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Revisiting Taste Change in Cost-of-Living Measurement

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Abstract

This paper derives conditional cost-of-living indexes (COLI) for the Constant Elasticity of Substitution model in the presence of taste change. Recent proposals to incorporate changing tastes reflect a different conceptual target (an unconditional COLI) from a consumer price index (a conditional COLI), and a strong implicit assumption (cardinal utility). Using Nielsen retail scanner data for food and beverage products, I find that tastes can dominate prices in unconditional COLI estimates, while they have smaller impacts on conditional COLI. Using CPI data, I find that category-level tastes have a relatively minor average effect on an all-items price index.

Keywords: Cost of living index; price index; taste change

JEL Codes: C43, D12, E31

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1 Introduction

The target of a consumer price index (CPI) is typically a theoretical construct known as a cost-of-living index (COLI), which measures the proportional expenditure change required for a consumer to be indifferent between two price situations, such as periods of time (Pollak, 1989). The consumer models underlying COLI formulas are often specified with constant preferences or tastes between periods.[1] As many have observed, this is unrealistic empirically.[2] In light of this, the conceptual target of a CPI is commonly (though not universally) agreed to be a conditional COLI, which aims to hold constant non-price factors including tastes.[3] For example, the Tornqvist formula used by the Bureau of Labor Statistics (BLS) for the Chained Consumer Price Index for All Urban Consumers (C-CPI-U) approximates the COLI that conditions on the set of average tastes between those pertaining to the index's reference and comparison periods (Caves, Christensen, and Diewert, 1982). An assumption of constant tastes is not required for this index to be relevant.

This paper shows that for the Constant Elasticity of Substitution (CES) model, variants of the formula proposed by Lloyd (1975) and Moulton (1996) are exact for COLI that condition on either the reference or comparison period tastes.[4] I also find that an average of these indexes is "flexible" in the sense of Diewert (1976), and approximates a COLI that conditions on an intermediate level of tastes. Using retail scanner data for food and beverage products, I estimate that COLI conditioning on comparison period tastes exceed those conditioning on reference period tastes by an average of 0.5 to 2.9 percentage points per year, depending on the category, with COLI conditioning on intermediate tastes falling roughly in the middle. Conditional COLI that fix individual product tastes at either the reference or comparison period levels (rather than averages) are infeasible within current data constraints at the BLS. However, I find that CPI aggregates that similarly account for

[1] E.g., Lloyd (1975).

[2] E.g., Heien and Dunn (1985)

[3] See, e.g., National Research Council (2002) or ILO (2004).

[4] Per Diewert (1976), a price index formula is exact if it equals a ratio of unit expenditure functions for a given set of preferences.

category-level tastes are affected relatively little by the choice of taste vector.

This paper also aims to clarify issues raised by the recent literature concerning taste change. Several papers have advocated constructing a COLI by evaluating its constituent expenditure functions at period-specific taste vectors, a notion previously considered by Balk (1989) and others.[5] In particular, Redding and Weinstein (2020) (henceforth RW) argue ignoring taste change causes a positive "taste-shock bias" in COLI estimates. Using the CES model and household scanner data, RW estimate this bias to be around 0.4 percentage points per year on average.[6] The potential implications are significant, placing taste change among the largest sources of bias in the CPI as estimated by Moulton (2018).[7] However, by incorporating pure taste effects, proposals like RW's target a different theoretical concept–the unconditional COLI. Findings of taste-shock bias, therefore, reflect this difference in scope. Indeed, my empirical analysis suggests the estimated taste effects dominate those of prices, turning low or moderate conditional COLI increases into unconditional COLI declines. In addition, capturing taste effects requires a strong implicit assumption about the intertemporal comparability of utility. Without this assumption, it is unclear how to interpret estimates of taste-shock bias.[8]

2 Existing literature

The economic approach to consumer price indexes, dating to Konüs (1924), is based on the expenditure function of an optimizing agent. The C-CPI-U, for example, uses the Tornqvist formula for upper-level aggregation, which corresponds to the translog expenditure function.[9]

[5] Among others, Redding and Weinstein (2020), Hottman and Monarch (2018), Lecznar and Smith (2018), Ehrlich et al. (2019), Zadrozny (2019) and Ueda, K. Watanabe, and T. Watanabe (2019) explore indexes of this type.

[6] An earlier version, Redding and Weinstein (2018), finds the average bias to be on the order of two to three percentage points per year. Redding and Weinstein (2020) uses more limited baskets of items for the common varieties indexes.

[7] Sources of bias include product turnover, quality change, new outlets, etc.

[8] Concurrent research also suggests estimates of taste-shock bias are sensitive to model fit and normalization of the taste parameters (Martin, 2019; Kurtzon, 2019).

[9] The final version of the C-CPI-U uses the Tornqvist formula (Cage, Greenlees, and Jackman, 2003), once requisite expenditure data are available. The Tornqvist index is superlative (Diewert, 1976), meaning

More modern treatments of cost-of-living theory (e.g., Pollak (1989)) are agnostic as to whether or not preferences change between periods—this is accommodated by a careful definition of the COLI itself. In addition, while the well-known theorems on exact and superlative indexes (e.g., in Diewert (1976)) are proven for the case of constant preference parameters, some indexes, including the Tornqvist, are still exact for a conditional COLI when preferences change.

