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## **Further Model-Based Estimates of U.S. Total Manufacturing Production Capital and Technology, 1949-2005**

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**FURTHER MODEL-BASED ESTIMATES OF U.S. TOTAL MANUFACTURING  
PRODUCTION CAPITAL AND TECHNOLOGY, 1949-2005**

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**ABSTRACT**

Production capital and technology (i.e., total factor productivity) in U.S. manufacturing are fundamental for understanding output and productivity growth of the U.S. economy but are unobserved at this level of aggregation and must be estimated before being used in empirical analysis. Previously, we developed a method for estimating production capital and technology based on an estimated dynamic structural economic model and applied the method using annual SIC data for 1947-1997 to estimate production capital and technology in U.S. total manufacturing. In this paper, we update this work by reestimating the model and production capital and technology using annual SIC data for 1949-2001 and partly overlapping NAICS data for 1987-2005.\*

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## 1. Introduction.

Production capital and total factor productivity or, more simply, capital and technology in U.S. manufacturing are fundamental for understanding output and productivity growth of the U.S. economy but are unobserved at this level of aggregation and must be estimated before being used in empirical analysis. Standard methods for estimating aggregate capital and technology are based on Solow (1957) and Jorgenson (1963). In level form, technology is usually called total factor productivity and, in percentage-growth form, the Solow residual. Chen and Zadrozny (2005) developed a model-based method for estimating unobserved capital and technology by, first, estimating a dynamic structural economic model by maximum likelihood and, then, using the estimated model, the data, and the Kalman filter to estimate capital and technology. Chen and Zadrozny (2005) applied the method to annual Standard Industrial Classifications (SIC) data for 1947-1997 and obtained estimates of capital and technology for U.S. total manufacturing. Current and previous applications illustrate the model-based method as a general method for accounting for the growth of output which is grounded in dynamic economic modelling and econometric practice.

The present paper extends Chen and Zadrozny (2005) by reestimating their model and reestimating capital and technology for U.S. total manufacturing using more contemporary data, a combination of annual Standard Industrial Classification (SIC) data for 1949-2001 and partly overlapping annual North American Industrial Classification System (NAICS) data for 1987-2005. The application shows: (1) small changes in the estimated model; (2) for 1949-2000, trends of model-based and standard capital and technology are broadly similar; (3) for 2000-2005, model-based and standard estimates diverge significantly: model-based capital continues its previous growth; standard capital levels off; model-based technology declines and levels off; and, standard technology continues its previous growth; and, (4) for 1949-2005, model-based capital is noisy and uncertain and model-based technology is smooth and certain. The overall continuity of present results with those in Chen and Zadrozny (2005) supports the model-based method as a general method for output-growth accounting.

The paper contributes in several ways. Updating model-based capital and technology using data through 2005 is significant not only because more and more recent data are used, but because during 2000-2005 manufacturing output and input quantities leveled off or declined, despite the fact that the

estimated parameters of the model changed little. Figure 1 shows these patterns: overall, output and research leveled off, materials and investment declined slightly, and labor declined significantly.

Standard capital and technology are based minimally on economic motivations for accumulating capital and technology. By contrast, the present model-based estimates, graphed in figure 3, aim to more fully incorporate these motivations. This is done by explicitly including the motivations in the estimated model which is, then, used to estimate capital and technology. Thus, unlike the present model-based method, standard methods are unsuitable for an economic analysis of the effects for policy changes on capital and technology estimates. For example, suitably expanded the present model could predict the effects of a change in tax rules on investment in capital and technology. Apparently, Slade (1989) is the only previous work that similarly applies Kalman filtering to an estimated economic model with explicit optimization in order to estimate technology (but not capital). However, by treating technology as exogenous, Slade's model disconnects technology from economic decisions and, thus, is unsuitable for such economic policy analysis of technology accumulation. Because it treats technology as an unexplained residual, the standard Solow-residual analysis is similarly unsuitable for such economic policy analysis.

Finally, in contrast to standard methods, the model-based method produces standard errors of capital and technology as a normal product of the Kalman filter, which quantify uncertainty about capital and technology due to model disturbances (but not uncertainty due to model and parameter uncertainty). The model-based capital and technology standard errors in figure 3 are about 1.02 and .036, respectively, so that the capital confidence bounds in figure 3 are about 28 times wider than the technology confidence bounds. Being in unit-free standardized form, the model-based capital and technology estimates and their confidence bounds are comparable. A similar numerical measure of uncertainty does not exist for standard capital and technology.

The partly overlapping SIC and NAICS data for 1987-2001 required some method for merging them. We solved this problem by averaging growth rates of SIC and NAICS data (additively and with equal weights) where they overlapped. Despite being based on extended, newly classified, partly revised, and partly averaged data, current model estimates and model-based capital and technology estimates for 1949-2005 are close to previous ones for 1947-1997, as is seen by comparing current and previous tables 1 and 2 and comparing current and

previous figures 3. Thus, at least for the present application, the new SIC and NAICS data are compatible with the previous SIC data.

The paper continues as follows. Section 2 describes the structure of the model. In the model, U.S. total manufacturing is treated as a single industry whose demand side is modeled by a conventional static demand curve but whose supply side is modeled in explicit dynamic detail. Following Lucas and Prescott (1971), the industry is considered to be competitive and its output supply is modeled by a representative firm which solves a dynamic optimization problem. The problem's explicit solution is used to impose overidentifying restrictions during estimation on the reduced-form parameters in terms of the structural parameters. The restrictions ensure that the estimated structural parameters are identified when key variables in the model (capital, technology, and demand state) are completely unobserved. Section 3 first discusses how, following Chen and Zadrozny (2005), the model's equations are assembled into a state representation for estimating the structural parameters and for computing filtered estimates of capital and technology based on the estimated model. Section 3 then discusses the application, first, sources and properties of the SIC and NAICS data and how they were merged and, then, properties of the estimated structural model and comparison of model-based and standard estimates of capital and technology. Section 4 contains concluding remarks.

## 2. Specification and Solution of the Model.

Every period,  $t$ , the representative firm of the industry being considered maximizes the expected present value of profits,

$$(2.1) \quad v_t = E_t \sum_{k=0}^{\infty} \delta^k \pi_{t+k} ,$$

with respect to a feedback decision rule, where the maximization is subject to equations to be specified,  $E_t$  denotes expectation conditional on the firm's information in period  $t$ ,  $\delta \in (0,1)$  denotes a constant real discount factor, and  $\pi_t = r_{qt} - (c_{qt} + c_{it} + c_{rt})$  denotes real profits or revenues minus costs, where  $c_{qt}$  is the cost of production and  $c_{it}$  and  $c_{rt}$  are direct (nonadjustment) costs of investment in capital and research in technology. Throughout, a real value is a nominal (current dollar) value divided by the GDP deflator. The firm's optimization problem is stated precisely in this section.

To obtain a competitive rational expectations equilibrium solution, following Lucas and Prescott (1971), we set revenues  $r_{qt} = \int_{x=0}^{q_t} p_q(x, d_t) dx$ , where  $p_q(\cdot)$  is the inverse output-demand curve,  $q_t$  is the production of saleable output, and  $d_t$  is the output-demand state. To obtain linear solution equations, which facilitate estimation and to which the Kalman filter can be applied, we specify  $r_{qt}$ ,  $c_{qt}$ ,  $c_{it}$ , and  $c_{rt}$  as quadratic forms. Accordingly, we assume the industry's inverse output-demand curve is

$$(2.2) \quad p_{qt} = -\eta q_t + d_t + \zeta_{pqt},$$

where  $\eta > 0$  is a constant slope parameter,  $d_t$  is the demand state generated by the 2nd-order autoregressive (AR(2)) process

$$(2.3) \quad d_t = \phi_{d1}d_{t-1} + \phi_{d2}d_{t-2} + \zeta_{dt},$$

and  $\zeta_{pqt}$  and  $\zeta_{dt}$  are disturbances. Preliminary experimentation with alternative specifications during model estimation showed that  $d_t$  is specified adequately as generated by an AR(2) process. The full set of distributional assumptions on disturbances is stated in section 3.

To specify  $c_{qt}$ , we assume that the firm uses capital ( $k$ ), labor ( $\ell$ ), and materials ( $m$ ), to produce saleable output ( $q$ ), installs investment goods ( $i$ ), and conducts research activities ( $r$ ) (subscript  $t$  is omitted sometimes). We assume that the "output activities,"  $q$ ,  $i$ , and  $r$ , are restricted according to the separable production function

$$(2.4) \quad h(q, i, r) = \tau g(k, \ell, m),$$

where  $\tau$  is the stock of technology or total factor productivity. Following Kydland and Prescott's (1982) treatment of the utility function, we assume  $g(\cdot)$  and  $h(\cdot)$  are the constant elasticity functions,

$$(2.5) \quad g(k, \ell, m) = (\alpha_1 k^\beta + \alpha_2 \ell^\beta + \alpha_3 m^\beta)^{1/\beta},$$

$$h(q, i, r) = (\gamma_1 q^p + \gamma_2 i^p + \gamma_3 r^p)^{1/p},$$

where  $\alpha_i > 0$ ,  $\alpha_1 + \alpha_2 + \alpha_3 = 1$ ,  $\beta < 1$ ,  $\gamma_i > 0$ ,  $\gamma_1 + \gamma_2 + \gamma_3 = 1$ , and  $\rho > 1$ . CES =  $(\beta-1)^{-1}$  is the constant elasticity of substitution among inputs, and CET =  $(\rho-1)^{-1}$  is the constant elasticity of transformation among outputs. Thus, we call (2.4)-(2.5) the CES-CET production function.

