The Quality of Seasonal Adjustment: An Empirical Comparison of Various Seasonal Adjustment Methods October 2012

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Abstract
Using two basic assumptions of time series analysis, a test is proposed to assess the quality of seasonal adjustment of a series. The test is applied to several seasonally adjusted series of the Consumer Price Index (CPI) using three methods of seasonal adjustment: State Space Model Based Method (SSMB), TRAMO SEATS Method and X12ARIMA method. The empirical results are presented along with comparative graphs.
Key Words: Intervention and Seasonally Adjusted Series, Trend, Smoothness.

1. Introduction

The quality of seasonal adjustment is very critical for policy purposes because a very large number of financial, economic and non-economic time series, after seasonal adjustment, are used to formulate business, economic and public policies. Currently several test statistics such as M1-M11 developed by Dagum (1988), Sliding Span diagnostics by Findley et al (1990), smoothness of trend-cycle and seasonal components, revision statistics and spectral diagnostics (see EuroStat (2009)), are used to judge the quality of seasonal adjustment of a time series. Most of these test statistics and diagnostics are used to judge the quality of individual components of a series. But even if each component estimate is validated by these test statistics, the quality of seasonal adjustment may still not be good because there is an implicit constraint among these components which must be tested for its validity. If one component estimate is unsatisfactory, it can affect the estimates of other components. It is this constraining relationship between various component estimates of a time series which is the basis of the graphical test outlined in the next section. This test called Component Consistency Graphical Test (CCGT) would be used to compare the quality of seasonal adjustment for four sample series. In addition, individual component estimates of each series estimated by all the methods are compared graphically to assess the numerical differences and quality of those estimates for each method.

2. The Component Consistency Graphical Test (CCGT):

Preliminaries

A time series is represented as:

(1)

Where

is the observed sample series at time ;

is the unobserved trend-cycle component at time ;
is the unobserved seasonal component at time ;
is the unobserved intervention component at time ;
is the error at time .

When estimated by *any* method, the above equation can be written as:

(2)

Where
the ^ on top of each component symbol represents an estimate of that component.
is obtained by subtracting all other estimated components from observed .

Two definitions we will use are given below:

INTERVENTION ADJUSTED SERIES (IAS)**:**

IAS = (3)

INTERVENTION AND SEASONALLY ADJUSTED SERIES (ISAS)*:*

ISAS = (4)

TWO PREMISES OF THIS TEST:

ISAS = TC + RESIDUAL ERROR (A)
TREND-CYCLE (TC) is smooth graphically like a local polynomial trend as in Kitagawa & Gersch (1984) (B)

THE CCG TEST IS DEFINED AS FOLLOWS**:**

The consistency constraint among the component estimates is satisfied and so CCGT is validated if

(i) The trend-cycle (TC) is smooth, and
(ii) Intervention and seasonally adjusted series (ISAS) is similar in shape, though not smooth because it includes residual errors.

The CCG Test is not validated if

(i) The trend-cycle (TC) is not smooth.

In this case, it does not matter if ISAS is similar in shape to TC or not.
The quality of seasonal adjustment is good if CCGT is validated by a seasonal adjustment method

3. Applications:

This test is applied to three methods of seasonal adjustment:

1. X12 ARIMA (X12A)
2. State Space Model Based Method (SSMB)
3. TRAMO-SEATS (TSW)

A brief description of each method is given below:

X12A: This is a non-model based method, hence not subject to statistical testing although statistics are frequently used in their evaluation. X12 ARIMA, or X12A, is an enhancement of X11A. As in the X11A method, the observed sample series to be seasonally adjusted is first extended beyond the sample period by ARIMA model forecasts. This is done because the moving average used to estimate the trend component reduces the size of the sample. The observed (or extended) series is then prior adjusted for interventions or calendar effects, if any, by regression models with ARIMA errors (regARIMA). Then moving averages are used to estimate the trend, which is removed from the prior adjusted series to obtain estimated seasonal and error components. Moving averages are used again to eliminate the residual errors to estimate the seasonal component. These steps are repeated several times to obtain seasonally adjusted series. (See Dagum (1988), Australian Bureau of Statistics (2008), Findley et.al(1998) and Jain (1989)).