Taste change received less attention prior to the seminal work of F. M. Fisher and Shell (1972) (henceforth FS).[10] FS observe that unless preferences are assumed constant, a motivating question like "how much would it cost in today's prices to make the consumer *just as well off* as he was yesterday?" (my emphasis) can only be answered by "arbitrary intertemporal weighting of utilities."[11] By referencing a specific indifference surface, however, the more rigorous conditional COLI is valid whether or not preferences change. On the other hand, expenditures associated with shifting preferences can only be unified by referencing a utility level. Capturing the pure effect of tastes, therefore, requires the cardinal properties of utility functions in addition to the ordinal.[12] For this reason, conditional COLI are sometimes called "ordinal", while unconditional COLI are sometimes called "cardinal."[13]

A conditional COLI precludes any analysis of pure taste change effects by fixing all welfare-determining factors save prices. In contrast, an unconditional COLI aims to track changes in expenditure whether driven by prices or by other factors like the environment. As a consequence, the index may increase or decrease even if prices are constant. Interested in the unconditional COLI concept, Balk (1989) proposes an index that attempts to hold constant some notion of well-being without fixing the cardinal utility level. The method

it is exact for an expenditure function (the translog) that is "flexible" in the sense of providing a second order approximation to an arbitrary expenditure function.

[10]Samuelson and Swamy (1974), Muellbauer (1975), Heien and Dunn (1985), among others, apply and extend the FS results.

[11]"What is meant by 'just as well off as he was yesterday' if the indifference map has shifted?" (FS, pp. 2)

[12]The assumption that utility is cardinal is generally considered outdated outside of specific applications (e.g., lotteries). See, for example, Hicks and Allen (1934).

[13]Muellbauer (1975) and Phlips and Sanz-Ferrer (1975) make the ordinal-cardinal distinction.

tracks the change in expenditure required to reach an indifference surface that passes through a fixed bundle. Gábor-Tóth and Vermeulen (2018) apply this method to European scanner data and find the average annual contribution of taste change to be −1.1 percentage points.[14] Unconditional COLI seek to answer interesting questions, and may be more comprehensive as *cost-of-living* concepts.[15] However, they do not contain any additional information on *prices* than what is already conveyed by a conditional COLI. Therefore, the conditional COLI is generally considered the more appropriate target for a consumer price index (National Research Council, 2002; ILO, 2004).

In order to isolate the issue of preference change, I focus my empirical analysis on matched-model indexes, i.e., those defined over a fixed set of specific product varieties with constant tangible attributes. RW's taste-shock bias is defined in association with a matched-model index. Preference change is different, in principle, from product turnover, which may be due to sample rotations or product entry and exit. Appendix B shows how with product turnover, it is possible to bound conditional COLI in the CES case. Mechanically, the unconditional COLI treats a change in preferences equivalently to a change in quality, which is to say, as if each item in the basket were substituted for a similar item with different attributes (F. M. Fisher and Shell, 1972). This has lead to the confusing analogy of taste-shock bias to quality bias stemming from comparing prices of items with different characteristics (Redding and Weinstein, 2020). By definition, price change for the matched model is measurable without quality adjustment, since the set of items and their associated bundles of attributes are constant.[16] Of course, in light of product turnover, we may wish to improve upon the matched model index. Product turnover may cause selection bias in matched model indexes (Pakes, 2003), or cause them to miss initial price declines for new items (Feenstra, 1994).

[14]A potential issue with this concept is that the choice between using the reference or comparison period bundles predetermines the direction of the pure taste change effect. See Corollary 5 of Balk (1989) and Section 2.2 of Gábor-Tóth and Vermeulen (2018).

[15]National Research Council (2002) gives several situations where a conditional COLI may be inadequate, including medical products whose quality is difficult to separate from general health status, and regional comparisons where fixing weather conditions may make little sense.

[16]If item definitions are not constant, then whether or not demand shifts are attributed to quality changes or taste changes can have large effects on index estimation (Nevo, 2003).

However, preference change is a separate phenomenon.

As noted by FS, deriving the relationship between tastes and either type of COLI is difficult without either assuming a specific parameterization of tastes (usually on a small subset of items only), or restricting attention to a particular utility function (as this paper does). Tastes do not pose much of a measurement challenge when the objective is a conditional COLI, however, even when the index formula does not appear to explicitly account for tastes. Caves, Christensen, and Diewert (1982), Diewert (2001), and Feenstra and Reinsdorf (2007) provide conditions under which the Tornqvist, Fisher, and Sato-Vartia price indexes, respectively, are exact for or approximate COLI that condition on some notion of average tastes. Section 3 discusses these results further, while Section 4 complements them by showing that variants of the Lloyd-Moulton index are also exact for conditional COLI in the CES case.