Including  $i$  and  $r$  in  $h(\cdot)$  is a parsimonious way of specifying internal adjustment costs. The idea is that positive rates of investment and research use capital, labor, and materials resources, which could otherwise be used to produce more output, and that this trade-off sacrifices ever more output per unit of increases in investment and research. Here "investment" means investment in production capital and research in technology. In the next step, we derive a quadratic approximation of the dual variable production cost function (DVPCF) from production function (2.4)-(2.5). The DVPCF includes convex, investment and research, adjustment costs.

Convex internal adjustment costs arise in (2.4)-(2.5) when, for given technology,  $\tau$ , and inputs,  $(k, \ell, m)$ , the transformation surfaces of the outputs,  $(q, i, r)$ , are concave to the origin. The adjustment costs are "convex" because the derived DVPCF is convex in  $(q, i, r)$ . Here,  $\rho > 1$  is a necessary and sufficient condition for the transformation surfaces to be concave. The transformation surfaces become more curved, hence, adjustment costs increase, as  $\rho$  increases. Similarly,  $\beta < 1$  is a necessary and sufficient condition for the input isoquants to be convex to the origin, and the isoquants become more curved, hence, input substitutability decreases, as  $\beta$  decreases (becomes more negative).

Let  $c_q = p_\ell \ell + p_m m$ , where  $p_\ell$  is the real hiring price of labor and  $p_m$  is the real purchase price of materials. Let  $c_i = p_i i$  and  $c_r = p_r r$ , where  $p_i$  and  $p_r$  are the real purchase prices of investment and research goods and services. Because  $\ell$  and  $m$  are variable (not subject to adjustment costs) and  $k$  and  $\tau$  are quasi-fixed (subject to adjustment costs), we refer to  $c_q$  as the variable cost and to  $c_i + c_r$  as the fixed cost. Let  $c_q(w)$  denote the dual variable cost function: given  $w = (w_1, \dots, w_7)^T = (q, i, r, k, \tau, p_\ell, p_m)^T$  (superscript T denotes transposition),  $c_q(w) = \text{minimum of } p_\ell \ell + p_m m$ , with respect to  $\ell$  and  $m$ , subject to production function (2.4)-(2.5).

In the standard approach to total factor productivity analysis (Bureau of Labor Statistics, 1997), all inputs are treated symmetrically, as variable flows. Accordingly,  $c_q$  would include all input costs as  $c_q = p_k k + p_\tau \tau + p_\ell \ell + p_m m$ , where  $p_k$  and  $p_\tau$  would be prices of renting capital and technology stocks,

obtained using Jorgenson's (1963) results for converting purchase prices of investment and research flows to rental prices of capital and technology stocks, under the more restrictive assumption that all inputs are variable. In this paper, we instead work with purchase prices of investment and research flows because this allows greater flexibility for handling adjustment costs in the firm's dynamic optimization problem. It is the explicit solution of this problem that generates the identifying restrictions that allow us to identify, hence, to estimate the structural parameters of the model when capital and technology stocks are completely unobserved.

Ignoring constant and linear terms, which contribute only an additional constant term to the optimal decision rule that is removed by mean adjustment of the data,  $c_q(w_t) \cong (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ , where  $\nabla^2 c_q(w_0)$  denotes the Hessian matrix of second partial derivatives of  $c_q$  evaluated at  $w = w_0$ .  $\nabla^2 c_q(w_0)$  is stated explicitly in terms of the  $\alpha$ 's,  $\beta$ ,  $\gamma$ 's, and  $\rho$  in Chen and Zadrozny (2005) for  $w_0 = (1, 1, 1, 1, 1, \alpha_2, \alpha_3)^T$ , a value which results in the simplest expression for  $\nabla^2 c_q(w_0)$  and works econometrically for the present and previous data. Therefore,

$$(2.6) \quad \pi_t = -(1/2)\eta q_t^2 + q_t(d_t + \zeta_{pq,t}) - (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t - p_{it}i_t - p_{rt}r_t.$$

The resulting symmetric Hessian matrix,  $\nabla^2 c_q(w_0)$ , has the standard local properties of homogeneity, convexity and concavity with respect to  $w$ .

We assume that input prices,  $p_i$ ,  $p_r$ ,  $p_\ell$ , and  $p_m$ , are exogenous to the industry being modeled and preliminary experimentation showed that the input prices are specified adequately as generated by independent, univariate, AR(2) processes,

$$(2.7) \quad p_{it} = \phi_{pi1}p_{i,t-1} + \phi_{pi2}p_{i,t-2} + \zeta_{pit},$$

$$p_{rt} = \phi_{pr1}p_{r,t-1} + \phi_{pr2}p_{r,t-2} + \zeta_{prt},$$

$$p_{\ell t} = \phi_{p\ell 1}p_{\ell,t-1} + \phi_{p\ell 2}p_{\ell,t-2} + \zeta_{p\ell t},$$

$$p_{mt} = \phi_{pm1}p_{m,t-1} + \phi_{pm2}p_{m,t-2} + \zeta_{pmt},$$



where  $\zeta_{pit}$ ,  $\zeta_{prt}$ ,  $\zeta_{p/t}$ , and  $\zeta_{pmt}$  are mutually uncorrelated disturbances. Input-price processes (2.7) are both structural and reduced form and have no particular structural interpretations. They are needed to provide forecasts of input prices for the firm's dynamic optimization problem.

Let  $\lambda$  and  $|\bar{\lambda}|$  denote a characteristic root and the largest absolute characteristic root of an input-price process. For example, a characteristic root of the investment-price process solves  $\lambda^2 - \phi_{pi1}\lambda - \phi_{pi2} = 0$  and is stationary if it is less than one in absolute value. Whether or not the estimated model has all stationary or some nonstationary roots (in practice, near unity), estimates of the structural parameters will be consistent and efficient and estimates of capital and technology will be linear least squares estimates based on the estimated model. The main difference when the estimated model has nonstationary roots is that the estimated structural parameters will no longer be asymptotically normally distributed, but will have a nonstandard asymptotic distribution. To solve the dynamic optimization problem, each input-price process must satisfy the growth condition  $|\bar{\lambda}| < 1/\sqrt{\delta}$  (Hansen and Sargent, 1980) and the estimated model satisfies this condition.

We assume that capital accumulates according to the stochastic perpetual inventory equation (PIE)

$$(2.8) \quad k_t = \phi_{k1}k_{t-1} + \phi_{i0}i_t + \zeta_{kt},$$

where  $0 < \phi_{k1} < 1$ ,  $\phi_{i0} = (\phi_{k1} - 1)/\ln(\phi_{k1})$ ,  $\zeta_{kt} \sim \text{NIID}(0, \sigma_k^2)$ , and  $\sigma_k^2 > 0$ . The restriction  $\phi_{i0} = (\phi_{k1} - 1)/\ln(\phi_{k1})$  comes from an antecedent continuous-time formulation and reflects the property that investments undertaken earlier in a period depreciate more by the end of the period than investments undertaken later in the period (Chen and Zadrozny, 2005). Similarly, we assume that technology accumulates according to the stochastic PIE

$$(2.9) \quad \tau_t = \phi_{t1}\tau_{t-1} + \phi_{r0}r_t + \zeta_{rt},$$

where  $0 < \phi_{t1} < 1$ ,  $\phi_{r0} = (\phi_{t1} - 1)/\ln(\phi_{t1})$ , and  $\zeta_{rt} \sim \text{NIID}(0, \sigma_r^2)$ , and  $\sigma_r^2 > 0$ .

The model's structural components have now been specified. It remains to explain how to solve the firm's dynamic optimization problem and how to assemble the specified laws of motion and the solved optimal decision rules

into a system of linear simultaneous equations that are the equilibrium equations of the model.