State Space Model Based (SSMB) Method: This is a model-based method in which unobserved component models are used to model trend and seasonal components and dummy variable models with random coefficients are used to estimate the intervention components. These models are set in a state space model format and estimated by Kalman filtering and Smoothing methods. The EM algorithm and Quasi-Newton maximum likelihood procedures are used to refine the estimates. A model thus estimated is then tested for Adequacy (Lung-Box Q*, BDS and MBDS statistics for uncorrelatedness of residuals), goodness of fit (RBARSQUARES, AIC and BIC) and forecasting performance of the model. Several models are estimated and tested for a series and the one that passes all tests and gives minimum AIC/BIC is used for seasonal adjustment of the series. A seasonally adjusted series is evaluated for the presence of stable and/or moving seasonality using M7 (see Dagum (1988)). SSMB is a one- step method in which all components of a model, trend, seasonal, and interventions are estimated simultaneously and there is no danger of misspecification. (See Harvey (1990), Jain (1992, 1993), Kitagawa and Gersch (1984), and Shumway and Stoffer (2000)).

TRAMO-SEATS(TSW)*:* In Spain, Victor Gomez and Augustin Maravall have developed a program, TRAMO-SEATS, under the auspicious of the Bank of Spain (www.bde.es/homee.htm). This is a two-step program in which TRAMO (Time Series Regression with ARIMA Noise, Missing Observations and Outliers) is used for prior adjustment of the observed series. The SEATS (Signal Extraction in ARIMA Time Series) is an ARIMA model based method of seasonal adjustment which estimates the trend, seasonal and irregular components of the prior adjusted series using signal extraction techniques applied to ARIMA models.

In summary, X12A and TRAMO-SEATS (TSW) are two-step procedures whereas the SSMB method is a one-step procedure. In the first step, in both X12A and TSW methods, regARIMA models, which are additive structural models with ARIMA errors, are used to obtain prior adjusted series. In the second step, however, these methods use either multiplicative X11A or ARIMA models, to decompose the prior adjusted series into components. In econometric parlance, this multiple specification leads to misspecification of the model underlying the observed series.

Four BLS series are used to illustrate the application of this test:

1. PPI of Industrial Electric Power (PEIND)
2. CPI of Fuel Oil and Other Fuels (S11)
3. CPI of Cheese and Related Products (S19)
4. Unemployment Level of Civilian Labor Force 16 to 19 years old Male (UM1)

4. Graphical Results for CCGT:

Graphs for each series by all three methods are presented on a single page for comparison. The graphs in each column are those of one method. The graphs in the first row of graphs on each of the four pages represent original series(OS) and the trend-cycle; in the second row the intervention adjusted series(IAS) and the trend cycle; and in the third row the intervention adjusted series(IAS) and the intervention and seasonally adjusted series(ISAS). See Appendix A for these graphs.

Inference From These Graphs:

(i) Comparing the first row of graphs, it is clear that the X12A method does not generate smooth TC graphs in any of the four series, and TSW does not generate smooth TC graphs in the first three series but does generate a smooth TC graph in case of the UM1 series, which has no interventions. On the other hand the SSMB method produces smooth TC graphs in all four series. For numerical measures of smoothness of trend see section 7. Table 1.

(ii) In the second row, the IAS graphs for X12A and TSW for the first three series, which have interventions, do not seem to be free of all intervention effects as compared to the graphs for the SSMB method. This is particularly obvious for the PEIND series. Due to the absence of interventions, the first two rows of UM1 graphs are identical.