3 Cost-of-living theory

A cost-of-living index is a ratio of two expenditure functions. It is helpful in this case to first specify a set of preferences rather than jump straight to a utility function. Consider an ordinal preference relation, denoted $\succeq$, on a commodity space $\mathcal{Q} \subseteq \mathbb{R}^N$, which is made up of bundles $\boldsymbol{q}$.[17] We assume:

Assumption 3.1 *The representative consumer's preference relation $\succeq$ is i) rational (complete and transitive), ii) continuous, iii) convex, and iv) monotone.*

Assumption 3.1 is sufficient for the existence of a utility function, $u : \mathcal{Q} \to \mathbb{R}$ which represents $\succeq$, in the sense that we have $\boldsymbol{q} \succeq \boldsymbol{q}' \Leftrightarrow u(\boldsymbol{q}; \succeq) \geq u(\boldsymbol{q}'; \succeq)$ (Mas-Colell, Whinston, Green, et al., 1995). Due to the ordinal nature of preferences, the function u, is not unique. Any positive monotone transformation of u will also represent $\succeq$. Let $\boldsymbol{p}$ denote a vector of prices. We then assume:

[17]E.g., for bundles $\boldsymbol{q}$, $\boldsymbol{q}'$ we read $\boldsymbol{q} \succeq \boldsymbol{q}'$ as "$\boldsymbol{q}$ is at least as preferred as $\boldsymbol{q}'$." Preferences are ordinal because they convey no information on the magnitude by which $\boldsymbol{q}$ is preferred to $\boldsymbol{q}'$.

Assumption 3.2 *Facing prices $\boldsymbol{p}$, the agent chooses $\boldsymbol{q}$ to maximize utility subject to a budget constraint, or equivalently, to minimize expenditure subject to a utility constraint.*

Let $\boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq) = \underset{\boldsymbol{q}}{\text{argmin}}\ \boldsymbol{p} \cdot \boldsymbol{q}$ s.t. $u(\boldsymbol{q}; \succeq) \geq \bar{u}$ denote the Hicksian demand function, which represents the quantities that minimize expenditure. The expenditure function is given as $C(\boldsymbol{p}, \bar{u}; \succeq) = \boldsymbol{p} \cdot \boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq)$.

3.1 Conditional COLI

A conditional or ordinal COLI is defined as the minimum expenditure required for an agent to be indifferent between two price situations. I label the reference situation 0 and the comparison situation 1. This paper focuses on intertemporal comparisons, but the general theory accommodates other possibilities (e.g., regional comparisons).

Definition 3.1 *(F. M. Fisher and Shell, 1972; Pollak, 1989) The class of conditional cost-of-living indexes is given by:*

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq)}{C(\boldsymbol{p}_0, \bar{u}; \succeq)}, \tag{1}$$

for a given $\bar{u}$ and $\succeq$.

The combination of $\succeq$ (preference relation) and $\bar{u}$ (location) determine the specific indifference surface on which Φ is based. Two immediate candidates for preferences to plug in are $\succeq_0$ and $\succeq_1$, corresponding to the reference and comparison periods, respectively. It is worth emphasizing that the oft-cited bounding results for the Laspeyres and Paasche indexes are one-way only, i.e., the Laspeyres is an upper bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_0; \succeq_0)$, and the Paasche is a lower bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_1; \succeq_1)$. In general, there is no talking about "the" COLI, except in the case of constant, homothetic preferences (Samuelson and Swamy, 1974). FS argue that from the standpoint of intertemporal compensation, the most interesting COLI is

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_1^*; \boldsymbol{\varphi}_1) = \frac{C(\boldsymbol{p}_1, \bar{u}_1^*; \boldsymbol{\varphi}_1)}{C(\boldsymbol{p}_0, \bar{u}_1^*; \boldsymbol{\varphi}_1)}, \tag{2}$$

where $\bar{u}_1^*$ is the hypothetical utility that the consumer would receive facing the period 0 budget constraint with period 1 preferences. FS and others argue that a COLI based on $\succeq_1$ is more relevant for public policy than one based on the obsolete preferences $\succeq_0$, but Pollak (1989) notes that in principle, $\succeq$ need not be linked to either the reference or comparison situations. Indeed, two of the parameter-free price indexes discussed in the next subsection are exact for COLI based on average indifference surfaces.

3.2 Parameter-free COLI

Under Assumption 3.2, the observed market expenditures $\boldsymbol{p}_0 \cdot \boldsymbol{q}_0$ and $\boldsymbol{p}_1 \cdot \boldsymbol{q}_1$ equal the expenditure levels $C(\boldsymbol{p}_0, \bar{u}_0, \succeq_0)$ and $C(\boldsymbol{p}_1, \bar{u}_1, \succeq_1)$, respectively. Since, Eq. 1 holds the indifference surface fixed, however, estimation generally requires knowledge of the expenditure function for the given set of preferences.

Nevertheless, some traditional price index formulas are exact for conditional COLI, precluding any need for structural estimation. I define these indexes and their components below. Let i index items or varieties, and denote the set of items as $\mathcal{I}$, which has dimension N.