To simplify the dynamic optimization problem, we eliminate  $q_t$  by maximizing  $\pi_t$  with respect to  $q_t$ . Because  $q_t$  is not a control variable in the laws of motion of  $k_t$  or  $\tau_t$ , conditional on  $i_t$  and  $r_t$  being at their optimal values, the optimal value of  $q_t$  is given by maximizing  $\pi_t$  with respect to  $q_t$ . The first-order condition,  $\partial\pi_t/\partial q_t = 0$ , yields the output supply rule

$$(2.10) \quad q_t = -(c_{11}+\eta)^{-1}(c_{12}i_t + c_{13}r_t + c_{14}k_t + c_{15}\tau_t + c_{16}p_{\ell t} + c_{17}p_{m t} - d_t) + \zeta_{qt},$$

where  $(c_{11}, \dots, c_{17})$  is the first row of  $\nabla^2 c_q$  and  $\zeta_{qt}$  is a disturbance added for statistical reasons.

Similar elimination of  $\ell_t$  and  $m_t$  from the dynamic optimization problem is justified because  $\ell_t$  and  $m_t$  are not control variables in the laws of motion of  $k_t$  or  $\tau_t$ . Optimal values of  $\ell_t$  and  $m_t$ , conditional on  $q_t$ ,  $i_t$  and  $r_t$  being at their optimal values, are obtained using the envelope theorem,

$$(2.11) \quad \ell_t = \partial c_{qt}/\partial p_{\ell t} = c_{61}q_t + c_{62}i_t + c_{63}r_t + c_{64}k_t + c_{65}\tau_t + c_{66}p_{\ell t} + c_{67}p_{m t} + \zeta_{\ell t},$$

$$(2.12) \quad m_t = \partial c_{qt}/\partial p_{m t} = c_{71}q_t + c_{72}i_t + c_{73}r_t + c_{74}k_t + c_{75}\tau_t + c_{76}p_{\ell t} + c_{77}p_{m t} + \zeta_{m t},$$

where  $(c_{61}, \dots, c_{67})$  and  $(c_{71}, \dots, c_{77})$  are the 6th and 7th rows of  $\nabla^2 c_q$ , and  $\zeta_{\ell t}$  and  $\zeta_{m t}$  are disturbances added for statistical reasons.

To solve the remainder of the firm's dynamic optimization problem, we restate it as a linear optimal regulator problem. We define the  $2 \times 1$  control vector  $u_t = (i_t, r_t)^T$  and the  $14 \times 1$  state vector  $x_t = (k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{m t}, d_t, k_{t-1}, \tau_{t-1}, p_{i,t-1}, p_{r,t-1}, p_{\ell,t-1}, p_{m,t-1}, d_{t-1})^T$ . We assemble the output-demand, input-price, capital, and technology processes, (2.3) and (2.7)-(2.9), as the state equation

$$(2.13) \quad x_t = Fx_{t-1} + Gu_t, \quad F = \begin{bmatrix} F_1 & F_2 \\ I_7 & 0_{7 \times 7} \end{bmatrix}, \quad G = \begin{bmatrix} G_0 \\ 0_{12 \times 2} \end{bmatrix},$$

where  $F_1 = \text{diag}[\phi_{k1}, \phi_{\tau1}, \phi_{p11}, \phi_{pr1}, \phi_{p\ell1}, \phi_{pm1}, \phi_{d1}]$ ,  $F_2 = \text{diag}[0, 0, \phi_{pi2}, \phi_{pr2}, \phi_{p\ell2}, \phi_{pm2}, \phi_{d2}]$ ,  $G_0 = \text{diag}[\phi_{i0}, \phi_{r0}]$ ,  $I_m$  is the  $m \times m$  identity matrix, and  $0_{m \times n}$  is the  $m \times n$

zero matrix. We may suppress disturbances in equation (2.13) because the regulator problem is certainty equivalent. We use output-supply rule (2.10) to eliminate  $q_t$  from  $\pi_t$  and write  $\pi_t$  as the quadratic form

$$(2.14) \quad \pi_t = u_t^T R u_t + 2u_t^T S x_{t-1} + x_{t-1}^T Q x_{t-1}.$$

The matrices  $R$ ,  $S$ , and  $Q$  are stated explicitly in terms of  $\eta$  and the elements of  $\nabla^2 c_q$  in Chen and Zadrozny (2005).

The regulator problem maximizes expected present value, (2.1), stated in terms of the quadratic form (2.14), with respect to the feedback matrix  $K$  in the linear decision rule  $u_t = Kx_{t-1}$ , subject to the state equation (2.13). Under concavity, stabilizability, and detectability conditions (Kwakernaak and Sivan, 1972), we can compute the unique optimal  $K$  matrix by solving an algebraic matrix Riccati equation using a Schur decomposition method (Laub, 1979). Finally, we write the optimal investment-research decision rule as

$$(2.15) \quad u_t = Kx_{t-1} + (\zeta_{it}, \zeta_{rt})^T,$$

where  $(\zeta_{it}, \zeta_{rt})^T$  is a  $2 \times 1$  disturbance vector added for statistical reasons.

Further details about the model's specification, including comparison with the more familiar translog DVPCF (Christensen, Jorgenson, and Lau, 1973), are discussed in Chen and Zadrozny (2005).

### 3. Estimation of the Model and Capital and Technology.

#### 3.1 Maximum Likelihood Estimation of the Model.

To estimate the model's structural parameters by maximum likelihood (MLE), using the Kalman filter, and, then, to estimate unobserved capital and technology, also using the Kalman filter, we express the reduced form of the model in a state representation. To this end, we collect the variables of the model in the  $13 \times 1$  vector  $y_t = (p_{qt}, q_t, \ell_t, m_t, i_t, r_t, k_t, \tau_t, p_{it}, p_{rt}, p_{ft}, p_{mt}, d_t)^T$  and their disturbances in the  $13 \times 1$  vector  $\zeta_t = (\zeta_{pqt}, \zeta_{qt}, \zeta_{\ell t}, \zeta_{mt}, \zeta_{it}, \zeta_{rt}, \zeta_{kt}, \zeta_{\tau t}, \zeta_{pit}, \zeta_{prt}, \zeta_{pft}, \zeta_{pmt}, \zeta_{dt})^T$ . We assume that the disturbances are generated by mutually uncorrelated, normally distributed, stationary processes, where the first 6 disturbances are generated by AR(1) processes and the last 7 disturbances are serially uncorrelated. The AR(1) and white-noise

specifications of the disturbance processes are based on preliminary experimentation and their adequacy is verified by the insignificant Q statistics in table 2. Thus, we assume that  $\zeta_t = (I_{13} - \Theta L)^{-1} \varepsilon_t$ , where  $\varepsilon_t \sim \text{NIID}(0, \Sigma_\varepsilon)$ ,  $L$  is the lag operator,  $\Theta = \text{diag}(\theta_{pq}, \theta_q, \theta_\ell, \theta_m, \theta_i, \theta_r, 0, 0, 0, 0, 0, 0, 0)$ , where the  $\theta$ 's  $\in (-1, 1)$  and  $\Sigma_\varepsilon = \text{diag}(\sigma_{pq}^2, \sigma_q^2, \sigma_\ell^2, \sigma_m^2, \sigma_i^2, \sigma_r^2, \sigma_k^2, \sigma_\tau^2, \sigma_{pi}^2, \sigma_{pr}^2, \sigma_{p\ell}^2, \sigma_{pm}^2, \sigma_d^2)$ .

The equations which form the basis of the parameter and capital-technology estimation are (2.2), (2.10)-(2.13), and (2.15). These 13 scalar-level equations constitute the complete set of linear simultaneous equations which, determine unique values of the 13 variables of the model. Concisely, the equations are

$$(3.1) \quad A_0 Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + (I_{13} - \Theta L)^{-1} \varepsilon_t,$$

where  $A_0$ ,  $A_1$ , and  $A_2$ , which depend on  $\eta$ , the  $\phi$ 's, and the elements of  $\nabla^2 c_q$  and  $K$ , are stated in detail in Chen and Zadrozny (2005). Rewriting (3.1), we obtain the reduced-form VAR(2) process

$$(3.2) \quad Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \xi_t,$$

where  $B_1 = A_0^{-1} (A_1 + \Theta A_0)$ ,  $B_2 = A_0^{-1} (A_2 - \Theta A_1)$ ,  $\xi_t = A_0^{-1} \varepsilon_t \sim \text{NIID}(0, \Sigma_\xi)$ , and  $\Sigma_\xi = A_0^{-1} \Sigma_\varepsilon A_0^{-T}$ .