(iii) A comparison of TC and ISAS graphs in the second and third rows respectively shows that the ISAS graphs generated by X12A or TSW methods for the first three series track the corresponding TC graphs; however CCG Test is not valid because none of the TC graphs for these series are smooth. Hence the quality of seasonal adjustment is suspect. In the case of the fourth series UM1, the ISAS is really the seasonally adjusted series (SAS) and SAS in this case tracks the TC graph for all three methods. The TC graph is relatively smooth for TSW but not X12A, arguing for the superiority of the quality of seasonal adjustment for TSW to X12A.

(iii) When using the SSMB method, the ISAS graphs track the TC graphs in all four series and the TC graphs are smooth.

On the basis of these observations, and focusing on the CCG test criteria in section 2, it is clear that the SSMB method generates smooth trend-cycle component estimate whereas the other methods do not, except TSW in the case of UM1 which has no interventions.. One may ask why the X12A and TSW methods do not satisfy the test criteria of CCG test and generate good quality seasonal adjustment. One possible explanation is already indicated above, that the underlying models for X12A and TSW are mis-specified at least for the first three series. Another source of mis-specification of the regARIMA models for the first three series is the following: In the first step, the regARIMA models for the first three series are used to estimate intervention component in order to obtain the prior/intervention adjusted series for X12A and TSW methods. These series are, however, seasonal and hence the regARIMA model should have regressors corresponding to the seasonal components; otherwise, it is a mis-specified model. If the analyst has correctly specified the model by including a seasonal regressor in the regARIMA model, then the estimated seasonal regressor can be used to obtain the seasonally adjusted series; hence the second step of using X12A or TSW becomes redundant.

6. Graphical Comparison of Component Estimates:

In the next set of graphs in Appendix B, each individual component of a time series estimated by the three methods is compared for each series. The reason for this comparison is to ascertain the validity of each method for the prior notions about each component an analyst may have. Also one may like to see if different methods of seasonal adjustment generate significantly different component estimates and if the comparison of these graphs can give us any clue to the superiority of any one or other method. Because of limitation of space, the component graphs for UM1 are omitted.

7. Inference from Component Graphs

In the above graphs, for each series, all estimates of each component obtained from various methods are graphed in a single graph .For each series four component graphs are printed on one page. The components from each method are distinguished by the color of the graph.

7.1 The Trend Cycle Component

The Trend-Cycle component estimates from each method are graphed in the top-left graph on each page for each series. Observing the graphs, it is clear that none of the TC graphs obtained from X12A is smooth and the same is true for TSW except for the TC graph for UM1. The SSMB graphs for TC are smooth except in the case of S11 it is less smooth. The smoothness of the TC graph or its reverse roughness (MR) is numerically estimated by the following formula suggested to me by John Greenlees (See Yao & Sloboda (2005)):

. The lower the value of MR, the smoother is the Trend-Cycle graph. This formula for MR is a special case of a general differencing function used as a constraint to obtain a smooth trend of function using Bayesian approach (See Kitagawa & Gersch (1984), Fogarty & Weber (2006)). The numerical values of MR for TC estimates for all three methods for all four series are presented in Appendix C, Table 1.

From the Table 1 Appendix C, it is clear that the MR estimates for SSMB method are lower than the corresponding estimates from TSW or X12A for every series.

7.2 The Seasonal Component

The Seasonal component estimates obtained from all three methods for each series are graphed in the top right quadrant. For PEIND, the component estimates from all three methods look very similar. The amplitude is about the same for all methods. This means the seasonally adjusted series would be very similar. For S11, the SSMB and TSW estimates indicate a moving seasonality. The movement for SSMB seasonality is, however, more pronounced than that for TSW as indicated by the higher amplitude for the SSMB seasonality. The seasonality for X12A has no single direction. For S19, the seasonal components for SSMB and X12A indicate the presence of stable seasonality but the TSW method does not estimate a seasonal component, indicating the presence of no or insignificant seasonality. The UM1 is a very seasonal series. The seasonal component estimates are different for different methods for this series.