Definition 3.2 *The Fisher price index*

$$P_F(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \sqrt{P_L P_P}, \tag{3}$$

where $P_L(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i \in \mathcal{I}} p_{i1} q_{i0}}{\sum_{i \in \mathcal{I}} p_{i0} q_{i0}}$ *is the Laspeyres index, and* $P_P(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i \in \mathcal{I}} p_{i1} q_{i1}}{\sum_{i \in \mathcal{I}} p_{i0} q_{i1}}$ *is the Paasche index.*

Definition 3.3 *The Tornqvist price index*

$$P_T(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i \in \mathcal{I}} \left(\frac{p_{i1}}{p_{i0}} \right)^{0.5(s_{i0} + s_{i1})}, \tag{4}$$

where $s_{it} = \frac{p_{it} q_{it}}{\sum_{j \in \mathcal{I}} p_{jt} q_{jt}}, t = 0, 1.$

Definition 3.4 *The Sato-Vartia price index*

$$P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i \in \mathcal{I}} \left(\frac{p_{i1}}{p_{i0}} \right)^{w_i}, \tag{5}$$

where $w_i = \left[\frac{s_{i1} - s_{i0}}{\ln s_{i1} - \ln s_{i0}} \right] / \left[\sum_{k \in \mathcal{I}} \frac{s_{k1} - s_{k0}}{\ln s_{k1} - \ln s_{k0}} \right]$.

Suppose tastes are represented by the vector $\boldsymbol{\varphi}$, as will be the case in the following section on CES preferences.[18] Diewert (2001) showed that there exists a u^* and $\boldsymbol{\varphi}^*$ such that $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, u^*; \boldsymbol{\varphi}^*)$ is bounded by the Laspeyres and Paasche indexes, where $\bar{u}_0 \leq u^* \leq \bar{u}_1$ and $\varphi_{i0} \leq \varphi_i^* \leq \varphi_{i1}$, $i = 1, \ldots, N$. If the Laspeyres and Paasche are close numerically, a symmetric average like the Fisher index approximates this COLI. In addition, under the assumption that the expenditure function is translog, Caves, Christensen, and Diewert (1982) showed that the Tornqvist price index is exact for the geometric average of the COLI based on period 0 preferences and the COLI based on period 1 preferences. Due to translog functional form, this is equivalent to the COLI evaluated at the geometric averages of the taste parameters and utilities, respectively.[19] The Tornqvist index is also attractive because the translog expenditure function approximates arbitrary expenditure functions to the second order. Finally, the Sato-Vartia index is exact for the CES COLI that conditions on intermediate levels of the tastes (Feenstra and Reinsdorf, 2007). Each of these results relates to an indifference surface that is, loosely speaking, an average of the base and current period indifference surfaces. Of course, the measurement of substitution effects (responses to relative price change), may change depending on which indifference surface the COLI is based, and so interpretations should be made carefully.

[18] Diewert (2001) and others refer to $\boldsymbol{\varphi}$ as "environmental variables" more generally, but make clear that tastes are included.

[19] The second-order parameters are assumed constant. A model with constant second-order parameters can be shown to match the parameterization used in RW.

3.3 Unconditional COLI

An unconditional or cardinal COLI measures the change in expenditure required for the consumer to achieve the same utility level in the comparison period as they experienced in the reference period.

Definition 3.5 *(Muellbauer, 1975) The class of cardinal or unconditional COLI is given by:*

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)}, \tag{6}$$

for some $\bar{u}$. RW and others estimate this ratio when $C(\boldsymbol{p}, \bar{u}; \succeq)$ is the CES unit expenditure function, but they derive it for other models as well. Unless preferences are constant, the associated quantities $\boldsymbol{h}(\boldsymbol{p}_0, \bar{u}; \succeq_0)$ and $\boldsymbol{h}(\boldsymbol{p}_1, \bar{u}; \succeq_1)$ do not lie on the same indifference surface, even though both are labeled $\bar{u}$. Therefore, the expenditure comparison is only meaningful if the utility levels can be compared. This amounts to starting from the following instead of Assumption 3.1.

Assumption 3.3 *The utility function $u(\boldsymbol{q}; \succeq)$ is a cardinal measure of the representative consumer's well-being.*

The following decomposition of an unconditional COLI is illustrative of the difference in intended scope.

$$\ln \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \ln \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_1) + \ln \left[\frac{C(\boldsymbol{p}_0, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)}\right] \tag{7}$$

Eq. 7 decomposes the unconditional COLI into two parts; a price effect equal to a conditional COLI, and a pure taste change effect $C(\boldsymbol{p}_0, \bar{u}; \succeq_1)/C(\boldsymbol{p}_0, \bar{u}; \succeq_0)$. It is straightforward to compare the unconditional COLI with other conditional COLI in a similar fashion. Equation 7 describes the sense in which Φ_U is "unconditional" in that the last term aims to capture the impact of factors other than prices (National Research Council, 2002). It is also apparent that the contribution of price change is completely captured by the ordinal index, and that the pure taste change component is what depends on cardinal utility.

4 CES Preferences

Section 3 described a few conditional COLI that can be estimated with prices and quantities only. In general, however, estimating a COLI requires specifying and estimating a model of preferences. For comparability to other studies, I focus on the CES model for the rest of this paper. The CES model is a workhorse for its tractability, though it implies significant restrictions on price and income elasticities. Appendix D derives some results for the homothetic translog expenditure function, which is more flexible, but requires estimating many more parameters. Specification error is a potential issue for an unconditional COLI, as well as COLI that condition on a specific period's tastes, as these depend on the model's ability to separate price responses from preference shifts (Martin, 2019).

