency decline and depreciation are different, their relationship depending on real interest rates. If one form is not geometric, neither is the other.

Thus it is not surprising that many economists have made attempts to establish whether geometric decay is consistent with the behavior of the real prices of used capital assets. These studies generally show that depreciation is relatively more rapid than straight line early in the life of an asset. These studies also often fail to reject the geometric form of depreciation statistically. They often conclude that this evidence suggests that geometric decay is not an unrealistic assumption for measuring depreciation or for measuring the efficiency of the capital stock. Jorgenson (1974) marshals a further argument based on renewal theory. Essentially, for a capital stock characterized by a stable age distribution and a stable growth pattern, replacement requirements tend to a constant rate regardless of the underlying efficiency patterns.

Counter to these views is the argument, based often on intuition, that the productive efficiency of assets does not decline nearly so rapidly as the geometric hypothesis requires. However, available studies of this type are vulnerable on theoretical grounds.

The purpose of this paper is to examine the issues related to the computation of a vintage aggregate. A central theme is the comparison of various alternative mathematical forms for the age/efficiency profile which must be assumed in order to build a vintage aggregaate. In Section II the theory of capital measurement is reviewed with particular attention to the duality between the efficiency and price of an asset. The age/efficiency profile must be used to estimate an asset's productive stock, to which services are proportional. The age/price profile must be used to estimate a stock of wealth, from which depreciation may be inferred. Both services and depreciation must be estimated for each type of asset in order to distinguish and aggregate asset classes of differing durabilities using modern procedures based on the rental price of capital. The assumption of geometric decay, which has been used in the past in conjunction with theoretically complete efforts to measure capital, avoids the distinction between productive capital and capital wealth. However, the existing vintage aggregation model is general enough that such simplification is unnecessary and incorrect if efficiency does not really decline geometrically.

In Section III we construct eight different efficiency functions and then calculate their dual age/price profiles under a variety of interest rate, asset life, and discard pattern assumptions. Several observations are made which are important for our examination of the issues in Section IV. There we evaluate arguments which have been made in favor of particular efficiency functions. A major point raised is that the empirical studies, which claim to support geometric decay, fall short of doing so. That work is based on estimating age/price profiles. A wide range of efficiency patterns which are concave to the origin are associated with convex age/price profiles. Existing empirical studies have by no means estimated age/price profiles well enough to identify the age/efficiency pattern. In addition we evaluate the renewal theory arguments which rationalize use of geometric decay even if assets do not decay geometrically. Using data for the U.S. Manufacturing sector, we demonstrate important problems which can result if efficiency does not in fact decay geometrically. We also introduce an analytic statistic, the ratio of implicit rent to capital cost, as a possible criterion for evaluating various efficiency assumptions, and, more fundamentally, for detecting when the entire vintage aggregation procedure is failing.

In Section V, we conclude that the existing literature and evidence offers little basis for preferring any one efficiency form. In this complicated area, it would seem common sense should carry as much weight as any evidence which economists have yet assembled.

II. Depreciation and the Theory of Capital Measurement

In this section we review a model for measuring capital services. This model consists of two major steps: the aggregation of past investments for a particular type of capital asset, and the aggregaton of different types of assets into an overall "measure" of capital. Regarding the first, or vintage aggregation step, we appeal to the duality between an asset's efficiency and real price in order to propose a distinction in types of vintage aggregates. One can construct a "productive stock" based on the age/efficiency function, and a "wealth stock" based on the age/price profile. This distinction evaporates only if we presume geometric decay. Regarding the asset aggregation step, we point out that the rental price formulation, which is used to weight differing assets, and which has previously been used only in tandem with the geometric assumption, can easily be used with other efficiency assumptions.

The capital stock of a particular type of asset is generally measured as a weighted average of past investments where the weights reflect the relative efficiencies of various capital vintages. These weights are conventionally normalized to one for new investment, with monotonic declines as the asset ages. We shall refer to this as vintage aggregation. Fisher (1965) has shown that capital augmenting technical change, or an equivalent assumption, is a precondition for consistent vintage aggregation. In one form of the assumption, it is evident that vintage weights must be independent of exogenous influences such as output demand and relative factor prices. More generally, Hall (1968) has shown that some normalization rule is required in order to distinguish the vintage weights from exponential rates of embodied and disembodied technical change. Hall suggests that fixing the relative efficiency of assets of different ages as time passes is the most useful rule, The process of constructing a vintage aggregate based on past investments weighted by relative efficiencies is frequently called the perpetual inventory method. We shall refer to the resulting stock measure as the "productive stock" of capital. It has a clear interpretation as the answer to the following question. "How much new investment would be required to produce the same present services as the existing stock of assets?" Age/efficiency functions should be constructed in such a way that the real capital services, and implicitly the rents, generated by capital of various vintages in a given period are proportional to their relative efficiencies. Under these conditions, the productive stock is proportional to capital services. Hence it is the correct measure of the real capital input quantity for a particular asset class for productivity measurement and econometric production function analysis.

In order to measure productivity or otherwise model production, it is necessary to construct a rental price for capital. If there were only one type of capital, we could use the assumption that factors of production are paid their marginal products, and simply divide the productive stock into total capital income (ie. before tax profits plus net interest plus depreciation plus subsidies plus indirect business taxes) to arrive at an estimate of the current rental price. Christensen and Jorgenson (1969) derive a more complicated method for the case where there are several assets of different durabilities. Essentially, relatively more of current capital income should be allocated to less durable assets because their rapid depreciation entails more cost. In other words, the faster the depreciation of an asset class, the higher its rental price.

Therefore, in order to construct rental prices, it is necessary to estimate depreciation by type of asset. Unfortunately, depreciation cannot generally be directly inferred from the productive capital stock measure. In order to measure depreciation, it is necessary to recognize the duality between an asset's age/efficiency function and its age/price function or profile. An asset's market price can be expected to equal the discounted stream of services (rents) which it will generate. As Arrow (1964) and Hall (1968) have shown, the age/price profile can be derived from the age/efficiency function by integrating the later (weighted by a discount factor) for assets of various ages. Jorgenson (1974) replicates this work for the discrete case. In Section III we study several simulations of age/price profiles. For now it is sufficient to point out that the age/price profile is different from the age/efficiency function unless geometric decay is assumed.

Once an age/price profile is derived, it is possible to perform a second perpetual inventory calculation, weighting past investments with the age/price profile. We refer to the resultant measure as the "stock of capital wealth" or the "wealth stock". This measure can be interpreted as the real value of the existing stock of assets, based on the discounted stream of future services expected from them. Depreciation can be inferred directly from this second perpetual inventory calculation as the drop in value of the previous period's wealth stock before adding in new investment. The following exemplifies the distinction between the productive and wealth stocks. Inflation and technological change are disregarded. Suppose your house is broken into and a five year old television, which is in excellent condition, is taken. Your insurance company may reimburse you for the "fair market value" of the item, that is, they will allow for inflation in TV prices, but will deduct for depreciation. If you wish to continue watching television, and lack access to or distrust the used television market, you may prefer to purchase a new set. The chances are the new set will cost more than you were reimbursed. The old set should have been included in the wealth stock at its fair market value. The old set should have been included in the productive stock at the value it costs to exactly replace its services with a new set.

Of course these distinctions between productive and wealth stocks and efficiency loss and depreciation are unnecessary in capital studies which presume geometric decay. The Christensen-Jorgenson (1969) study, and subsequent studies including Gollop-Jorgenson (1980) and Frameni-Jorgenson (1980) have made the simplifying assumption of geometric decay. However, the Hall (1968) or Jorgenson (1974) papers assume a general form of efficiency decline, and formulate rental prices in terms of the rate of depreciation as distinct from the rate of efficiency decline.

In the following Section we compute and discuss a variety of age/price profiles for a variety of age/efficiency profiles. Then in Section IV we evaluate various studies and arguments which have attempted to identify the "correct" efficiency pattern. In the process, we introduce new evidence based on an experimental study of capital in the U.S. Manufacturing sector.