One question which could be raised by critics is that the quality of seasonal component estimates obtained by the SSMB method for all four series may not be good because of the way the trend is estimated as a smooth curve by using the local polynomial as a trend model. However, the estimates for the M7 identifiability test for seasonality, as seen in Appendix C, Table 2, are within the acceptable range of zero and one for SSMB method for all series. For the X12A method M7 is greater than one for the S19 series. The TSW software used here does not compute M7.

The other ANOVA based tests for stable and moving seasonality also give consistent results for the SSMB method but the software for the other two methods do not give such results.

7.3 The Intervention Component

The intervention component estimates obtained from three methods are very different for each of the three series which are affected by intervention. Since there is no model underlying any intervention, it is difficult to judge the reasonableness of any estimate.

7.4 The Residual Component

The residual component estimates are the lowest, essentially zero in three out of four series for the SSMB method. Even for the fourth series PEIND, residuals throughout the sample period are smaller than those of the other two methods. This indicates that SSMB models fit the series very well. But the same is not true for X12A and TSW methods.

8. Conclusions

One advantage of this test is that one graph of both TC and ISAS components near the numerical output of the seasonal adjustment of a series would give an analyst the option of accepting the results of seasonal adjustment if CCGT is valid and rejecting the results otherwise.

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Any opinions, expressed in this paper are those of the author and do not constitute policy of the Bureau of Labor Statistics. All comments and suggestions will be highly appreciated.

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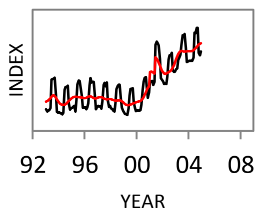
Appendix A

PEIND

ORIGINAL SERIES (OS) AND TREND CYCLE (TC)

X12-ARIMA



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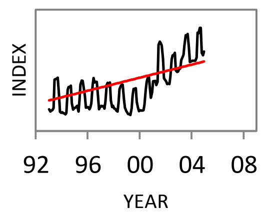
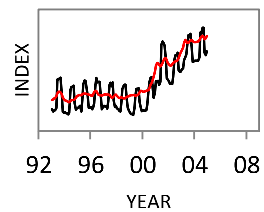
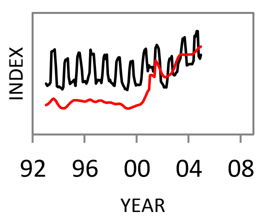


TRAMO-SEATS



INTERVENTION-ADJUSTED SERIES (IAS) AND TREND CYCLE (TC)

X12-ARIMA



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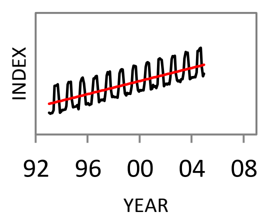

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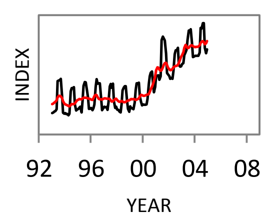
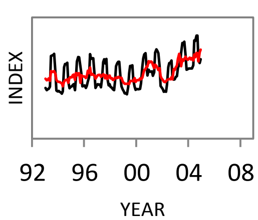


INTERVENTION-ADJUSTED SERIES (IAS), AND INTERVENTION AND SEASONALLY ADJUSTED SERIES (ISAS)

X12-ARIMA



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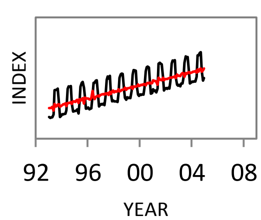

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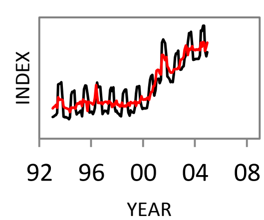

S11

ORIGINAL SERIES (OS) AND TREND CYCLE (TC)