We now assume:

Assumption 4.1 *The representative agent's expenditure function has the form:*

$$C(\boldsymbol{p}, \bar{u}; \boldsymbol{\varphi}) = \bar{u} \left[\sum_{i \in \mathcal{I}} \left(\frac{p_i}{\varphi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

For the purposes of a COLI, we take $\bar{u} = 1$ without further loss of generality (preferences are homothetic) and suppress the argument from further notation. The parameter $\sigma \neq 1$ is the elasticity of substitution, which we assume is constant over time, and so the notation now refers to preferences through the vector of demand shifters $\boldsymbol{\varphi}$.[20] The agent's optimal expenditure shares are given by

$$s_i(\boldsymbol{p}; \boldsymbol{\varphi}) = \frac{p_i h_i(\boldsymbol{p}; \boldsymbol{\varphi})}{\sum_{j \in \mathcal{I}} p_j h_j(\boldsymbol{p}; \boldsymbol{\varphi})} = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j \in \mathcal{I}} (p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p}; \boldsymbol{\varphi})]^{1-\sigma}}, \; i = 1, \ldots, N. \tag{9}$$

[20] Allowing σ to also vary over time would be more general, but is often infeasible empirically.

Under Assumptions 3.2 and 4.1, the observed expenditure shares $s_{it} = \frac{p_{it}q_{it}}{\sum_{j \in \mathcal{I}} p_{jt}q_{jt}}$ equal the optimal expenditure shares $s_i(\boldsymbol{p}_t; \boldsymbol{\varphi}_t)$. The indexes in the following subsections make use of this equation to estimate conditional and unconditional COLI.

Eq. 10 shows that under Assumption 4.1, the log expenditure share of item i in period t can be decomposed into its log price, the log expenditure function, and the log of the taste parameters.

$$\ln s_{it} = (1 - \sigma) \ln p_{it} + (\sigma - 1) \ln [c(\boldsymbol{p}_t; \boldsymbol{\varphi}_t)] + (\sigma - 1) \ln \varphi_{it} \tag{10}$$

As RW note, the taste parameters provide a source of idiosyncratic error which is necessary for empirical analysis.

4.1 Exact Price Indexes for CES Preferences

As previously mentioned, the index proposed by Sato (1976) and Vartia (1976) (see Definition 3.4) is exact for the CES COLI that conditions on an intermediate taste vector $\bar{\boldsymbol{\varphi}}$ (Feenstra and Reinsdorf, 2007). The salient question then is how do price comparisons using $\bar{\boldsymbol{\varphi}}$ compare to price comparisons using $\boldsymbol{\varphi}_0$, $\boldsymbol{\varphi}_1$ or some other tastes?

Exact price indexes for reference period or current period tastes already exist for the CES model, though to my knowledge, their interpretation as such is novel. Lloyd (1975) and Moulton (1996) developed the following price index in the setting of constant tastes.[21]

Definition 4.1 *Lloyd-Moulton Index*

$$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{11}$$

Similarly, the time-antithesis (I. Fisher, 1922), or "backwards" version of the Lloyd-Moulton index can be formed.

[21] The advantage of the Lloyd-Moulton index to statistical agencies is that it does not require current-period expenditure shares. Since 2015, the BLS has used a modified version of this index for initial estimates of the C-CPI-U (Klick, 2018), the modification being that shares correspond to an earlier biennial period, though they are updated to reflect prices in a pivot month preceding month 0.

Definition 4.2 *Backwards Lloyd-Moulton Index*

$$P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma} \right\}^{\frac{-1}{1-\sigma}} \tag{12}$$

The Lloyd-Moulton and Backwards Lloyd-Moulton are exact for the COLI that condition on reference period tastes and comparison period tastes, respectively. To see this, start with Eq. 4.1 for $P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$. Use the right hand side of Eq. 9 to substitute for s_{i0}, re-arrange, and use Eq. 8. We then have

$$\begin{aligned} P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) &= \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\ &= \left\{ \sum_{i \in \mathcal{I}} \frac{(p_{i0}/\varphi_{i0})^{1-\sigma}}{[C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)]^{1-\sigma}} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\ &= \frac{\left\{ \sum_{i \in \mathcal{I}} (p_{i1}/\varphi_{i0})^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\ &= \frac{C(\boldsymbol{p}_1, \boldsymbol{\varphi}_0)}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\ &= \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0) \end{aligned}$$

The case of $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$ is very similar. The following summarizes the result.

Proposition 1 *Under Assumption 4.1,*
$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$, *and* $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$.

This result implies an additional superlative index may be of interest in the context of changing tastes. Consider the geometric mean of the Lloyd-Moulton indexes, denoted P_{LMM}.

It has the form:

$$P_{LMM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = [P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)]^{\frac{1}{2}}$$
$$= \left[\frac{\sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma}}{\sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma}} \right]^{\frac{1}{2(1-\sigma)}} \tag{13}$$

Eq. 13 shows P_{LMM} is, in fact, the Quadratic Mean of Order r price index, where $r = 2(1-\sigma)$ Diewert (1976). This implies the following result.

Proposition 2 *Under assumption 4.1, the Quadratic Mean of Order r price index is exact for the geometric mean of two CES conditional COLI, $[\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)]^{\frac{1}{2}}$, where $r = 2(1 - \sigma)$.*

This is important because superlative indexes like this one have been shown to approximate each other to the second order (Diewert, 1978). This implies P_{LMM} should be somewhat robust to errors in estimation of σ or departures from CES functional form.[22] Additionally, the availability of both P_{LMM} and P_{SV} for the CES case offers an interesting potential contrast. One averages COLI evaluated at different tastes, while the other is a COLI evaluated at an average of the tastes. A priori, we would not necessarily expect them to give identical answers, though their estimates in Section 5 are very similar.