III. Alternative Patterns of Decline

In this section we generate depreciation patterns for a variety of patterns of efficiency decline, interest rates, and asset lives. This work forms the foundation for a critical comparison of the various fixed assumptions about decline which underlie a large number of economic studies. It also serves as a tool by which to evaluate the conclusions of econometric studies of market declines in asset prices.

In each of our trials, exhibited in Tables 1 thru 5, we compare real price patterns associated with assumed efficiency patterns. These patterns represent most of the methods commonly used by economists. They fall into three major types: the hyperbolic family, double declining balance, and geometric. The hyperbolic methods are formulated as

$$S_t = (L-t) / (L-Bt) \qquad (1)$$

where s_t is the relative efficiency of a t-year old asset. One hoss shay (gross stock), and straight line are special cases where B = 1 and B = 0, respectively. Regardless of B, $S_0 = 1$ and $S_L = 0$, that is a new asset's value is indexed to one and an asset's value reaches 0 at the end of its life. In order to ensure that S_t is not less than S_{t-1}, B must not exceed 1. For geometric decay,

$$S_t = S_{t-1} * (1-2/L) \qquad (2)$$

In other words, an asset's efficiency declines at a constant rate. We follow Jorgenson's usual convention of using the double declining balance rate. Lastly, we define double declining balance efficiency decay as the same as geometric until the straight line decay rate becomes faster than the double declining rate. At that point, a straight line is used to ensure that an asset's efficiency vanishes at time t=L. This form provides us with a critical comparison for geometric decay, since it permits us to evaluate how heavily the special simplifying properties for geometric decay depend on its infinite tail.

On each of these tables, for each efficiency pattern, we have estimated an age/price profile using the following formula

$$P_t = \frac{\sum_{\tau=t}^{\infty} S_\tau (1-r)^{\tau-t}}{\sum_{\tau=0}^{\infty} S_\tau (1-r)^{\tau}} \qquad (3)$$

where r is an assumed real discount rate which is the same in all periods. Summations were truncated after 200 years with negligible effects.

Table 1, our basic case, is based on an assumed own interest rate of 4% and an asset life of ten years. Tables 2 and 3 are based on different interest rates (0% and 8%), each with the ten year life. Table 4 is based on 4% interest and a 50 year asset life. In Table 4, real asset prices are presented only every five years to permit ready comparison with Table 1. Finally, Table 5 makes use of the same service life (10 years) and interest rate (4%) as Table 1, but postulates a normally distributed pattern of discards, with a mean life of 10 years, and a standard deviation of 2 1/2 years, truncated at 5 and 15 years.

In the tables, decay assumptions are arrayed in order of increasingly rapid patterns of early efficiency decline. We will refer to decline and depreciation patterns which are slower than straight line as concave, and patterns which are more rapid as convex. These terms refer to the usual plot of efficiency and market value against the age of an asset (see Figure 1). We have carefully selected eight patterns of efficiency decline for study, which appear in the upper half of each of Tables 1 thru 5. Straight line depreciation is used by BEA (with a discard function) to determine capital consumption allowances. Double declining balance frequently appears in tax accounting, and one hoss shay has sometimes been used by economists to represent efficiency. Denison (1979) uses a 3-1 weighted average of one hoss shay and straight line stocks. Jorgenson and many others use geometric decay at a double declining balance rate. Faucett, in work done with the U.S. Bureau of Labor Statistics (1979) Office of Economic Growth uses hyperbolic decay with B values of .9 for structures and .75 for equipment. We also present results for B values of .5 since this is intermediate between one hoss shay (B=1) and straight line (B=0). Finally we present hyperbolic decay for B= -1.25. This particular value was chosen because it yields an efficiency function (Table 1) which tracks closely with geometric and double decline balance patterns, for an asset with a ten year life, until the seventh or eight year.

In the lower half of each Table (1 thru 5) we present the duals to the upper half: the relative price functions.

Some general observations regarding the tables are in order. First, except for geometric decay, depreciation always proceeds faster than efficiency decline. This result is independent of choice of interest rate or asset life,

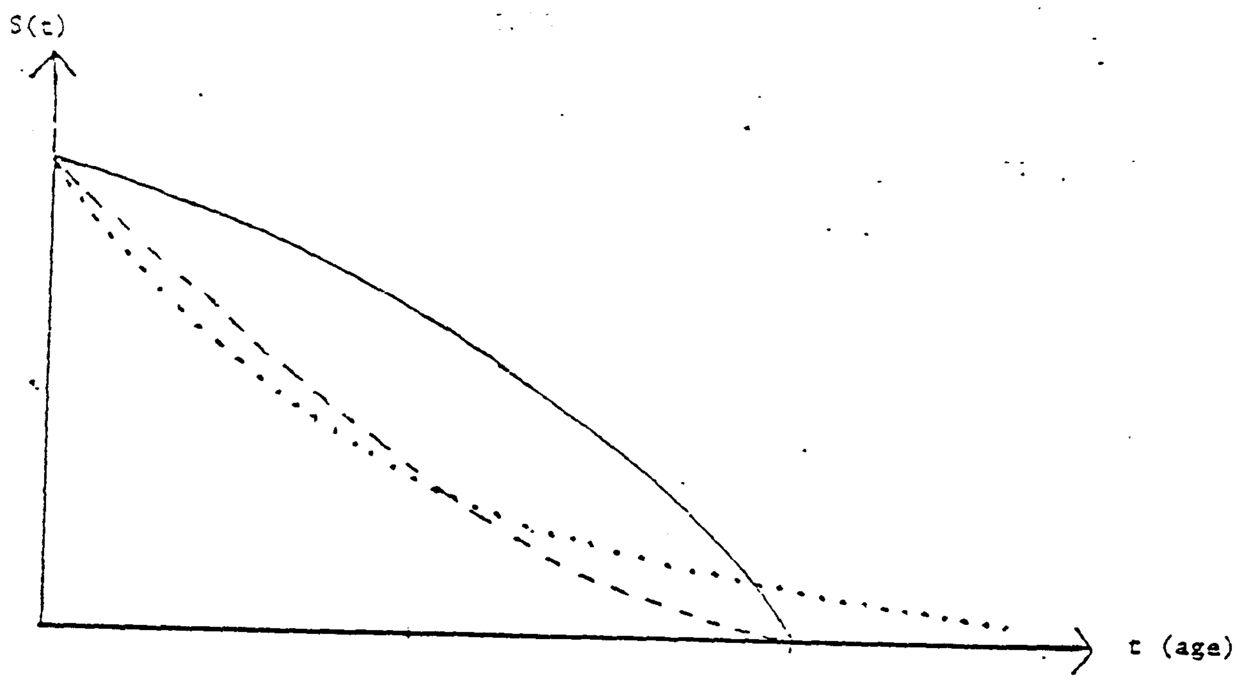

Figure 1. Asset Age/Efficiency and Age/Price Profiles.

For an asset with a ten year life and four percent real interest, the solid line represents efficiency, and the dashed line represents real market price for an asset based on hyperbolic efficiency decline, defined as S(t) = 10-t/(10-.5t), ie. with B = .5. The dotted line represents both efficiency and market price based on geometric efficiency decline, defined as S(t)=S(t-1).(1-2/10), ie at the double declining balance rate.