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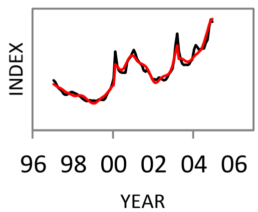

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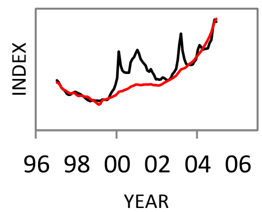

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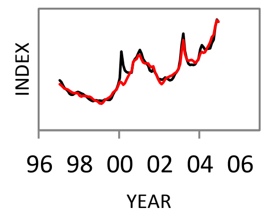
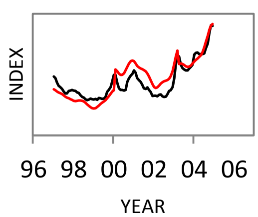


INTERVENTION-ADJUSTED SERIES (IAS) AND TREND CYCLE (TC)

X12-ARIMA



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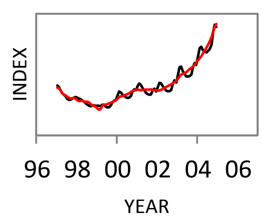

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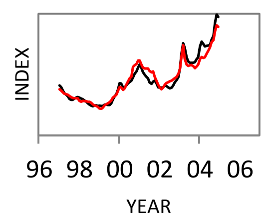

INTERVENTION-ADJUSTED SERIES (IAS), AND INTERVENTION AND SEASONALLY ADJUSTED SERIES (ISAS)

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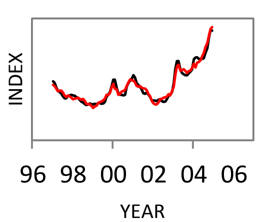

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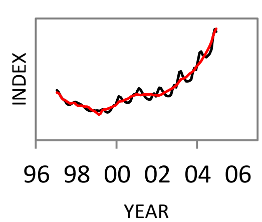

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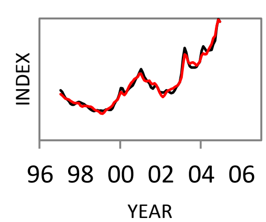
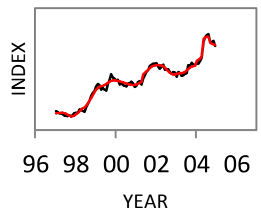


S19

ORIGINAL SERIES (OS) AND TREND CYCLE (TC)

X12-ARIMA



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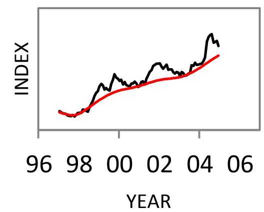
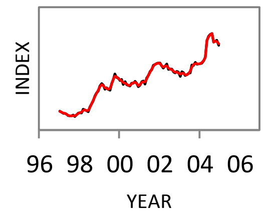
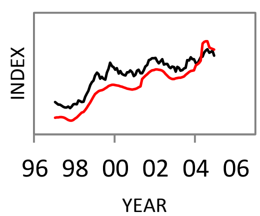


TRAMO-SEATS



INTERVENTION-ADJUSTED SERIES (IAS) AND TREND CYCLE (TC)

X12-ARIMA



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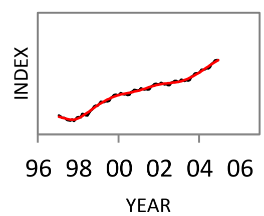

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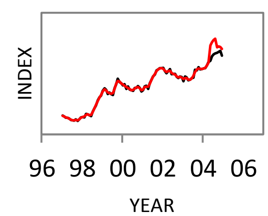

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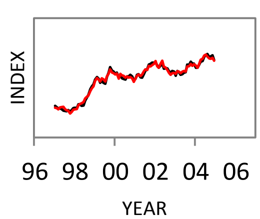