4.2 RW's CES Common Varieties Index

RW propose a price index to target the CES unconditional COLI. A practical challenge concerns the scale of tastes. Given knowledge of σ, Eq. 9 implies that observed expenditure shares and prices identify φ_{it} up to a time-varying scale factor.[23] The CES conditional COLI are invariant to the scale of the tastes, but the unconditional is not. RW address this issue by normalizing the φ_{it} to have a constant geometric mean over time, in effect allowing only

[22] This is provided that $|\sigma|$ is not too large (Hill, 2006).
[23] For issues surrounding identification of σ, see Feenstra (1994), Hausman (1996), or Soderbery (2010).

relative taste changes. Although CES expenditure shares do not depend on the scale of tastes, expenditure levels do, and so this normalization is not free.[24] Changes in the scale of tastes between periods affect the magnitude and possibly the direction of the unconditional COLI, an observation made by Kurtzon (2019).

Using the normalization, RW derive the following estimator for $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$:

Definition 4.3 *RW's CES Common Varieties Index (CCV)*[25]

$$P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \exp\left[\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{p_{i1}}{p_{i0}}\right) + \frac{1}{\sigma-1}\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{s_{i1}}{s_{i0}}\right)\right], \qquad (14)$$

The time-constant scale factor precludes some potentially interesting situations. First, it prohibits systematic increases or decreases in the agent's "efficiency" as a producer of utility (Muellbauer, 1975). This rules out, among other phenomena, the "hedonic treadmill" hypothesis discussed in National Research Council (2002), whereby the agent needs to consume higher quantities over time to remain as well-off. Such general trends in well-being would clearly affect $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ conditional on a set of relative taste changes. Second, the normalization of the unweighted geometric mean is just one of an infinite number of equivalent normalizations, on which the observed expenditure and price data bear no informational content (Kurtzon, 2019). Appendix C finds restricting the set $\mathcal{I}$ affects P_{CCV} through implicitly changing the normalization.

Table 1 summarizes the price index formulas discussed in this and the previous section. The following two sections compare them empirically, first using retail scanner data, and then using CPI elementary item-area indexes.

[24]The function $C(\boldsymbol{p}; \boldsymbol{\varphi})$ is homogeneous of degree -1 in $\boldsymbol{\varphi}$, which varies between numerator and denominator of Φ_U.

[25]RW's proposed CES Unified Price Index consists of the CCV plus a product turnover adjustment in the style of Feenstra (1994).

5 Application to Retail Scanner Data

5.1 Data and model estimation

I estimate quarterly price indexes for food and beverage product categories using the CES model introduced in Section 4 and Scantrack, a point-of-sale scanner dataset from The Nielsen Company.[26] The data are similar in scope to Nielsen's household panel, which RW use. The Neilsen retail scanner data has been proposed for use in the CPI by Ehrlich et al. (2019), and similar data is used by the Australian Bureau of Statistics to estimate some food components of its CPI. The data cover the fourth quarter of 2005 through the second quarter of 2010, and include expenditures and quantities for roughly 600,000 universal product codes (UPC) sold by participating grocery, drug, and mass merchandise store chains.[27] Because items are defined by UPC, their characteristics and quality are arguably constant over time (Broda and Weinstein, 2010; Redding and Weinstein, 2020). UPCs are classified according to a structure defined by Nielsen. For instance, UPC 003800040500 is described as "Kellogg's Eggo Round Chocolate Chip 10 count." It belongs the brand module "Kellogg's Eggo," product module "Frozen Waffles/Pancakes/French Toast," product group "Breakfast Foods - Frozen," and department "Frozen Foods." Like RW, I calculate quarterly expenditure shares (within product group) and unit value prices by UPC, treating the continental United States as one market.[28]

Table 2 describes some basic attributes of the dataset. Just over 54% of food and beverage

[26]I use data for food and beverage products only, though Scantrack data also covers general merchandise, personal care, and other non-food grocery items sold in grocery and drug stores. Scantrack expenditures on nonfood goods equal only about 19% and 12% of comparable Consumer Expenditure Survey and Personal Consumption Expenditure estimates, respectively, suggesting the majority of consumption on these products originates from non-covered retailers (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019). Furthermore, the degree to which the simple CES model is a suitable approximation for the data may differ between food and nonfood categories. The model assumes no dynamic behavior, i.e., stockpiling or durable goods, and expenditure on a nonfood product (e.g., "Kitchen gadgets") may be a relatively poor proxy for consumption of that product, even at a quarterly frequency.

[27]According to a Nielsen representative, the sample covers 90% of such retail chains and is weighted to be nationally representative. Potential selection bias is a limitation of this and other studies using convenience samples of transactions.