TABLE 1. EFFICIENCY AND REAL PRICE: BASIC ASSUMPTIONS

INTEREST=0.04 ASSET LIFE= 10

YEAR	ONE HOSS SHAY	HYPERBOLIC B=.9	HYPERBOLIC B=.75	HYPERBOLIC B=.5	STRAIGHT LINE	GEOMETRIC	DOUBLE DECLINING	HYPERBOLIC B=-1.25
				RELATIVE EFFICIENCY				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	100.0	98.9	97.3	94.7	90.0	80.0	80.0	80.0
2	100.0	97.6	94.1	88.9	80.0	64.0	64.0	64.0
3	100.0	95.9	90.3	82.4	70.0	51.2	51.2	50.9
4	100.0	93.7	85.7	75.0	60.0	41.0	41.0	40.0
5	100.0	90.9	80.0	66.7	50.0	32.8	32.8	30.8
6	100.0	87.0	72.7	57.1	40.0	26.2	26.2	22.9
7	100.0	81.1	63.2	46.2	30.0	21.0	19.7	16.0
8	100.0	71.4	50.0	33.3	20.0	16.8	13.1	10.0
9	100.0	52.6	30.8	18.2	10.0	13.4	6.6	4.7
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0
				RELATIVE PRICE				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	91.7	90.1	88.4	86.2	82.9	80.0	77.6	76.8
2	83.1	79.9	76.7	72.8	67.1	64.0	59.6	58.1
3	74.2	69.5	65.0	59.8	52.9	51.2	45.0	42.9
4	64.8	59.0	53.5	47.5	40.2	41.0	33.3	30.8
5	55.1	48.2	42.2	36.0	29.1	32.8	23.8	21.1
6	44.9	37.4	31.3	25.6	19.6	26.2	16.1	13.6
7	34.4	26.8	21.1	16.4	11.9	21.0	9.8	7.9
8	23.4	16.5	12.1	8.8	6.1	16.8	5.0	3.8
9	11.9	7.1	4.7	3.1	2.0	13.4	1.7	1.2
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0

TABLE 2. EFFICIENCY AND REAL PRICE: ZERO REAL INTEREST

INTEREST=0.00 ASSET LIFE= 10

YEAR	ONE HOSS SHAY	HYPERBOLIC B=.9	HYPERBOLIC B=.75	HYPERBOLIC B=.5	STRAIGHT LINE	GEOMETRIC	DOUBLE DECLINING	HYPERBOLIC B=-1.25
				RELATIVE EFFICIENCY				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	100.0	98.9	97.3	94.7	90.0	80.0	80.0	80.0
2	100.0	97.6	94.1	88.9	80.0	64.0	64.0	64.0
3	100.0	95.9	90.3	82.4	70.0	51.2	51.2	50.9
4	100.0	93.7	85.7	75.0	60.0	41.0	41.0	40.0
5	100.0	90.9	80.0	66.7	50.0	32.8	32.8	30.8
6	100.0	87.0	72.7	57.1	40.0	26.2	26.2	22.9
7	100.0	81.1	63.2	46.2	30.0	21.0	19.7	16.0
8	100.0	71.4	50.0	33.3	20.0	16.8	13.1	10.0
9	100.0	52.6	30.8	18.2	10.0	13.4	6.6	4.7
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0
				RELATIVE PRICE				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	90.0	88.5	86.9	84.9	81.8	80.0	77.0	76.1
2	80.0	77.1	74.2	70.6	65.5	64.0	58.6	57.1
3	70.0	65.9	61.9	57.2	50.9	51.2	43.8	41.8
4	60.0	54.9	50.0	44.8	38.2	41.0	32.1	29.7
5	50.0	44.1	38.8	33.4	27.3	32.8	22.6	20.1
6	40.0	33.6	28.4	23.4	18.2	26.2	15.1	12.8
7	30.0	23.6	18.8	14.7	10.9	21.0	9.1	7.3
8	20.0	14.3	10.6	7.8	5.5	16.8	4.5	3.5
9	10.0	6.1	4.0	2.7	1.8	13.4	1.5	1.1
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0

TABLE 3. EFFICIENCY AND REAL PRICE: HIGH REAL INTEREST

INTEREST=0.08 ASSET LIFE= 10

YEAR	ONE HOSS SHAY	HYPERBOLIC B=.9	HYPERBOLIC B=.75	HYPERBOLIC B=.5	STRAIGHT LINE	GEOMETRIC	DOUBLE DECLINING	HYPERBOLIC B=-1.25
				RELATIVE EFFICIENCY				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	100.0	98.9	97.3	94.7	90.0	80.0	80.0	80.0
2	100.0	97.6	94.1	88.9	80.0	64.0	64.0	64.0
3	100.0	95.9	90.3	82.4	70.0	51.2	51.2	50.9
4	100.0	93.7	85.7	75.0	60.0	41.0	41.0	40.0
5	100.0	90.9	80.0	66.7	50.0	32.8	32.8	30.8
6	100.0	87.0	72.7	57.1	40.0	26.2	26.2	22.9
7	100.0	81.1	63.2	46.2	30.0	21.0	19.7	16.0
8	100.0	71.4	50.0	33.3	20.0	16.8	13.1	10.0
9	100.0	52.6	30.8	18.2	10.0	13.4	6.6	4.7
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0
				RELATIVE PRICE				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	93.3	91.6	89.7	87.4	83.8	80.0	78.1	77.3
2	86.1	82.6	79.1	74.8	68.7	64.0	60.4	59.0
3	78.2	73.1	68.1	62.4	54.8	51.2	46.1	44.0
4	69.6	63.0	56.9	50.3	42.1	41.0	34.4	31.9
5	60.3	52.4	45.6	38.7	30.9	32.8	24.9	22.1
6	50.1	41.4	34.4	27.8	21.1	26.2	17.0	14.3
7	39.1	30.1	23.6	18.1	13.0	21.0	10.5	8.4
8	27.2	18.9	13.7	9.8	6.7	16.8	5.4	4.1
9	14.1	8.3	5.4	3.6	2.3	13.4	1.8	1.4
10	0.0	0.0	0.0	0.0	0.0	10.7	0.0	0.0

TABLE 4. EFFICIENCY AND REAL PRICE: LONG ASSET LIFE

INTEREST=0.04 ASSET LIFE= 50

YEAR	ONE HOSS SHAY	HYPERBOLIC B=.9	HYPERBOLIC B=.75	HYPERBOLIC B=.5	STRAIGHT LINE	GEOMETRIC	DOUBLE DECLINING	HYPERBOLIC B=-1.25
				RELATIVE EFFICIENCY				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
5	100.0	98.9	97.3	94.7	90.0	81.5	81.5	[illegible]
10	100.0	97.6	94.1	88.9	80.0	66.5	66.5	[illegible]
15	100.0	95.9	90.3	82.4	70.0	54.2	54.2	[illegible].9
20	100.0	93.8	85.7	75.0	60.0	44.2	44.2	40.0
25	100.0	90.9	80.0	66.7	50.0	36.0	36.0	30.8
30	100.0	87.0	72.7	57.1	40.0	29.4	28.8	22.9
35	100.0	81.1	63.2	46.2	30.0	24.0	21.6	16.0
40	100.0	71.4	50.0	33.3	20.0	19.5	14.4	10.0
45	100.0	52.6	30.8	18.2	10.0	15.9	7.2	4.7
50	0.0	0.0	0.0	0.0	0.0	13.0	0.0	0.0
				RELATIVE PRICE				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
5	96.6	94.3	92.1	89.4	85.3	81.5	80.2	78.2
10	92.5	87.7	83.4	78.2	71.1	66.5	63.7	60.2
15	87.4	80.0	73.7	66.6	57.5	54.2	50.0	45.5
20	81.2	71.1	63.0	54.6	44.8	44.2	38.3	33.2
25	73.5	60.8	51.5	42.6	33.1	36.0	28.2	23.2
30	64.1	49.2	39.4	30.8	22.7	29.4	19.3	15.0
35	52.6	36.2	26.9	19.8	13.8	24.0	11.7	8.7
40	38.5	22.3	15.0	10.3	6.7	19.5	5.7	4.0
45	21.2	8.7	5.1	3.2	2.0	15.9	1.7	1.1
50	0.0	0.0	0.0	0.0	0.0	13.0	0.0	0.0