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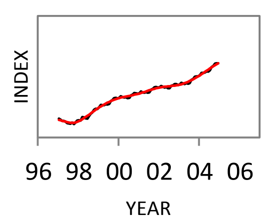
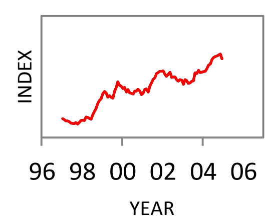


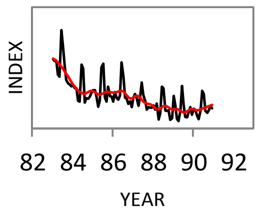
TRAMO-SEATS



UM1

ORIGINAL SERIES (OS) AND TREND CYCLE (TC)

X12-ARIMA



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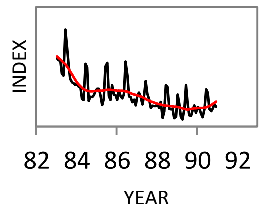

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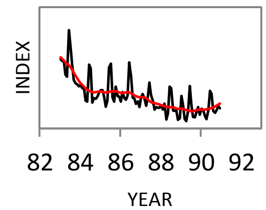

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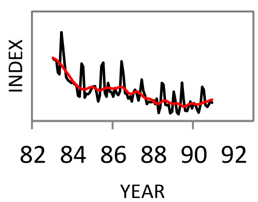

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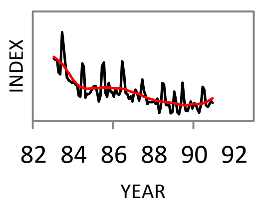
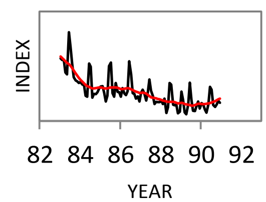
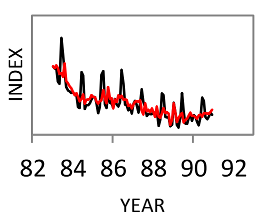


TRAMO-SEATS

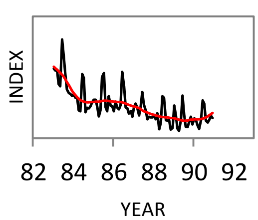


INTERVENTION-ADJUSTED SERIES (IAS), AND INTERVENTION AND SEASONALLY ADJUSTED SERIES (ISAS)

X12-ARIMA



SSMB



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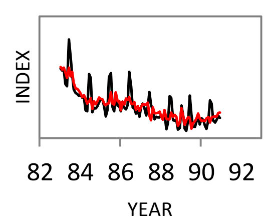
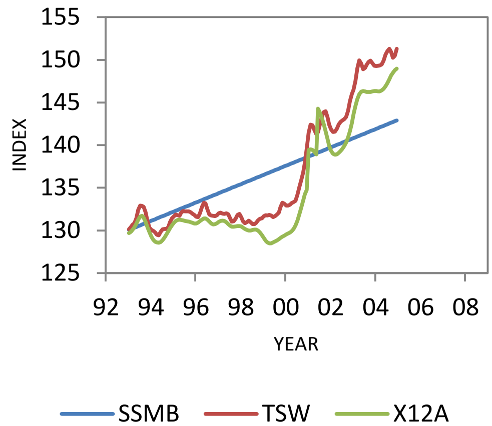