Finally, following Chen and Zadrozny (2005), we write process (3.2) in state-space form and use this form, in conjunction with the missing-data Kalman filter (MDKF), to compute the normal likelihood function of the observations. Let  $L(\vartheta)$  denote the resulting likelihood function, where  $\vartheta$  is the vector of the 39 structural parameters, defined by  $\vartheta = (\vartheta_1^T, \vartheta_2^T)^T$ ,  $\vartheta_1 = (\delta, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma_{pq}^2, \sigma_\ell^2, \sigma_m^2)^T$ , and  $\vartheta_2 = (\phi_{pi1}, \phi_{pr1}, \phi_{p\ell1}, \phi_{pm1}, \phi_{pi2}, \phi_{pr2}, \phi_{p\ell2}, \phi_{pm2}, \sigma_{pi}^2, \sigma_{pr}^2, \sigma_{p\ell}^2, \sigma_{pm}^2, \theta_{pq}, \theta_q, \theta_\ell, \theta_m, \theta_i, \theta_r, \eta, \beta, \rho, \phi_{k1}, \phi_{t1}, \phi_{d1}, \phi_{d2}, \sigma_q^2, \sigma_i^2, \sigma_r^2, \sigma_k^2, \sigma_\tau^2, \sigma_d^2)^T$ . Under the following identifying restrictions on  $\vartheta_1$ , we used the MDKF to compute  $L(\vartheta)$  in maximum likelihood estimation (MLE). We needed to use the MDKF as opposed to the ordinary KF, which requires that all variables are observed in every sample period, because variables in the model are either not observed over the same periods or, like  $k_t$ ,  $\tau_t$ , and  $d_t$ , are not observed at all. A future

extension of the present analysis could further exploit the MDKF by using a sample of mixed-frequency observations, analogous to Zadrozny (1990). Details about implementing the MDKF accurately and quickly in the MLE are discussed in Anderson and Moore (1979), Zadrozny (1988, 1990), and references therein.

Because the 39 parameters in  $\mathcal{Q}$  are not identified without further a priori restrictions, we imposed the following identifying restrictions on  $\mathcal{Q}_1$ , which ensure that  $\mathcal{Q}_2$  is locally identified and, hence, is estimatable by MLE. We set  $\delta = .935$ , which corresponds to a real interest rate of  $\delta^{-1} - 1 = .0695$ . In production function (2.4)-(2.5), we set  $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1/3$ . It would seem that we need to set one disturbance variance for each unobserved variable. Capital ( $k_t$ ), technology ( $\tau_t$ ), and the output-demand state ( $d_t$ ) are actually unobserved and materials ( $m_t$ ) was treated as unobserved (for reasons explained in Chen and Zadrozny, 2005). However, it turned out that setting  $\sigma_{pq}^2 = \sigma_l^2 = \sigma_m^2 = 10^{-10}$  was sufficient to identify the unrestricted and estimated structural parameters. We set the 3 variances to small positive values, rather than to zero, because doing so resulted in more accurate computations using the MDKF. We considered different identifying restrictions and obtained different structural-parameter estimates but obtained similar reduced-form-parameter and capital and technology estimates.

Because input prices are assumed to be generated by exogenous univariate AR(2) processes (2.7), the processes can be estimated (asymptotically) efficiently and individually using ordinary least squares (OLS), which is much simpler than using MLE to estimate simultaneously all parameters in  $\mathcal{Q}_2$ . Thus, in the application, first, we estimated input-price parameters using OLS and, then, conditional on these estimates, used MLE to estimate the remaining parameters in  $\mathcal{Q}_2$ .

Two separate general identification conditions must be satisfied in order to estimate the structural parameters and, then, to estimate capital and technology for given estimated parameters. The first parameter-identification condition is the standard one that the Hessian matrix of  $L(\mathcal{Q}, \bar{Y}_N)$  with respect to estimated parameters in  $\mathcal{Q}_2$ , evaluated at restricted and estimated parameter values in  $\mathcal{Q}$ , is positive definite. The second capital-and-technology-identification condition is that the model's state representation is reconstructible. See Chen and Zadrozny (2005) and references therein for further details. Both the parameter identification and reconstructibility conditions were verified numerically in the application.

### 3.2. Description of the Data.

We used annual U.S. total manufacturing data on prices and quantities of output and inputs from 1949-2005. Investment and GDP-deflator data were obtained from the Bureau of Economic Analysis (BEA), research data from the National Science Foundation (NSF), and all other data from the Bureau of Labor Statistics (BLS). All data were obtained from their producers in annual form, even though all except research price and quantity data are also available monthly or quarterly and seasonally adjusted or not. All data were previously released to the public and are not confidential. Thus, we obtained data on 10 of 13 variables in the model:  $p_{qt}$ ,  $q_t$ ,  $l_t$ ,  $m_t$ ,  $i_t$ ,  $p_{it}$ ,  $p_{it}$ , and  $p_{mt}$  for 1949-2005 and  $p_{rt}$  and  $r_t$  for 1953-2005.

Until 1997, U.S. industries were classified according to the Standard Industrial Classification (SIC), with the highest (cross-sectionally) aggregated manufacturing industries numbered 20-39. After 1997, the North American Industry Classification System (NAICS) replaced SIC, with the highest aggregated manufacturing industries numbered 31-33. As expected, data for more aggregated industry groups were affected less by the switch than data for less aggregated industry groups and, for each degree of aggregation, levels data were affected more than growth-rate data. After the switch, BLS produced SIC manufacturing data until 2001 and NAICS manufacturing data back to 1987. Thus, overlapping SIC and NAICS manufacturing data are available for 1987-2001.

In MLE with the MDKF, variables can have any pattern of missing observations subject only to having enough observations so that parameters can be identified and estimated. The MDKF can automatically process multiple observations per variable when a state representation is set up to allow this possibility. However, it comes at the cost of specifying an observation-error process, imposing additional restrictions on parameters to ensure identification, and estimating additional observation-error parameters. Instead, we more simply transformed the two observations per variable to a single one in the overlapping period 1987-2001 as follows. When both SIC and NAICS observations were available, we arithmetically averaged their growth rates, with equal weights, to a single observation; when one observation was available, we chose it as the single observation.

Except for labor quantity measured by the number of production workers, all prices and quantities were computed as indexes based on given nominal price indexes, real quantity indexes, and nominal expenditures, as follows. Real

quantities were computed as nominal expenditures divided by nominal price indexes and nominal prices were computed as nominal expenditures divided by real quantity indexes. Then, all given or computed nominal price indexes were converted to real form by dividing them by the GDP deflator.

Resulting real prices and real quantities of U.S. total manufacturing output and inputs are depicted in figure 1. For viewing convenience, the data were scaled to lie between 0 and 4. The graphs suggest the following brief economic interpretation: increasing demand for output driven partly by a declining real price of output induced manufacturers to increase production capacity. Increasing quantities of investment and research built increasing stocks of capital and technology, hence, increased production capacity. As the price of labor increased, manufacturers used approximately the same labor input and more materials, capital, and technology inputs, which resulted in increased labor productivity. Current and previous figures 1 show that current merged SIC and NAICS data and previous SIC data are similar in the overlapping 1949-1997 period. The notable exception is research prices which in the previous SIC data are slightly noisier and follow a mostly concave path but in the current SIC and NAICS data initially follow a convex path which later switches to a concave path.

[Put figure 1 approximately here]

### 3.3. Properties of the Estimated Model.

Following Chen and Zadrozny (2005) and for reasons explained there, when estimating the model, we treated materials inputs as unobserved, because including their observations in the estimation resulted in a very poor fit of the labor equation. Table 1 reports OLS estimates of input-price processes (2.7): estimated coefficients, their absolute t ratios in parentheses, implied absolute characteristic roots,  $R^2$  statistics, and Q statistics for testing absence of residual autocorrelations at lags 1-10, with marginal significance levels or p values in parentheses. Estimated equations fit typically well for level-form data, having  $R^2 \geq .81$ . Residuals show no significant autocorrelations, having p values of Q statistics greater than 1%. Except for the clearly stationary materials price equation, the other price equations have borderline unit roots. All characteristic roots satisfy the growth condition  $|\bar{\lambda}| < 1/\sqrt{\delta}$ , which is required for solving the firm's dynamic optimization problem. The only notable difference between current and previous

table 1 estimates of the input-price processes is that the maximum absolute characteristic root of the estimated research-price equation increased from a borderline stationary value of .949 to a firmly unit-root value of .999.

[Put table 1 approximately here]

Table 2 reports MLE of non-input-price parameters in  $\mathcal{G}_2$  (conditional on  $\mathcal{G}_1$  set by identifying restrictions and on OLS-estimated input-price parameters in  $\mathcal{G}_2$ ) and fit statistics of implied reduced-form equations of observed endogenous variables. Standard errors of ML-parameter estimates were very large and are not reported because the nonuniqueness of the structural parameter estimates vitiates the usual meaning of their standard errors. In particular, experimentation showed that different identifying restrictions resulted in different structural-parameter estimates, but similar reduced-form-parameter, capital, and technology estimates. However, if the principal goal is obtaining model-based capital and technology estimates, then, as happened, obtaining essentially unique reduced-form-parameter, capital, and technology estimates is primary and obtaining nonunique structural-parameter estimates is secondary. Thus, because nonunique structural-parameter estimates depend on somewhat arbitrary identifying restrictions, their dubious standard errors are not reported.