TABLE 5. EFFICIENCY AND REAL PRICE: NORMALLY DISTRIBUTED DISCARDS

INTEREST=0.04 ASSET LIFE= 10

YEAR	ONE HOSS SHAY	HYPERBOLIC B=.9	HYPERBOLIC B=.75	HYPERBOLIC B=.5	STRAIGHT LINE	GEOMETRIC	DOUBLE DECLINING	HYPERBOLIC B=-1.25
				RELATIVE EFFICIENCY				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	100.0	98.8	97.1	94.4	89.4	80.0	79.1	81.1
2	100.0	97.3	93.6	88.1	78.9	64.0	62.8	65.9
3	100.0	95.4	89.3	80.9	68.3	51.2	49.6	53.7
4	100.0	92.7	83.6	72.7	57.8	41.0	38.6	43.9
5	100.0	88.1	76.2	63.0	47.2	32.8	29.4	35.7
6	96.6	80.2	66.2	52.1	37.0	26.2	21.7	28.5
7	90.3	69.5	54.4	40.7	27.5	21.0	15.3	22.1
8	80.2	56.2	41.6	29.7	19.2	16.8	10.2	16.2
9	66.3	41.8	29.2	19.9	12.3	13.4	6.4	11.0
10	50.0	28.0	18.5	12.1	7.2	10.7	3.6	6.8
11	33.7	16.5	10.3	6.4	3.7	8.6	1.8	3.6
12	19.8	8.3	4.9	2.9	1.6	6.9	0.8	1.6
13	9.7	3.3	1.3	1.0	0.5	5.5	0.3	0.5
14	3.4	0.7	0.4	0.2	0.1	4.4	0.0	0.1
15	0.0	0.0	0.0	0.0	0.0	3.5	0.0	0.0
				RELATIVE PRICE				
0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1	92.1	90.1	88.3	86.1	82.8	80.0	76.7	79.3
2	83.9	80.0	76.6	72.7	67.1	64.0	58.1	62.4
3	75.4	69.6	65.0	59.8	53.0	51.2	43.3	48.7
4	66.5	59.1	53.6	47.7	40.6	41.0	31.5	37.3
5	57.2	48.5	42.5	36.6	30.0	32.8	22.2	28.0
6	47.5	38.1	32.2	26.8	21.1	26.2	15.0	20.3
7	37.9	28.4	23.1	18.5	14.1	21.0	9.7	14.0
8	28.6	19.9	15.5	12.0	8.8	16.8	5.8	9.1
9	20.1	12.3	9.5	7.1	5.1	13.4	3.3	5.5
10	13.0	7.4	5.3	3.8	2.6	10.7	1.7	2.9
11	7.5	3.8	2.6	1.8	1.2	8.6	0.7	1.4
12	3.7	1.6	1.0	0.7	0.5	6.9	0.3	0.5
13	1.5	0.5	0.3	0.2	0.1	5.5	0.1	0.2
14	0.4	0.1	0.1	0.0	0.0	4.4	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	3.5	0.0	0.0

at least in the meaningful range. Furthermore, a wide variety of efficiency patterns give rise to convex price patterns, including some of the concave efficiency patterns. In particular, straight line decay and hyperbolic decay with B=.5, yield convex patterns in all of our trials. Even very high values of B give convex price patterns for shorter asset lives and lower interest rates.

On Tables 1 thru 3, the price functions for the geometric double declining balance, and hyperbolic (with B= -1.25) efficiency functions are generated from efficiency patterns which are nearly identical for the first five years. For each interest rate, geometric decay gives significantly higher asset prices, due to lower depreciation, than the other two, even in the early years when efficiency patterns are the same. The differences range from 2.5 -5% in the first year and are as high as 35% in the fourth year. This disparity is remarkable, and we will argue later that the high levels of geometric efficiency near the end of the asset's life (still 10.7% at the assumed end of its life), and the nonzero levels of efficiency which extend well beyond the asset's life, account for this difference.

Some additional observations are in order. In general, higher interest rates lead to slower depreciation patterns. This is true for all the patterns tested in Tables 1 thru 5, except for geometric, which is independent of interest rates. This observation corresponds to the common sense notion that future declines in efficiency should be of less importance when higher interest rates deflect more weight to the early part of the efficiency pattern.

For similar reasons, the longer service life (Table 4) produces slower depreciation for identical efficiency patterns and identical positive interest rates (Table 1). Under positive interest, events in the distant future have less effect on current depreciation. Again, geometric decay is unaffected due to its special shape. (Actually, geometric efficiency and price patterns in Table 4 are slightly higher than in Table 1. This is because of our strict adherence to the double declining balance rate (equation 2). Additional compounding, from extra subdivisions of the asset's life, causes this effect.)

The observation that for nongeometric forms, depreciation tends to become slower as the asset life lengthens is important. Empirical studies of depreciation, such as Hulten and Wykoff (1980), find more rapid depreciation, relative to the asset life, for equipment than for structures. Therefore, a single nongeometric form of the efficiency function can be consistent with both the observed slower depreciation associated with longer lived assets such as structures and with the relatively faster depreciation associated with equipment. A geometric form can produce the slow depreciation patterns observed for structures only by assuming a slow decay rate. This slow rate implies extraordinary levels of capital services well beyond the presumed end of the asset's useful life.

Careful examination of the hyperbolic series for B=.9 in Table 4 reveals a curious possiblity. Initially real asset prices are higher than the straight line efficiency pattern, but later in the life they become lower. In other words, the asset price curve, of the type plotted in Figure 1, is inflected. It is characterized by early concavity, and later convexity. This can be attributed to the interaction of two effects. The interest rate effect combined with long asset life (which we discussed above) dominates in early years. In later years, the general tendency for asset prices to take a convex form dominates. Indeed, under zero interest rates, one hoss shay efficiency based asset prices take a straight line form (Table 2) while all other forms are convex. This tendency toward convex price patterns seems to dominate the interest effect for all but the longest asset lives, highest interest rates, and highest B values. Even then, convexity of prices occurs in the later stages of an asset's life. It is little wonder that researchers find convex prices in market data on used assets.

One might imagine that the observations we have made about efficiency and price in this section would be different if we considered the separate life histories of each member of a cohort of investment goods. To address this question, we have constructed Table 5 in a manner similar to Table 1, except that each of the finite efficiency functions (those other than geometric) are constructed presuming a normally distributed pattern of discards. The discard pattern itself can be seen by looking at the one hoss shay column of Table 5, where only discards, and not relative efficiency losses, come into play. The asset is presumed to have a mean service life of 10 years. The distribution is truncated at 2 standard deviations (5 and 15 years). In columns 2-5 and 7-8, the same discard pattern is presumed, and, in addition, a separate efficiency function is calculated using the given formula for efficiency, for each possible lifetime between 5 and 15 years. The various efficiency functions are then integrated (using a discrete approximation) into a single, cohort efficiency function using the normal density function to weight the various cohort elements.

Our observation is that Table 5 closely resembles Table 1. The two are strikingly similar during the first five years, differing somewhat in the late years as they obviously must. The major observations, such as that price decline is relatively faster than efficiency except in the geometric case, are unaffected. We note that we have not subjected geometric decay to a discard pattern. Dick McDonald, of the BLS Office of Research and Evaluation, has pointed out that the efficiency pattern for such a cohort would only be geometric if the distribution of discards were geometric. That is, it is unlikely that a cohort efficiency function would be geometric, even if members of the cohort have geometric efficiency patterns. Therefore there is no equivalence of efficiency and price patterns unless a cohert's distribution of discards is geometric.

IV. An Evaluation of Alternate Efficiency Assumptions

In this section we review and evaluate various arguments which have been put forth in support of particular mathematical forms for the efficiency function. We find reasons to prefer a concave function, or relatively slow form of decay, based on several arguments. In evaluating arguments favoring convex forms, particularly the geometric form, we find the evidence or reasoning is inadequate to support a conclusion of convex efficiency. Our overall assessment of the situation is that evidence is inadequate to conclusively establish any particular form.

First we review the common sense argument for a concave form. Then we evaluate the possibility that obsolescence can account for a convex form. Next we compare the opinions of various major productivity measurement experts. We observe that the work of those who have used highly concave forms can be challenged because of its failure to address the requirements of theory, not because of the concave forms. Next we evaluate several attempts at ascertaining the shape of the efficiency function by indirect means, and find that, if anything, they point toward concave efficiency. Next we review defenses of geometric decay based on renewal theory, and find them groundless unless cohorts actually decay geometrically. Finally we introduce a new analytic ratio, the ratio of rent to cost, as an empirical criterion by which to judge the success of the capital measurement process.