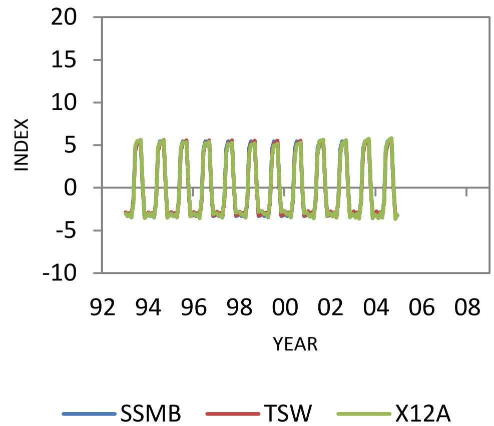
Appendix B

PEIND

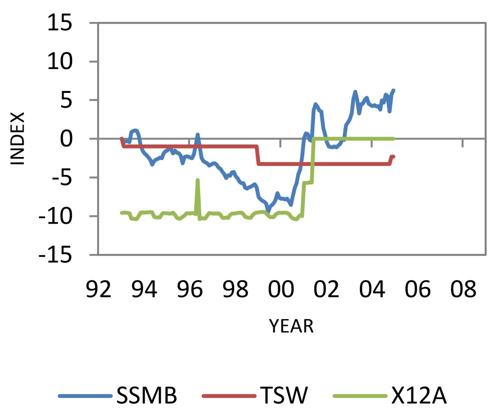
TREND COMPONENT (TC)



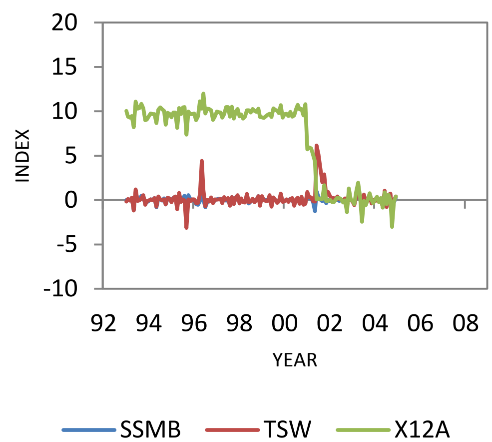
SEASONAL COMPONENT (SC)



INTERVENTION COMPONENT (IC)

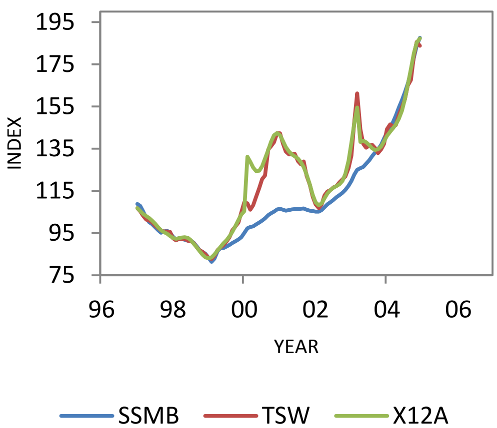


RESIDUAL COMPONENT (RC)

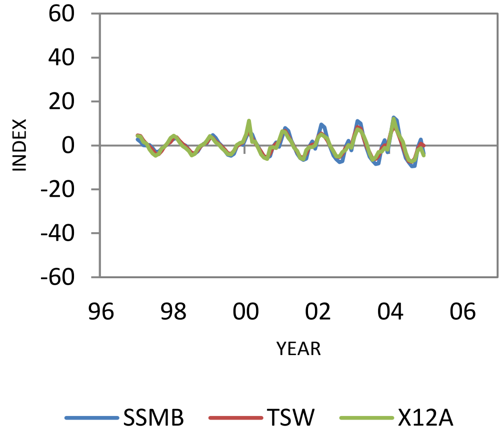


S11

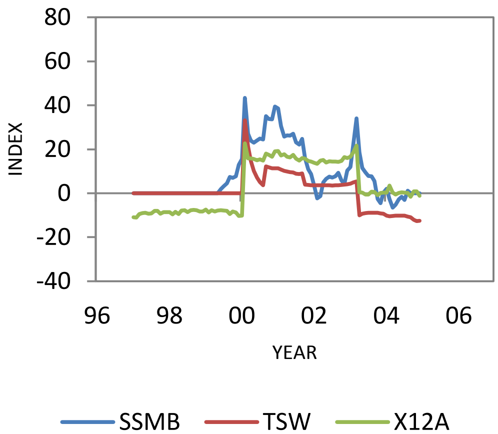
TREND COMPONENT (TC)