[28]As in RW, I winsorize by dropping items whose change in price or value were in the top or bottom one percentile for a given quarter.

expenditures are from the Dry Grocery department, comprising about two-thirds of the total number of UPCs. Dairy (15%) and Frozen Foods (11%) are the next largest departments by expenditure. Use of these data for consumer price indexes treats retail sales as proxies for consumer expenditures, but they also include purchases by non-households. Total food and beverage expenditures in Scantrack exceed the BLS's Consumer Expenditure Survey (CE) estimates over the same time period by about 66%, while they exceed the Bureau of Economic Analysis's Personal Consumption Expenditure (PCE) estimates by about 9% (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019).[29]

For each product group, I calculate a series of indexes of the form $P(\boldsymbol{p}_{t-4}, \boldsymbol{p}_t, \boldsymbol{q}_{t-4}, \boldsymbol{q}_t)$, where $P()$ is one of the formulas given in Section 3 or 4. The index for quarter t uses the same quarter in the year prior as its base period, so index values reflect year-over-year price changes. Table 3 presents summary statistics for the four-quarter price relatives $p_{it}/p_{i,t-4}$ pooled over the sample period. Average price relatives exceed one for all departments, ranging from 1.021 for Alcoholic Beverages to 1.037 for Dairy. The distributions are quite dispersed however, with standard deviations within departments ranging from 0.106 for Packaged Meat to 0.165 for Fresh Produce. Relatives are positively skewed in all departments, as one might expect if prices follow an upward trend over time. Compared to a normal distribution (which has kurtosis equal to 3), the distributions of price relatives have higher kurtosis, which indicates thicker tails.

Estimation of the substitution elasticities follows the "double-differencing" method of Feenstra (1994), using panel variation in prices and expenditure shares. This method assumes $\sigma > 1$, which is reasonable for indexes over similar product varieties. I mainly follow the weighting and estimation procedure of Broda and Weinstein (2010).[30] Start with Eq. 10 and difference over time and with respect to a reference variety k, which is chosen to be

[29] CE and PCE cover slightly different target populations and rely on different survey methods. See Passero, Garner, and McCully (2014) for a discussion.

[30] Like RW, my procedure differs from Broda and Weinstein (2010) in that I do not distinguish between within-brand and across-brand substitutions. I am grateful to the authors for providing their Stata code on the *American Economic Review* website.

the variety with the largest average market share. Denote the doubled-difference of variable x_{it} as $\Delta^k x_{it} = (x_{it} - x_{i,t-1}) - (x_{kt} - x_{k,t-1})$. For varieties $i = 1, \ldots, N$ and time periods $t = 1, \ldots, T$, the double-differenced demand equation is:

$$\Delta^k \ln s_{it} = -(\sigma - 1)\Delta^k \ln p_{it} + \Delta^k \ln u_{it}, \tag{15}$$

where $\Delta^k \ln u_{it}$ is the double-differenced error. The inverse supply equation is derived by assuming each variety is produced by a distinct firm in monopolistic competition, leading to a pricing equation that is linear in log-expenditure share, with slope depending on the inverse supply elasticity parameter ω. The double-differenced inverse supply equation is then:

$$\Delta^k \ln p_{it} = \frac{\omega}{1+\omega}\Delta^k \ln s_{it} + \Delta^k \ln v_{it}, \tag{16}$$

where $\Delta^k \ln v_{it}$ is the double-differenced supply error. The primary identifying assumption is that for each i, the double-differenced supply and demand errors are uncorrelated.[31] The parameters can then be estimated using Generalized Method of Moments (Hansen, 1982) based on the moment conditions

$$E\left[\left(\Delta^k \ln p_{it}\right)^2 - \theta_1 \left(\Delta^k \ln s_{it}\right)^2 - \theta_2 \Delta^k \ln p_{it} \Delta^k \ln s_{it}\right] = 0, i = 1, \ldots, N, \tag{17}$$

where $\theta_1 = \frac{\omega}{(1+\omega)(\sigma-1)}$ and $\theta_2 = \frac{\omega(\sigma-2)-1}{(1+\omega)(\sigma-1)}$. As in Feenstra (1994), Eq. 17 can be written as a regression of time averaged variables and estimated using weighted nonlinear least squares.[32] When analytical estimates are outside of theoretical bounds (e.g., $\sigma < 1$), parameters are estimated by a grid search over the parameter space.[33]

[31] We also assume that the demand and supply errors are drawn from stationary distributions with variances that differ by product variety (Feenstra, 1994).

[32] From Broda and Weinstein (2006), I include the time average of $\left(q_{it}^{-1} + q_{i,t-1}^{-1}\right)$ as additional regressor to control for measurement error introduced by aggregating transaction prices into quarterly unit values. This means a product group must have at least four varieties for estimation.

[33] Following the code used for Broda and Weinstein (2010), the grid search, e.g., searches for the value of $\sigma \in [1.04, 50.5]$, at 4% increments, that minimizes the sample objective function.

Two product groups in the Dairy department have too few varieties for estimation and are dropped from the analysis. Of the remaining, the procedure yields 55 analytical and 15 grid-searched estimates, the summary of which is presented in Table 4. The overall median elasticity is 4.32, which is lower than what RW found using the Feenstra method and Homescan data (6.48), but reasonable given data and time period differences.[34] There is heterogeneity in estimates by product group, as the interquartile range is nearly three.