[Put table 2 approximately here]

$R^2$  statistics in table 2 show that reduced-form equations of observed endogenous variables have typically good fits for level-form data: moderate ( $> .65$ )  $R^2$  of the labor equation and high ( $> .93$ )  $R^2$ s of the other equations reflect labor's noisiness and the other variables' smoothness. Estimated residual autocorrelation parameters,  $\theta$ , are high, which raises the question of whether residual autocorrelations or the economic part of the model account for most of the sample variations of the observed endogenous variables. However, reestimation with all  $\theta$ 's set to zero produced  $R_{pq}^2 = .921$ ,  $R_q^2 = .929$ ,  $R_\ell^2 = .604$ ,  $R_i^2 = .905$ , and  $R_r^2 = .961$  for the reduced-form equations, which indicates that the economic part of the model accounts for most of the sample variations of the endogenous variables.



Comparing current and previous tables 2, only  $\hat{\phi}_{t1}$  differs significantly, declining from .161 to .043, which indicates increased per annum depreciation of technology from 83.9% to 95.7%. Although differences in other parameter estimates might seem insignificant, they imply quantitatively different behavior: although current and previous impulse responses in figures 2 are qualitatively similar, variance decompositions in current and previous tables 3 are quantitatively different.

Following Chen and Zadrozny (2005), we used a likelihood ratio to test the validity of the model's overidentifying restrictions. The test is important because if the estimated model's overidentifying restrictions are not rejected, then, the estimated model and the capital and technology estimates derived from it can be considered consistent with the data. To test the economic or behavioral implications of the model, we excluded exogeneity zero restrictions on the input-price processes. Under standard assumptions on the data generating process and the assumption that the null hypothesis that the overidentifying restrictions are valid, the likelihood-ratio statistic, LR, is distributed asymptotically as  $\chi^2(\kappa)$ , where  $\kappa = 118$  denotes the degrees of freedom and the number of overidentifying restrictions being tested. For details on how LR was constructed, see Chen and Zadrozny (2005) and Sims (1980, p. 17, fn. 18).

For the 1960-1990 SIC-NAICS data (the sample period for the LR test in Chen and Zadrozny, 2005), under conventional  $\chi^2(118)$  evaluation, LR = 151 with  $p = .0205$  indicates either weak rejection ( $p < 5\%$ ) or weak nonrejection ( $p > 1\%$ ) of the null hypothesis. For the 1960-2005 SIC-NAICS data, under conventional  $\chi^2(118)$  evaluation, LR = 181 with  $p = .000169$  indicates strong rejection of the null hypothesis. Although unit roots might affect these  $p$  values (see the discussion at the end of section 3.4), it is unclear in which direction. More significantly, because 118 degrees of freedom is large relative to 57 sample periods and 13 variables, the distribution of LR might not be close to  $\chi^2(118)$ . Despite these doubts, LR is a useful quantitative indicator of the consistency of the model with the data. In sum, given good fit (high  $R^2$ 's and high  $p$  values of  $Q$ 's) and economically rationalizable impulse responses in terms of adjustment costs, we accept the estimated model and the capital and technology estimates derived from it as an acceptable economic-statistical accounting, surely for the 1960-1990 SIC-NAICS data, more tentatively for the 1991-2005 SIC-NAICS data.

[Put figure 2 approximately here]

Figure 2 illustrates some adjustment-cost features of the estimated model in terms of responses to unit impulses in output-demand and technology disturbances. Upper graphs 2a depict responses to a unit one-period shock to output demand ( $\zeta_{dt}$ ) in period 1, starting from an initial long-run equilibrium at the origin. High adjustment costs arising from  $\hat{\rho} = 267$  imply a steep marginal-cost-of-production curve, so that after the output-demand shock occurs, the price of output rises sharply but output increases only slightly. Initially, the extra output is produced using additional freely-adjusted labor and materials inputs and pre-shock stocks of capital and technology. Because the shocked demand state declines sufficiently slowly, firms have an incentive to increase their production capacities, by increasing their investment and research rates and substituting capital and technology for labor and materials. Eventually, all variables return to the origin. Lower graphs 2b depict responses to a unit one-period shock to technology ( $\zeta_{\tau t}$ ) in period 1, again starting from an initial long-run equilibrium at the origin. Output-demand conditions remain unchanged so there is little change in price or quantity of output. The shock mainly causes technology to be substituted for labor and materials until the windfall addition to technology has depreciated fully. Again, eventually all variables return to the origin.

[Put table 3 approximately here]

Table 3 reports variance decompositions (Sims, 1986) of the estimated model. Each number in the table, in percentage form, indicates the fraction of the variance of one of the 8 endogenous variables (rows 2-9 indicated by  $s_{10,pq,j}, \dots, s_{10,\tau,j}$ ) or the sum of the variances of the 8 endogenous variables (row 10 indicated by  $\bar{s}_{10,j}$ ) accounted for by the variance of a structural disturbance of an endogenous or exogenous variable, which is not set to near zero as an identifying restriction but is estimated (columns 2-11 indicated by  $\sigma_q^2, \dots, \sigma_d^2$ ). The particular values of the variance decompositions in table 3 depend on the particular values of the set and estimated structural parameters, which, for the given model structure, are nonuniquely determined by the data, but depend somewhat arbitrarily on identifying restrictions. Nevertheless, the variance decompositions in table 3 are broadly similar to alternative variance decompositions resulting from alternative identifying restrictions and resulting parameter estimates.

Table 3 is summarized as follows. If, somewhat arbitrarily, a number in the table less than 5, between 5 and 10, and more than 10 is considered insignificant, moderately significant, and strongly significant, then, the numbers in columns 6 and 9, reflecting  $\zeta_{rt}$  and  $\zeta_{p/rt}$ , are insignificant; the numbers in columns 2 and 8, reflecting  $\zeta_{qt}$  and  $\zeta_{p/rt}$ , are at most moderately significant; and, one or more numbers in each of columns 3, 4, 5, 7, 10, and 11, reflecting  $\zeta_{it}$ ,  $\zeta_{rt}$ ,  $\zeta_{kt}$ ,  $\zeta_{pit}$ ,  $\zeta_{pmt}$ , and  $\zeta_{dt}$ , are strongly significant. There are notable symmetries. For example, investment and research disturbances have symmetrical effects (columns 3-4) and labor and materials respond very similarly in most cases (rows 4-5). Columns 3-6 indicate that investment and capital disturbances account for somewhat more of variations of individual endogenous variables (rows 2-9) and account for 7.47 times more of overall variations of endogenous variables (row 10) than do research and technology disturbances and, accordingly, counter the "real business cycle" premise that technology disturbances are the primary source of variations in variables. As an accounting of output growth, row 2 indicates that 94.5% of the variations of output are accounted for in decreasing order by variations in disturbances of investment, price of investment, research, output demand, price of materials, price of research, and output. Current and previous tables 3 are consistent.

#### **3.4. Model-Based and Standard Capital and Technology Estimates.**

The model was estimated as described in section 3, using standardized levels of the data described in section 3.2. As in Chen and Zadrozny (2005), the MDKF was applied to the estimated model and the 1949-2005 data, which produced filtered state estimates,  $\hat{z}_{t|t}$ , and their error covariance matrices,  $E(z_t - \hat{z}_{t|t})(z_t - \hat{z}_{t|t})^T$  for 1949-2005. Then, elements 7 and 8 of  $\hat{z}_{t|t}$  were picked as the model-based estimates of capital and technology,  $\hat{k}_{t|t}$  and  $\hat{\tau}_{t|t}$ , and the square roots of diagonal elements 7 and 8 of the error covariance matrices were picked as their estimated standard errors. Figure 3 depicts the model-based and standard estimates of capital and technology for U.S. total manufacturing industries for 1949-2005. Solid lines depict model-based estimates and their 2-standard-error confidence bounds. Standard errors of technology estimates for 1949-1952 are large due to initialization effects in the MDKF. For capital, dashed lines depict weighted sums of BLS stock estimates of equipment, structures, inventories, and land, based on nonstochastic PIEs (equations (2.8)-(2.9) without disturbances are nonstochastic PIEs of capital

and technology). BLS also produces service-flow estimates of equipment, structures, inventories and land, but weighted sums of these estimates are very similar and are, thus, not depicted or considered further. For technology, dashed lines depict BLS estimates of total factor productivity (TFP) based on Solow residuals. The BLS capital stock and TFP estimates are graphed in figure 3 as examples of standard capital and technology estimates.

Because MLE is tractable only if the data are scaled similarly, the data were standardized prior to estimation, by subtracting sample means and dividing by sample standard deviations. Being based on standardized data, the model-based capital and technology estimates and standard errors are implicitly, but not exactly, in standardized form. Standard (BLS) capital (stock) and technology (TFP) estimates are in different and essentially arbitrary units. To make the capital and technology estimates and their standard errors comparable, before graphing them, we standardized them. Also, to make the estimates and confidence bounds look more sensible by being positive, before graphing them, we shifted them all up by the same amount. However, because the graphed values are in fundamentally arbitrary units, vertical differences between them should not be interpreted in percentage terms.