Realism

There are a number of important productivity researchers, including Denison (1979), Diewert (1980), and Kendrick (1980), who have questioned the geometric form on the grounds that it is either unrealistic or represents an extreme view. If one selects a fast rate of geometric decay, as does Jorgenson, who generally assumes the double declining balance rate, a capital good must lose half of its efficiency in the first third of its life. Whether you consider a car, a typewriter, or a building, this seems unreasonable. Although some of these items may lose value this quickly, it is difficult to imagine how some combination of physical deterioration and increased maintenance costs would cut their efficiency so quickly. For example, in a majority of cases a two-year-old car can provide substantially the same transportation services as a new car even considering realiability, current maintenance and appearance. At the same time the older car will sell for 20% to 40% less because of the accumulated, though hidden, wear and tear which will require major maintenance two years sooner. If one selects a slower rate of geometric decay, as Hulten and Wykoff (1980) suggest, one is confronted with a substantial tail of services well beyond the end of an asset's presumed life span. With a concave efficiency form, one does not need to juggle these two unrealistic possibilities.

One hoss shay (gross stocks) and geometric seem to represent opposite extremes on the range of likely shapes. Capital goods rarely gain efficiency as they age, as would be implied were one hoss shay not concave enough. On the other hand, it would be debatable whether goods were durable enough to be considered capital if they yielded the majority of their services much more quickly than the geometric assumption requires.

Obsolescence

Technological obsolescence might be regarded as a reason for assuming a geometric deterioration pattern. The reasoning is that new capital goods are relatively more efficient than older ones since their design reflects advances in technology. This view cannot be reconciled with the vintage aggregation model. Another perspective on this phenomenon can be gained by considering technological progress instead of obsolescence. Progress is reflected in improved capital goods in the form of acquisition prices which are relatively higher than those of older models. That is, if our real investment figures are properly measured, they reflect real improvements in capital's design which allow it to be more efficient in production. For example, if expenditures on capital remain constant while innovations are improving capital, so that one dollar's worth provides more services, then the investment deflator should be declining and the real investment measure should be increasing. Thus, real investment figures should reflect adjustment for technology-induced quality change of capital goods.

Of course, there may be measurement error in our investment deflators which cause them not to capture all innovation. There are well known difficulties involved in detecting and measuring innovations and linking them when estimating our price deflators. We should not correct for this by using a faster deterioration pattern. The correction should be made when separating investment expenditures into real quantity and price. Equivalently, we could change our relative efficiency normalization rule (that new goods have a value of one) so as to place more weight on newer vintages. However, to use a relatively faster deterioration pattern in order to represent improvements in technology would bias our capital measures toward slower growth. What is essentially an improvement in new capital goods would be misrepresented as a loss in the productive capabilities of older goods.

A theoretical problem arises when older capital goods become systematically underutilized. For example, the electric power industry tends to use its oldest plants only during periods of peak demand. The annual service flow from these plants is diminished because of their technological inferiority to new plants, even though the oldest plants may resemble one hoss shay's in the sense that they could still do the same job they originally did.

A violation of the vintage aggregation assumptions is inherent in this scenario. In this example, utilization responds to exogenous influences differently depending on the age of the asset. Whether those influences include technology, relative prices (such as that of energy) or other external forces, they contradict one or more of the equivalent forms of the vintage aggregation conditions discussed by Fisher (1965). When reality contradicts a maintained hypothesis, we must either modify the hypothesis or awknowledge the problem. Hall (1968) has shown that we may make a correction, within the vintage aggregation framework, to the extent that embodied technical change, disembodied technical change, and efficiency contain exponential (geometric) components. That is, if underutilization is systematically a function of age (disembodied technical change) and efficiency has a geometric component, we may correct for underutilization, within the vintage framework, by speeding up the geometric efficiency decay rate. Later in this section we present evidence that suggests underutilization is a function of other variables besides age. What is more, since the finite efficiency forms we have considered do not contain geometric factors, they would not become geometric functions even if they were scaled by geometric factors.

Our conclusion on this issue is that a geometric form cannot be justified by reference to the obsolescence issue. Systematic underutilization due to age is unlikely to be a large enough effect to make the large adjustment one would need to get from a concave to a convex efficiency form. Even if it were, the resulting function, though concave, would not be geometric, and would not have the special properties of geometric decay. Nonsystematic (function of variables other than age) underutilization cannot be accounted for under the vintage aggregation assumption. Technological effects which do not result in efficiency losses for used assets should be counted as improvements in new capital, and accounted for in the deflator for new capital goods. Thus consideration of technology may result in serious biases in the capital measure, only part of which can be represented by scaling the efficiency function, and none of which can be represented by assuming a geometric efficiency function when fully utilized goods do not decay geometrically.

Major Productivity Studies

Unfortunately, the capital work of productivity economists who disbelieve geometric decay has proven vulnerable in its failure to recognize the distinction between efficiency decline and depreciation. Although each of these researchers reveals in his writings that he is aware of the distinction, none makes adequate use of it in his empirical work. Denison (1979) weights gross and straight line stocks 3-1 for his estimate of the productive stock of capital, distinguishing it from the straight line depreciation used to compute capital consumption in the NIPA. If he were then to use NIPA adjusted capital consumption allowances as an estimate of depreciation when pricing capital, he might have reasonably consistent measures of productive capital and depreciation. Unfortunately, he commits a serious theoretical error by failing to count depreciation at all when determining capital's share. Kendrick (1980) has been disuaded from this position by Jorgenson. Since Kendrick uses gross stocks for productive capital, and since, implicitly, straight line depreciation underlies his estimate of capital's price and share, his estimates of the effects of capital on productivity reflect measurement practices that are reasonably consistent with the theory. However Kendrick does not distinguish assets by rental price. The BLS/Faucett (1979) stocks exhibit a similar rate of decline to the Denison/Kendrick ones, except that they are formulated from the hyperbolic method. This work does not address the effects of capital on multifactor productivity.

A weakness of the work of Kendrick, Denison, and Faucett is that they aggregate capital assets directly. Jorgenson and his associates, and more recently Norsworthy, Harper, and Kunze (1979), distinguish between capital stock and capital services by weighting assets, with different rates of depreciation by their rental prices. However, Norsworthy and Harper (1981) fail to distinguish between asset efficency and real asset price. Their productive capital stocks, and the depreciation element in their rental price equations, are both based on straight line decline patterns as computed by BEA. This just follows the work of BEA, as outlined by Young and Musgrave (1980), which also fails to make a distinction. However, this is not a mistake from the BEA point of view since their purpose is only to measure depreciation and wealth, not productivity and productive capital. Jorgenson avoids this theoretical problem by using geometric decay. Here both depreciation and efficiency decline are identical.

Therefore, only users of the Jorgenson approach have empirical work on capital and productivity which is fully consistent with state of the art economic theory. Kendrick's work at least recognizes the duality issue. Jorgenson and Kendrick stand at the two extremes regarding the speed at which deterioration occurs. As we have mentioned in Section II, it is unnecessary to assume a (possibly) unrealistic efficiency form in order to fully conform with existing theory. It is unclear whether it is worse to use an incorrect efficiency form or to ignore theory. It is clear that neither mistake is necessary under the maintained model.

Indirect Measures of Efficiency

Lack of data on the relative efficiency of various vintages in production has lead to attempts to observe efficiency by indirect means. Two major groups of such indirect attempts exist. In one group, depreciation is measured in used asset markets with the hope of infering efficiency by using the duality of efficiency decline and depreciation. In another group, investment demand functions are fitted for various presumed patterns of deterioration.