SEASONAL COMPONENT (SC)



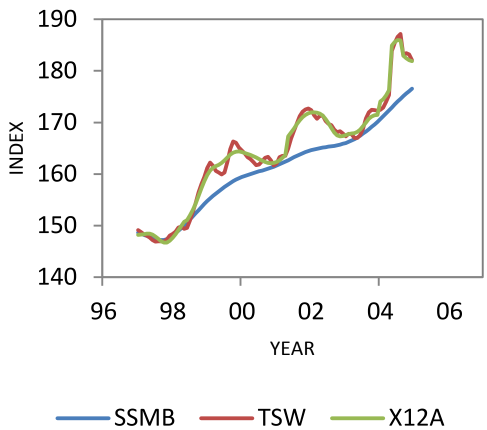
INTERVENTION COMPONENT (IC)



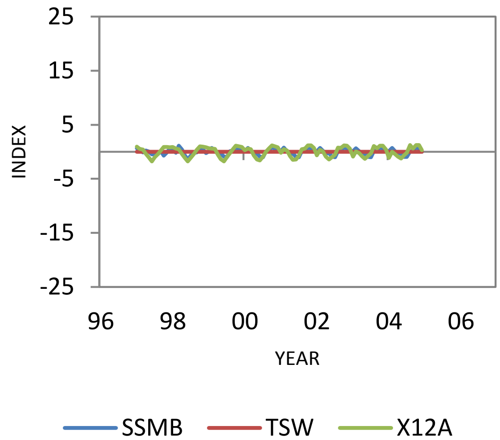
RESIDUAL COMPONENT (RC)

S19

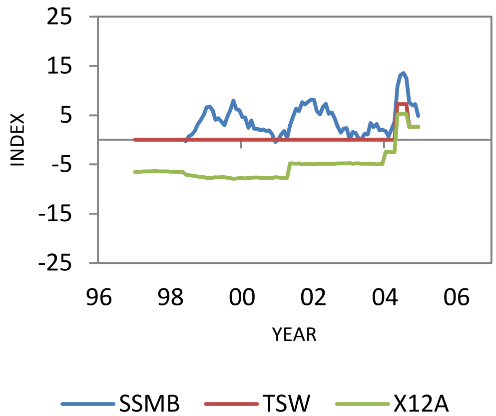
TREND COMPONENT (TC)



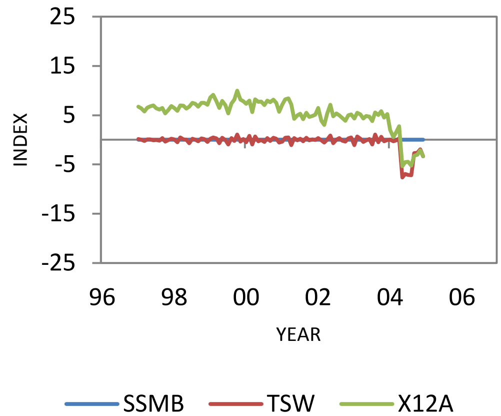
SEASONAL COMPONENT (SC)



INTERVENTION COMPONENT (IC)



RESIDUAL COMPONENT (RC)



Appendix C

Table 1: MR Statistics for Smoothness of Trend

Methods / ***Series***	SSMB	TSW	X12A
PEIND	6.5991E-08	0.030536598	0.075300818
S11	0.08531347	0.32539625	0.413981785
S19	0.00209494	0.077502144	0.074511223
UM1	0.01515822	0.035067395	0.054420244

Table 2: M7 Statistics

Methods / ***Series***	SSMB	TSW	X12A
PEIND	0.06	-	0.16
S11	0.04	-	0.46
S19	0.39	-	1.14
UM1	0.33	-	0.39