[Put figure 3 approximately here]

Graphs of the model-based and standard capital and technology for 1949-2005 in figure 3 are now summarized. (1) For 1949-2000, trends of model-based and standard capital and technology are broadly similar. (2) For 2000-2005, model-based and standard estimates diverge significantly: model-based capital continues its previous growth; standard capital levels off; model-based technology declines and levels off; and, standard technology continues its previous growth. (3) For 1949-2005, model-based capital is noisy and uncertain and model-based technology is smooth and certain. To the extent that current results duplicate previous ones (Chen and Zadrozny, 2005), they support the model-based method as a general method for estimating capital and technology. The results have the following implications and interpretations.

The similar trends of the model-based and standard estimates for 1949-2000 make them mutually reinforcing. Being produced by government agencies and commonly used, the standard estimates are usually considered the "truer" ones. The intention here is not to challenge this view but to consider alternative estimates of capital and technology based on an estimated dynamic structural economic model which has features that are now standard in dynamic

economic modeling: the variables of primary interest are endogenous in the model, in this case, capital and technology; agents in the model solve an explicitly considered dynamic optimization problem; the resulting dynamics of the endogenous variables arise naturally from elementary structural components, in this case, adjustment costs from the CES-CET production function; and, the model is identified and estimated using real (not simulated) data.

Suppose "short run" means cycles with periodicities less than about 5.6 years long (about the average business cycle length in the U.S. after World War II) and "long run" means longer cycles. Some short-run variations of capital and technology, either model-based or standard, are correlated with and, hence, may be considered explained by large known events such as the Vietnam War (1965-73) or oil-price shocks (1973, 1979). Remaining unexplained short-run variations may, then, be considered random noises. Figure 3 shows that model-based capital has more, larger, and noisier short-run variations than model-based technology. Consequently, model-based capital appears more uncertain than model-based technology, a conclusion which is supported by the 2-standard-error confidence bounds in figure 3 produced by the Kalman filter. Standard errors of model-based capital and technology are, respectively, about 1.01 and .036, which means that capital confidence bounds are about 28 times wider than technology confidence bounds. Being in unit-free standardized form, model-based estimates and confidence bounds are comparable. In essence, the residual role of technology in standard estimates switches to capital in model-based estimates, which is more realistic because capital is the residual-income earner and knowledge, the basis of technology, is presumably mostly invariant to higher-frequency economic variations.

An important question is whether investment and capital or research and technology better account for variations in endogenous variables, in particular, output (Gordon, 2000; Oliner and Sichel, 2000; Stiroh, 2001). Columns 3-6 of table 3 indicate that the estimated model's investment and capital disturbances account for somewhat more of variations of individual endogenous variables (rows 2-9) and account for 7.47 times more of overall variations of endogenous variables (row 10) than do research and technology disturbances.

The time-series properties of standard capital estimates depend entirely on the time-series properties of investment and on capital depreciation in its PIE. For given time-series properties of investment, standard capital estimates are smoother and more trendlike when capital depreciates more slowly. Time-

series properties of model-based capital and technology estimates likewise depend on time-series properties of investment and research and on depreciation rates, but also on Kalman-filter estimates of disturbances in stochastic PIEs. For example, Kalman-filter estimates of capital based on equation (2.8) are

$$(3.6) \quad \hat{k}_{t|t} = \phi_{k1} \hat{k}_{t-1|t} + \phi_{i0} i_t + \hat{\zeta}_{kt|t},$$

where  $\hat{x}_{s|t}$  denotes estimated or expected  $x_s$ , conditional on data through period  $t$ . Thus, the time-series properties of model-based capital and technology estimates also partly depend on the time-series properties of estimated disturbances,  $\hat{\zeta}_{kt|t}$ .

To consider how much noise the relatively large estimated capital disturbance variance of .99 passes to the model-based capital estimates through equation (3.6), as in Chen and Zadrozny (2005), we recomputed the capital and technology estimates with the capital disturbance variance set to  $1.0 \times 10^{-6}$ , the value of the estimated technology disturbance variance, and left the other parameters at their estimated values. The resulting graphs (not shown here; cf., Chen and Zadrozny, 2005, figure 4) of model-based capital and technology are very similar to those in figure 3, except that both capital and technology are as smooth as technology in figure 3. Thus, the large capital disturbance variance in (2.8) appears to make model-based capital estimates noisy.

Estimated annual capital and technology depreciation rates of  $1 - \phi_{k1} = .39$  and  $1 - \phi_{r1} = .96$  are very high (cf., Jorgenson and Stephenson, 1967). To check whether system-wide MLE somehow caused high estimated depreciation rates, we reestimated capital and technology equations (2.8)-(2.9) separately by nonlinear least squares, using the model-based and BLS capital-stock and technology estimates as data, and obtained very similar estimated depreciation rates (not shown here; cf., Chen and Zadrozny, 2005, tables 2 and 4). For given depreciation, investment, and research rates, the model-based capital and technology estimates and their estimated disturbances,  $\hat{\zeta}_{kt|t}$  and  $\hat{\zeta}_{rt|t}$ , are smoother and more trendlike when system characteristic roots are near one. A system characteristic root,  $\lambda$ , is a solution of the characteristic equation,  $|I_{13}\lambda^2 - B_1\lambda - B_2| = 0$ , of equation (3.2), where  $|\cdot|$  denotes a determinant and  $I_{13}$  denotes the  $13 \times 13$  identity matrix. The system has 26 roots: 2 exogenous roots arising from demand-state process (2.3), 8 exogenous roots arising from input-price processes (2.7), 6 exogenous roots arising from residual autocorrelation

coefficients  $\theta$ , and 10 endogenous roots depending on the solution of the dynamic optimization problem. An accounting based on the parameter estimates in tables 1 and 2 indicates 9 near-unit roots (within .02 of 1): 8 exogenous near-unit roots and 1 endogenous near-unit root. The estimate  $\hat{\rho} = 275$  in table 2 implies that output transformation in production is near zero (CET = .004), so that adjustment costs are very high and an endogenous root is near one, which is confirmed by the root accounting above. Model-based capital and technology estimates should be smoother and more trendlike to the extent that estimated  $\rho$  is high, estimated adjustment costs are high, and an endogenous root is near one. However, an unexplained paradox remains why technology estimates are very smooth or unit-root-like while their primary determinant, technology equation (2.9), has a very un-unit-root-like high depreciation rate.

#### 4. Conclusion.

The paper has described and applied an economic method for estimating unobserved stocks of production capital and technology or total factor productivity of U.S. total manufacturing industries for 1949-2005. The method was applied to the merged 1949-2001 SIC and 1987-2005 NAICS data. The method involves using the data to estimate a dynamic structural economic model and, then, using the data, the estimated model, and the Kalman filter to compute filtered estimates of capital and technology. The estimated model gives the data and the capital and technology estimates an economic-statistical rationale missing from standard estimates based entirely on Solow-residual and perpetual-inventory accounting.

Despite an equivocal result of a likelihood-ratio test of overidentifying restrictions, we accept the estimated model as an economic-statistical accounting, surely for 1960-1990 SIC-NAICS data, more tentatively for 1991-2005 SIC-NAICS data. The variance decompositions of the estimated model in table 3 indicate that investment and capital disturbances account for somewhat more of individual variations of endogenous variables (rows 2-9) and account for 7.47 times more of overall variations of endogenous variables (row 10) than do research and technology disturbances. The model-based capital and technology estimates in figure 3 indicate the following. For 1949-2000, trends of model-based and standard estimates of capital and technology are broadly similar and, thus, reinforce each other. For 2000-2005, the estimates diverge significantly: model-based capital continues its previous growth; standard capital levels off; model-based technology declines and levels off; and,

standard technology continues its previous growth. For 1949-2005, model-based capital is noisy and uncertain and model-based technology is smooth and certain. Estimated capital and technology depreciation rates are very high. Specifying capital and technology PIEs as more general rational distributed lags, instead of (2.8)-(2.9), could result in lower estimated capital and technology depreciation rates.

If possible, we should distinguish between degrees of effective allocation and utilization rates of capital and technology, although, without further detailed modelling, capital and technology estimates must, as usual, be considered fully effective and fully utilized. Both model-based and standard estimation methods effectively treat investment and research as fully successful, regardless of any misallocations and market valuations, and do not adjust the estimates for utilization rates. Thus, an optimally allocated factory adds the same amount to a capital estimate as a misallocated one built using the same resources. However, adjusting the capital estimates for effectiveness and utilization rates (also, possibly the technology estimates) would require expanding the model significantly beyond its present form.