A number of researchers, including Feldstein and Foot (1971), Hall (1971), Jorgenson (1974), and Hulten and Wykoff (1981), have attempted to test for the rate and form of economic depreciation as observed in used asset markets. Feldstein and Foot, and Hall both reject constant rates of depreciation for their data on real prices. However, Jorgenson points out that the Feldstein and Foot work is theoretically flawed. Jorgenson also redoes the Hall statistical tests in a nested testing structure and fails to reject a constant rate of depreciation. Hulten and Wykoff test for the form of depreciation in a Box-Cox model which places only weak restrictions on that form. The model takes on one hoss shay, straight line, and geometric price patterns as special cases. Usually, all three special cases are statistically rejected, but parameter estimates come closest to those which would indicate a geometric form.

On these grounds, Jorgenson and Hulten and Wykoff have concluded that the geometric form is probably a good approximation to asset price behavior and hence relative efficiency. Here our numerical experiments in the previous section become important. The price patterns corresponding to even concave forms of efficiency decline tend to be convex. Indeed, they can be virtually indistinguishable from the geometric form to the eye (as in Figure 1) and to the statistical model. Failure to reject geometric decay against a general form is too weak a statistical result to establish the superiority of geometric decay.

Theoretically other specific forms would have to be rejected in carefully conducted head to head tests in order to establish geometric decay. Since existing tests even tend to reject the geometric form, geometric decay is far from established. A particular criticism we have of the Hulten-Wykoff Box-Cox test is that it fails to recognize duality between efficiency and price. The three specific special cases of the age/price profile their model assumes are one-hoss shay, straight line, and geometric. Few who examine Tables 1 thru 5 would think it reasonable to expect to find a one hoss shay age/price profile, even in the extreme case of a one hoss shay age/efficiency profile. Yet this is one of the two alternatives against which Hulten and Wykoff find geometric to be the best. The geometric pattern is the only one of the three specific cases they consider with a convex age/price profile. It is little wonder that it gives the best results. As we have seen, most forms of efficiency lead to convex asset prices. All that studies of asset price data have done is verify the expectation of convex prices. Given the similarity between the age/price profiles depcited in Figure 1, and the stochastic forces which play a role in generating used asset prices, it would seem improbable that studies of used asset prices will ever decisively identify efficiency patterns.

Why do radically different forms of efficiency decline give rise to similar depreciation forms? As we noted in the previous section, double declining balance and hyperbolic forms, which are faster than straight line, can assume patterns of efficiency decline which are remarkably similar to the geometric pattern, and yet give rise to much faster depreciation. The source of this discrepancy is clearly the failure of geometric efficiency to decline to zero at the end of the asset's presumed life. Therefore, the infinite tail on the geometric curve in Figure 1 allows geometric efficiency decay to be associated with significantly slower depreciation rates than those which characterize similar patterns which are finite.

Thus the infinite tail on the geometric pattern is associated with its special properties, namely the equality of efficiency decay and depreciation, and the independence of depreciation from interest rates. Finite efficiency functions with much slower rates of decline give rise to depreciation patterns similar to geometric. As previously mentioned, this tail becomes particularly large when a slow rate of geometric decay is assumed. After their Box-Cox test, Hulten and Wykoff maintain the geometric form and, in the case of structures, find very slow rates of decay. These slow rates imply substantial services in some cases 100-200 years hence. The Hulten-Wykoff sample is inadequate to verify this prediction. A hyperbolic efficiency assumption, which predicts slow depreciation in the early years and faster depreciation later and which predicts slower early depreciation as asset lives lengthen, would seem a much more successful interpretation of the fine Hulten-Wykoff dataset.

We must conclude from this that economists interested in estimating efficiency from studies of used asset prices would have to carefully study the behavior of asset prices late in the life of a cohort in order to verify geometric efficiency. For this approach to be reasonable, significant proportions of the assets in a cohort must provide significant economic services well beyond the average asset life. In addition, these economist would need to design statistical tests which are capable of distinguishing remarkably similar convex forms.

Clearly studies of depreciation are an indirect and ambiguous way to go about establishing relative efficiency. Even though there are almost no data on relative efficiency, data on price are far from adequate enough to permit economists to measure efficiency by using duality theory.

Major studies were performed by Robert Coen (1975, 1980) using the investment demand approach to examine various efficiency patterns. Coen carefully observes the distinction between efficiency decline and depreciation in the formulation of his investment demand model. He compares the predictability of investment behavior which results from using each of several deterioration and asset life assumptions. He tries one hoss shay, straight line, geometric, truncated geometric, and sum of years digits efficiency functions for various asset lives. Then, for 21 2-digit manufacturing industry for equipment and structures, he picks the method and life which fits investment behavior best. In the 1975 paper, truncated geometric and straight line, applied to equipment, each fit best for 8 industries. For structures, one hoss shay fit bests for 11 industries, straight line 5 and truncated geometric 4. Geometric was best in only one of 42 cases. In the 1980 paper, truncated geometric dominated for 11 industries for equipment, and 11 for structures. Geometric and one hoss shay were best in only one case each.

The Coen results are very interesting, and the 1980 revision seems to verify the "realists" position that one hoss shay and geometric decay are extreme positions. Coen's work does have limitations as a way of measuring efficiency patterns. The specification of the investment demand model is the most important. Many factors affect the demand for investment besides the coefficients of the Cobb-Douglass production function. Inelastic factor substitution, biased technical change, increasing returns to scale, and systematic exogenous factors can all disrupt this comparison. A second problem is built into the measures of equipment and structures: no allowance for changing composition. A third problem is with the selection criterion which is to pick the best one. No efficiency pattern intermediate to one hoss shay and straight line is represented, while 3 fast methods appear. Also, in a comment on the 1980 paper, Fabricant raises questions as to the stability of these estimates when the various trials are compared. Results are often obtained for long lives with fast efficiency decay which are similar to results obtained for short lives and slow efficiency decay. Thus the Coen work does not appear to allow decisive discrimination between functional forms of efficiency.

Renewal Theory

Jorgenson (1974) offers a second line of defense for geometric decay. According to renewal theory, a population of capital assets, characterized by a steady state or by a stable growth investment pattern, will tend to a constant rate of decline, regardless of the shape of the relative efficiency function for a particular asset. Therefore, under stable growth conditions, the vintage aggregation model can reproduce any decay pattern with some geometric function.

Jorgenson predicts two possible problems with using the geometric form in practice when the assets themselves do not exhibit geometric efficiency declines. First of all, the geometric based capital stock estimates will be biased when the capital population does not conform to stable growth. Thus, by imposing a geometric form when it is unrealistic, we could systematically mismeasure capital during business cycles or slowdowns. Secondly, he points out that even under stable growth, efficiency decline rates do not, in general, equal depreciation rates. (See Jorgenson, 1974, p. 209).

In order to evaluate the practical consequences of these possible problems, we draw on some capital data development work being done at BLS by Harper and Rosenthal (1982). Their draft paper presents in detail the sources and methods used to construct these experimental data. The results are regarded as unofficial by BLS, but are more than adequate to address the issues posed in this paper. Essentially, productive and wealth stocks in the U.S. Manufacturing Sector are estimated separately for 18 types of equipment and 3 types of structures from investment data provided by the U.S. Bureau of Economic Analysis from the National Income and Product Accounts (NIPA). These are aggregated and presented in Table 6, for four separate efficiency assumptions: one hoss shay, hyperbolic (B=.5), straight line, and geometric.

There are several observations on Table 6. First the level of the capital stock depends substantially on the efficiency assumption. The distinction between productive and wealth stocks is important in terms of levels if we dismiss the geometric alternative. Secondly, the year to year growth rates are observed to differ by over 1% over the course of a typical business cycle. Essentially, stocks starting from a smaller base are more sensitive to shocks in the investment data. Third, only when we view the long term trends does the prediction of renewal theory, that the rates of growth will be similar, take force. Thus renewal theory does not appear to justify using a geometric form if efficiency is not in fact geometric, at least not for preparing annual capital measures.