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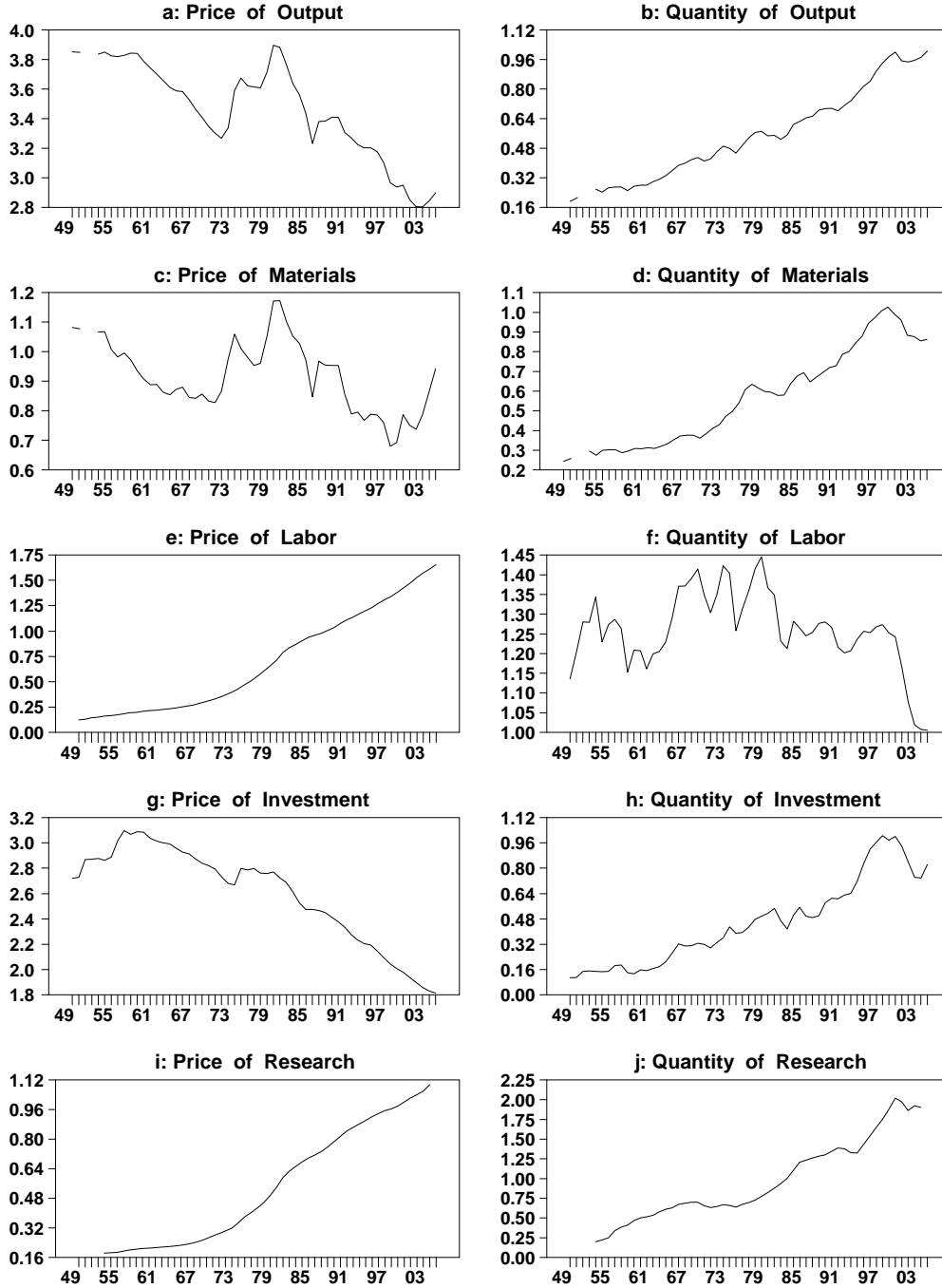
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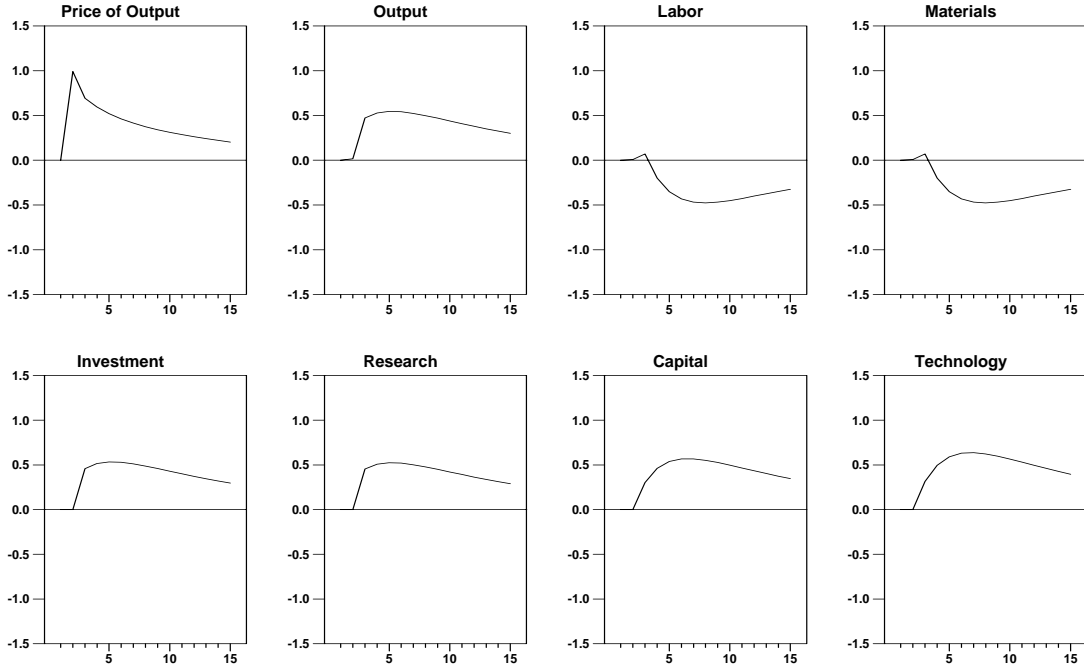
Figure 1: U.S. Total Manufacturing, Real Prices and Quantities of Output and Inputs, 1949-2005



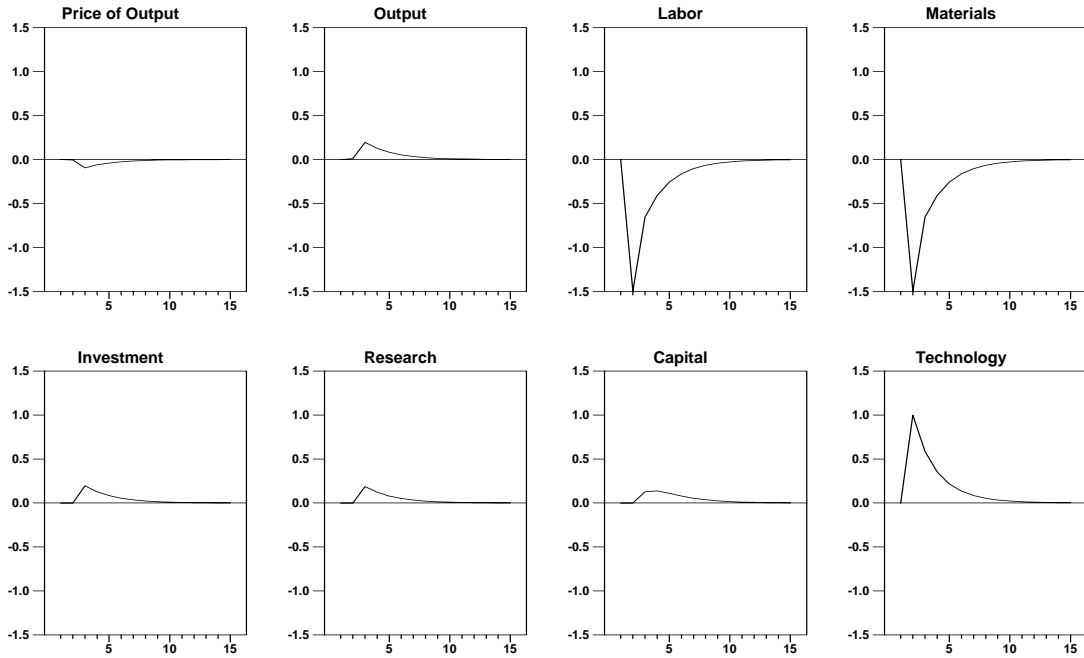
The dates on the horizontal axes refer to 1949-2003.

Figure 2: Impulse Responses of the Estimated Model

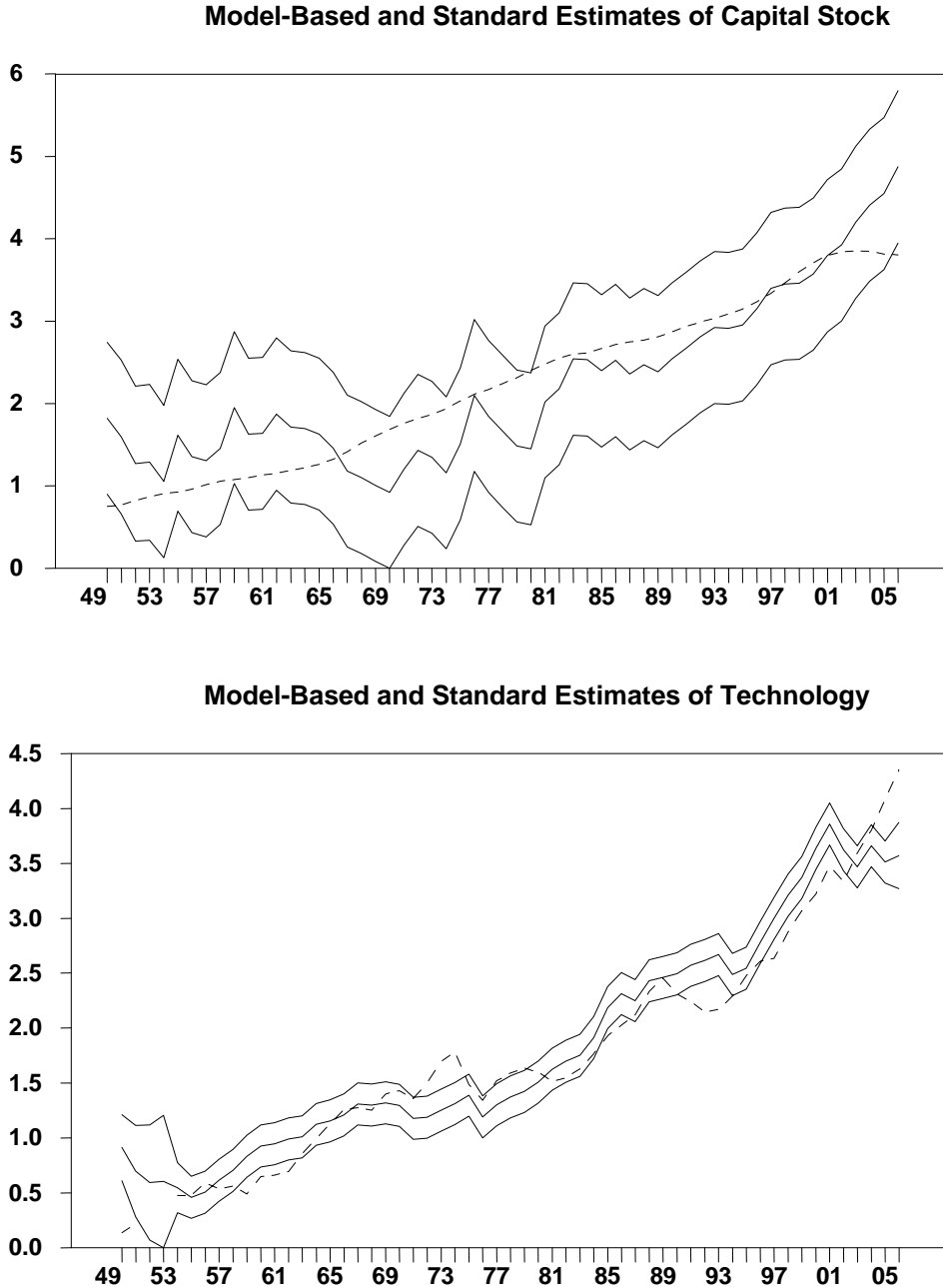
a: To an Impulse in the Output-Demand Disturbance



b: To an Impulse in the Technology Disturbance



**Figure 3: Model-Based and Standard Capital (Stock) and Technology (TFP)  
Estimates for U.S. Total Manufacturing, 1949-2005**



Solid lines depict model-based capital and technology estimates and 2-standard-error confidence bounds produced by the Kalman filter. Dashed lines depict standard capital stock estimates and technology estimates as total factor productivity, produced by BLS. The dates on the horizontal axes refer to 1949-2005.

Table 1: Ordinary Least Squares Estimates of Input-Price Process Parameters

Variable	Parameter Estimates			Fit Statistics	
	$\hat{\phi}_{.1}$	$\hat{\phi}_{.2}$	$ \bar{\lambda} $	$R^2$	Q
$P_i$	1.37 (10.5)	-.356 (2.61)	1.02	.988	6.76 (.563)
$P_r$	1.90 (28.2)	-.900 (13.1)	.999	.999	17.6 (.024)
$P_\ell$	1.88 (25.6)	-.874 (11.7)	1.03	.999	13.3 (.101)
$P_m$	1.16 (8.34)	-.319 (2.32)	.712	.812	4.40 (.820)

Columns 2-6 show estimates of  $\hat{\phi}_{.1}$  and  $\hat{\phi}_{.2}$ , with absolute t statistics in parentheses, implied maximum absolute characteristic roots (solutions of  $\lambda^2 - \hat{\phi}_{.1}\lambda - \hat{\phi}_{.2} = 0$ ), unadjusted  $R^2$ , and Ljung-Box Q statistics for testing absence of residual autocorrelations at lags 1 to 10, with marginal significance levels or p values in parentheses.

**Table 2: Maximum Likelihood Estimates of Non-Input-Price Structural Parameters**

<p><b>Production Function Parameters</b></p> $\hat{\beta} = -5.48 \text{ (CES} = -.182), \hat{\rho} = 275 \text{ (CET} = .004)$
<p><b>Output-Demand Curve Parameters</b></p> $\hat{\eta} = .932, \hat{\phi}_{d1} = 1.18, \hat{\phi}_{d2} = -.367$
<p><b>Capital and Technology Equation Coefficients</b></p> $\hat{\phi}_{k1} = .610, \hat{\phi}_{i0} = .789, \hat{\phi}_{r1} = .043, \hat{\phi}_{r0} = .304$
<p><b>Residual Autocorrelation Coefficients</b></p> $\hat{\theta}_{pq} = .999, \hat{\theta}_q = .675, \hat{\theta}_\ell = .999, \hat{\theta}_m = .999, \hat{\theta}_i = .848, \hat{\theta}_r = .942$
<p><b>Structural Disturbance Standard Deviations</b></p> $\hat{\sigma}_q = .144, \hat{\sigma}_i = .246, \hat{\sigma}_r = .106, \hat{\sigma}_k = .995, \hat{\sigma}_\tau = .001, \hat{\sigma}_d = .207$
<p><b>Reduced-Form Equation Fit Statistics</b></p> $R_{pq}^2 = .932, R_q^2 = .942, R_\ell^2 = .651, R_i^2 = .938, R_r^2 = .990$ $Q_{pq} = 2.52, Q_q = 2.55, Q_\ell = 13.1, Q_i = 13.6, Q_r = 12.9$ $(.989) \quad (.990) \quad (.218) \quad (.194) \quad (.230)$

The sample span is 1949–2005 (57 years).  $R^2$  and  $Q$  statistics and  $p$  values in parentheses are as in table 1.

Table 3: Structural Variance Decomposition of the Estimated Model

	$\sigma_q^2$	$\sigma_i^2$	$\sigma_r^2$	$\sigma_k^2$	$\sigma_\tau^2$	$\sigma_{pi}^2$	$\sigma_{pr}^2$	$\sigma_{pl}^2$	$\sigma_{pm}^2$	$\sigma_d^2$
$\mathbf{s}_{10,pq,j}$	6.6	17.7	16.3	3.4	.00	17.4	7.6	1.6	8.0	21.4
$\mathbf{s}_{10,q,j}$	7.2	19.2	17.7	3.7	.00	18.8	8.3	1.8	8.6	14.7
$\mathbf{s}_{10,l,j}$	2.2	3.7	3.3	80.3	.00	1.5	.82	.01	5.7	2.4
$\mathbf{s}_{10,m,j}$	2.0	3.2	2.9	70.7	.00	1.3	.73	.88	16.1	2.1
$\mathbf{s}_{10,i,j}$	.00	54.3	3.5	.54	.00	16.4	6.8	1.7	5.8	11.0
$\mathbf{s}_{10,r,j}$	.00	1.6	41.9	3.5	.00	19.5	9.2	1.9	8.2	14.2
$\mathbf{s}_{10,k,j}$	.00	23.5	1.5	57.8	.00	6.8	2.5	.63	2.6	4.8
$\mathbf{s}_{10,\tau,j}$	.00	1.6	41.9	3.4	.00	19.5	9.2	1.9	8.2	14.2
$\bar{\mathbf{s}}_{10,j}$	1.4	19.8	8.3	42.2	.00	9.7	4.1	.99	5.6	7.8

Rows 2-9 give percentage decompositions of 10-year-ahead forecast-error variances of the 8 endogenous variables in terms of the 10 estimated structural-disturbance variances. For example,  $s_{10,pq,2} = 6.6$  is the percentage of the variance of  $p_{qt}$  accounted for by the variance ( $\sigma_q^2$ ) of the structural disturbance of output ( $\zeta_{qt}$ ). Row 10 gives the percentage decomposition of the sum of the variances of the 8 endogenous variables. Each row sums to 100.