In Table 7, we present the rates of efficiency decline and depreciation implied by the various assumptions. The rate of efficiency decline is always measured against the productive stock, and the rate of depreciation against the wealth stock. Jorgenson's second acknowledged problem is certainly demonstrated. What's equally important is to observe that the rate of depreciation differs depending on the method selected. Since the rate of depreciation enters the rental price formulation, an error here could bias the cost share weights assigned to individual assets.

TABLE 6. CAPITAL IN PLACE AT THE END OF THE PREVIOUS YEAR

U.S. MANUFACTURING SECTOR
** DEPRECIABLE ASSETS ONLY **

	ONE HOSS SHAY		HYPERBOLIC (B=.5)		STRAIGHT LINE		GEOMETRIC
	PRODUCTIVE STOCK	WEALTH STOCK	PRODUCTIVE STOCK	WEALTH STOCK	PRODUCTIVE STOCK	WEALTH STOCK	PRODUCTIVE & WEALTH STOCKS
	BILLIONS OF 1972 DOLLARS						
1948	171.2	117.5	115.8	90.3	99.4	82.7	94.4
1965	296.3	194.8	194.6	143.5	162.3	128.4	147.1
1973	394.4	263.5	264.7	197.5	222.2	177.6	200.7
1974	405.8	270.6	271.9	202.6	228.1	182.2	206.1
1975	421.9	282.4	283.8	212.4	238.8	191.4	216.1
1976	434.4	290.3	291.9	218.0	245.3	196.3	221.7
1977	447.6	298.5	300.4	223.9	252.2	201.4	227.6
1978	464.2	309.6	311.8	232.4	261.8	209.2	236.3
1979	482.1	321.4	323.8	241.3	272.0	217.4	245.4
	PERCENT CHANGE FROM PREVIOUS YEAR						
1974	2.87	2.69	2.71	2.59	2.65	2.57	2.69
1975	3.97	4.37	4.40	4.84	4.66	5.07	4.88
1976	2.96	2.78	2.85	2.62	2.74	2.54	2.59
1977	3.06	2.85	2.92	2.69	2.81	2.61	2.68
1978	3.70	3.71	3.78	3.81	3.82	3.87	3.81
1979	3.85	3.79	3.87	3.86	3.88	3.89	3.83
	AVERAGE ANNUAL RATE OF CHANGE						
1948-1965	3.29	3.04	3.13	2.79	2.94	2.65	2.66
1965-1973	3.65	3.86	3.91	4.10	4.03	4.17	3.98
1973-1978	3.31	3.28	3.33	3.31	3.34	3.33	3.33

NOTE: INCLUDES EQUIPMENT AND STRUCTURES ONLY

SOURCE: BUREAU OF LABOR STATISTICS
OFFICE OF PRODUCTIVITY AND TECHNOLOGY
EXPERIMENTAL DATA

JANUARY, 1982

TABLE 7. AVERAGE RATES OF EFFICIENCY DECLINE AND DEPRECIATION
(PERCENTAGES OF PRODUCTIVE OR WEALTH STOCK RESPECTIVELY)

U.S. MANUFACTURING SECTOR
** DEPRECIABLE ASSETS ONLY **

	ONE HOSS SHAY		HYPERBOLIC (B=.5)		STRAIGHT LINE		GEOMETRIC
	RATE OF DECLINE	RATE OF DEPR.	RATE OF DECLINE	RATE OF DEPR.	RATE OF DECLINE	RATE OF DEPR.	RATE OF DECLINE DEPR.
			PERCENT				
1948	3.64	6.46	6.32	9.57	8.21	11.12	9.99
1965	4.28	7.80	7.81	11.33	9.77	12.87	10.86
1966	4.25	7.78	7.74	11.33	9.75	12.91	11.02
1967	4.16	7.72	7.64	11.30	9.70	12.93	11.20
1968	4.08	7.69	7.57	11.31	9.68	12.96	11.31
1969	4.05	7.69	7.58	11.31	9.69	12.95	11.24
1970	4.04	7.71	7.60	11.33	9.70	12.95	11.22
1971	4.07	7.75	7.67	11.37	9.75	12.98	11.19
1972	4.15	7.85	7.78	11.47	9.85	13.06	11.17
1973	4.21	7.92	7.85	11.56	9.92	13.16	11.23
1974	4.26	7.97	7.88	11.64	9.97	13.26	11.33
1975	4.23	7.97	7.85	11.67	9.98	13.32	11.46
1976	4.22	8.03	7.90	11.80	10.07	13.48	11.57
1977	4.19	8.11	7.97	11.95	10.17	13.65	11.69
1978	4.15	8.20	8.04	12.12	10.30	13.85	11.88
1979	4.13	8.31	8.14	12.30	10.44	14.05	12.05

NOTE: INCLUDES EQUIPMENT AND STRUCTURES ONLY

SOURCE: BUREAU OF LABOR STATISTICS
OFFICE OF PRODUCTIVITY AND TECHNOLOGY
EXPERIMENTAL DATA

JANUARY, 1982

Tables 6 and 7 provide defense to our contention that the choice of mathematical form of efficiency loss is important. It might be argued that by suitable adjustments to presumed asset lives, just about any level of capital stock can be achieved. For example, geometric decay might be assumed to occur at 1/L instead of 2/L where L is the estimated useful life. Where such reasoning fails is that an erroneous mathematical form of efficiency, even if it achieves an accurate estimate of the level and growth rate of the productive stock, will lead to an erroneous estimate of the stock of wealth and depreciation. Besides damaging these measures, such an error would damage the asset aggregation process, which depends on the estimate of depreciation. Thus there is not a free tradeoff between mathematical form and presumed life.

In Table 8 we present estimates of the growth rate of Tornquist indexes of all capital assets, including the nondepreciable ones, together with "composition effects", based on comparing Tornquist and direct aggregates of productive stocks. Our observations are similar to those for Table 6. Table 8 will permit the reader to assess the overall implications of these issues for a final measure of capital input.

Clearly the renewal theory argument is of no value. It makes no sense to impose the implications (geometric decay) of a stable growth rate for investment if investment does not exhibit such a pattern. After all, the whole idea is to measure capital. Defense of any choice of efficiency function, including a geometric one, must rest entirely on whether it is right.

The Ratio of Rent to Cost

The Harper Rosenthal (1982) work provides a new criterion by which to evaluate relative efficiency functions, and, for that matter, the validity of

TABLE 8. RATES OF GROWTH OF CAPITAL SERVICES AND COMPOSITION EFFECTS

U.S. MANUFACTURING SECTOR
** ALL PHYSICAL ASSETS **

	ONE HOSS SHAY		HYPERBOLIC (B=.5)		STRAIGHT LINE		GEOMETRIC	
	SERV.	COMP.	SERV.	COMP.	SERV.	COMP.	SERV.	COMP.
	PERCENT CHANGE FROM PREVIOUS YEAR							
1965	3.01	.26	3.13	.37	3.38	.44	3.63	.43
1966	4.70	.33	5.50	.55	5.97	.65	6.28	.68
1967	6.33	.51	7.58	.67	8.20	.77	8.49	.82
1968	6.23	.23	7.27	.23	7.70	.29	7.33	.35
1969	4.05	.13	4.17	.09	4.03	.00	3.50	-.03
1970	4.07	.13	4.09	.05	4.01	-.03	3.91	.00
1971	2.90	.19	2.52	.07	2.30	.01	2.16	.06
1972	1.57	.11	.82	-.03	.46	-.04	.27	.00
1973	2.43	.37	1.99	.33	1.83	.35	1.78	.36
1974	3.18	.09	3.13	.06	3.16	.08	3.29	.17
1975	4.03	-.52	4.73	-.25	5.11	-.15	5.39	-.04
1976	2.35	-.32	2.38	-.12	2.33	-.07	2.23	-.06
1977	2.34	.12	2.11	.24	1.94	.26	1.70	.17
1978	3.31	.38	3.79	.39	3.75	.38	3.65	.31
1979	4.10	.49	3.97	.44	3.90	.40	3.76	.32
	AVERAGE ANNUAL RATE OF CHANGE							
1948-65	3.78	.58	3.59	.52	3.42	.47	3.21	.43
1965-73	4.04	.27	4.24	.25	4.32	.25	4.32	.27
1973-78	3.14	-.05	3.24	.07	3.26	.10	3.25	.11

NOTE: INCLUDES SERVICES OF EQUIPMENT, STRUCTURES, INVENTORIES, AND LAND

SOURCE: BUREAU OF LABOR STATISTICS
OFFICE OF PRODUCTIVITY AND TECHNOLOGY
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the vintage aggregation assumption. Jorgenson and his colleagues construct a rental price as a means of allocating capital income to assets. In order to ensure that this happens, a rate of return is constructed from NIPA capital income data in such a way that this allocation occurs exactly. Such a method allowing for taxes and inflation, yields an estimate of the internal rate of return on capital. In earlier literature on the user cost, or rental price, of capital, a discount rate usually assumed to be a market interest rate is used. Of course nobody believes that capital markets are in equilibrium every year as the user cost model assumes. But if they were in equilibrium, if our vintage aggregation procedure faithfully represented the implicit rental values of capital, and if perfect competition and foresight existed, it might be reasonable to predict that the rents implied by the rental price equations and by market interest rates would equal current capital costs.

Using the Moody AAA bond rate, and the rental price equations described in Harper and Rosenthal (1982), we estimate the implicit rental value of capital by summing the products of rental prices and productive stocks. We then divide this implicit rent into the actual current dollar capital income reported in the NIPA. As we have argued, this "ratio of rent to costs" should be about one if all the assumptions we make are really correct. Harper and Rosenthal (1982) discuss the possible relationship between this ratio and "Tobin's Q", or the ratio of market value to book value of the stock market. Essentially, Tobin's Q may reflect investor's discounted expectations of future values of the ratio of rent to cost.

In Table 9 we present estimates, for the various efficiency assumptions, from 1948-1979. This ratio appears to confirm the view that existing capital

TABLE 9. RATIO OF IMPLICIT RENTS TO CAPITAL COSTS

U.S. MANUFACTURING SECTOR

	ONE HOSS SHAY	HYPERBOLIC (B=.5)	STRAIGHT LINE	GEOMETRIC
1948	.60	.69	.71	.58
1949	1.59	1.54	1.52	1.35
1950	.67	.67	.65	.52
1951	.32	.46	.49	.38
1952	1.00	1.01	.99	.84
1953	1.04	1.00	.96	.81
1954	1.26	1.19	1.14	.96
1955	1.13	1.05	1.00	.84
1956	.33	.47	.50	.36
1957	.99	.97	.94	.78
1958	1.36	1.27	1.21	1.02
1959	1.39	1.25	1.18	1.00
1960	1.32	1.20	1.13	.94
1961	1.33	1.24	1.16	.97
1962	1.20	1.08	1.02	.85
1963	1.08	.98	.93	.77
1964	1.01	.92	.87	.72
1965	.94	.86	.81	.68
1966	.87	.81	.77	.65
1967	.93	.88	.84	.71
1968	.89	.83	.79	.67
1969	.84	.80	.77	.64
1970	.95	.90	.86	.72
1971	.94	.87	.83	.70
1972	.99	.88	.83	.70
1973	.72	.66	.62	.50
1974	.09	.14	.14	.04
1975	.35	.40	.40	.31
1976	.92	.80	.75	.64
1977	.63	.58	.55	.46
1978	.49	.48	.46	.38
1979	.34	.34	.33	.25
ROOT MEAN SQUARE DIFFERENCES FROM 1.0				
1948-1979	.37	.34	.36	.41
1952-1972	.24	.19	.18	.27

SOURCE: BUREAU OF LABOR STATISTICS
OFFICE OF PRODUCTIVITY AND TECHNOLOGY
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stocks experienced a large loss of value beginning in 1973, the year of the Arab Oil embargo. After the 1974-5 recession, during which rents nearly vanished, it would appear that the existing stock of capital remained incapable of generating rents consistent with any of the efficiency assumptions.

From this we must judge that the vintage aggregation assumption, based on any presumed fixed efficiency pattern, has failed miserably since 1972. The assumpton that capital services are proportional to stocks would appear wrong. Some downward adjustment in capital services appears to be indicated.

Regarding the earlier years, it would appear that conventional methods are fairly accurate from 1952-1972. If we accept the reasoning that the ratio of rent to cost should approximate one, we can compute the root mean square differences from one for each efficiency assumption as a criterion for evaluating their performance. These statistics have been presented for the entire time period, and for the period during which the model seems reasonably well behaved. We observe that the one hoss shay assumption appears to predict rents which tend to be higher than capital income, while the geometric form predicts rents which are almost always lower than capital income.

We recognize that this test is limited by data and assumptions, but we take comfort anyway that it appears to demonstrate that one hoss shay and rapid geometric decay are opposite extremes when confronted with actual capital cost data.

V. Summary and Conclusions

In this study we have examined how real capital input should be measured in light of the economic theory of replacement and depreciation and in light of evidence on the patterns and rates of deterioration and depreciation.

Our first conclusion is that evidence on the correct form of decay is lacking. The lack of direct evidence on rental values as an asset ages is the major problem. Indirect approaches to the question cannot be regarded as conclusive. Worse, it is unclear how direct or indirect methods could be dramatically improved. Direct approaches are burdened by the lack of rental markets. When rental markets do exist, there are long term leases which obscure time relationships and factor in tax incentives. Indirect approaches face sensitive statistical identification problems while dealing with a variable, price, which is affected by all kinds of things besides the efficiency function we wish to measure.

Our second conclusion is that the correct specification for efficiency is important. These first two conclusions echo the assessment of Diewert (1980, p-478). We have found additional evidence that an incorrect specification will distort the relationship between efficiency and depreciation. Neither renewal theory, discard patterns, obsolescence, nor empirical evidence can justify the use of an incorrect mathematical form of the efficiency function, because even if productive stocks are adequately estimated, other variables, such as wealth and depreciation, will be seriously biased.

However, the contributions of this paper are by no means negative. First of all, we have distinguished two concepts of capital stock, productive and wealth, which have distinct and interesting interpretations. We have argued that this distinction follows from the Hall (1968) and Jorgenson

(1974) papers, and is logically necessary within the framework of that model, if we are to elect a nongeometric efficiency assumption. We have done so empirically for four alternative efficiency assumptions in the U.S. Manufacturing sector.

Secondly, we have identified a particular mathematical form, hyperbolic with B=.5, which is associated with an age/price profile extremely similar to the geometric pattern. Besides having the convex shaped price pattern found in the Hulten-Wykoff data, this form has a concave shaped efficiency pattern which many researchers would consider more realistic. This "realism" stems from its gradual rate of efficiency decline in early years coupled with elimination of the large troublesome tail found on a geometric form which declines slowly in the early years. The choice of a precise value for B seems to be no more arbitrary than the choice of a rate of decline for the geometric form. Such a choice must be made to yield a model which is as realistic as possible. A high value of B tends to generate a concave price function, while if B=0 we have straight line efficiency decline which may be too fast. However, in spite of identifying B=.5 as an appealing alternative, we emphasize our conclusion that evidence on which form is correct is lacking. Such a choice rests largely on scientifically indefensible grounds, one's instincts regarding realism.

Thirdly, we have identified a new statistic, the ratio of rent to costs, which, with proper refinement, could become one criterion by which to discriminate various efficiency assumptions, and to identify when the traditional vintage aggregation procedures fail. In our test data, two observations were made. First, one hoss shay and geometric appear to be extreme views on the high and low side, as some suspect based on intuition.

Secondly, the vintage aggregation procedure seems to be a failure at representing capital input during recessions, and in the post-1973 period, as many have argued. Again, we caution that conclusions based on this statistic are sensitive to many assumptions. However, the statistic does seem to affirm the need to adjust our capital stock estimates. Failure to address the problems imposed on our capital stock measures by the vintage aggregation conditions seems to leave us with capital measures, multifactor productivity measures, and models of production which are obsolete.

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