5.2 Results

As described in the previous subsection, I calculate CES price indexes for 70 food and beverage product groups in the Scantrack dataset. For ease of presentation, figures and tables show statistics that are weighted by expenditure share. Figure 1 shows four-quarter percent changes averaged across all product groups, while Figure 2 and Table 5 break these out by department. Table 6 gives the percentiles of the distributions of differences between price indexes, while Table 7 presents average differences by department.

As discussed in Sections 3 and 4, these indexes can be derived from the same CES model with time-varying taste parameters. Differences between them reflect differing theoretical objectives as opposed to biases stemming from improper modeling assumptions. Comparisons among P_{LM}, P_{BLM}, P_{LMM}, and P_{SV} shed light on the degree to which the estimate of pure price change is affected by the choice of conditioning taste vector. The difference between P_{CCV} and conditional COLI like P_{SV} is an estimate of the partial effect of tastes on the unconditional COLI (like in Eq. 7), provided one accepts the comparison of cardinal utility levels.[35]

The Scantrack data show that the contributions of tastes dominate those of prices in

[34] Previously, Kurtzon (2016) found estimated elasticities from Scantrack to be lower than those estimated using Homescan. The Feenstra method is based on large-T asymptotic arguments, so estimates may have finite-sample bias from the relatively sample period. See Soderbery (2010) and Soderbery (2015). As discussed in Section 4, P_{LMM} is robust to small changes in σ used, and comparisons among P_{CCV}, P_{LM}, and P_{BLM} are qualitatively similar when using alternative values of σ (e.g. setting all equal to 6.48).

[35] I omit this qualifier for the rest of this section, thought it and the normalization issue discussed in Appendix C invite caution in interpreting any results involving P_{CCV}.

determining P_{CCV}, driving it to be negative in most quarters. From Figure 1, the conditional COLI estimates tend to be positive, with average four-quarter percent changes ranging from 1.31 for P_{LM} to 3.61 for P_{BLM}. In contrast, P_{CCV} implies annual unconditional cost-of-living declines between 0.29 and 6.65 percentage points from late 2006 to mid 2010, implying a strong negative contribution from taste changes. Similar to RW, I find P_{CCV} tends to imply substantially lower inflation than P_{SV}, though I find a larger average difference of 5.64 percentage points in magnitude. In more than 75% of observations, the difference is greater than 2 percentage points.[36] Figure 1 shows P_{CCV} also tends to be much lower than the other conditional COLI estimates, though Figure 2 and Table 7 suggest considerable heterogeneity by department. The average $P_{SV} - P_{CCV}$ spread ranges from 1.51 percentage points for Dairy products to 11.59 percentage points for Frozen Foods. For the latter category, P_{CCV} estimates an average annual unconditional COLI decline of 9.65%, while the conditional COLI estimates range from 1.4% to 2.4% on average. With the exception of Dairy, the gap between the conditional COLI estimates and P_{CCV} persists over time.

Turning to the conditional COLI, the Scantrack estimates indicate that the choice of taste vector can impact the measurement of price change, but to a smaller degree than tastes affect the unconditional COLI. From Table 6, P_{BLM} exceeds P_{LM} by 2.3 percentage points on average and by 1.21 percentage points at the median. Looking across departments (Table 7), the average difference ranges from 0.54 for Packaged Meat, to 2.92 for Dry Grocery. As Figures 1 and 2 illustrate, the average $P_{BLM} - P_{LM}$ gap tends to be smaller than the gap between P_{CCV} and any one of the other indexes. Conditioning on an intermediate level of tastes, P_{SV} in most cases is very close to P_{LMM}, the geometric average of reference-taste and comparison-taste indexes. Overall, they differ by 0.24 percentage points on average, and

[36] My results are most comparable to Redding and Weinstein (2018), which uses the same definition of common varieties and finds average differences between P_{SV} and P_{CCV} on the order of 2-4 percentage points. Redding and Weinstein (2020) finds the same sign for $P_{CGG} - P_{SV}$, but an average magnitude of only 0.4 percentage points. However, the basket of varieties is limited to items that have lifespans of at least six years and are not within three quarters of their introduction or exit in the reference or comparison quarter. I also find smaller $P_{CGG} - P_{SV}$ differences when I similarly limit the basket. Appendix C shows also that restricting the set of varieties has a large effect on the result by implicitly changing the normalization imposed on the CES taste parameters.

0.03 percentage points at median, with average differences by department spanning -0.01 (Fresh Meat) to 0.425 (Dry Grocery) percentage points. Conditioning on current preferences (as FS prefer), P_{BLM} implies that consumers, on average, needed to increase expenditure by 3.61% to be indifferent between current and year-ago food and beverage prices over the late 2000s. This is 0.93 percentage points greater than if measured using intermediate tastes (P_{SV}).[37]

As discussed in Section 3, it is known in the literature that neither functional form assumptions nor structural parameter estimates are required to estimate conditional COLI based on intermediate tastes. In fact, the literature finds that even the Sato-Vartia formula, while not technically superlative, approximates superlatives like the Fisher and Tornqvist.[38] Appendix A shows this holds for the Scantrack data as well. In contrast, functional form does matter when estimating either the impact of tastes on a conditional COLI (e.g., $P_{BLM} - P_{LM}$) or the pure taste effect on the unconditional COLI $P_{CCV} - P_{SV}$. The reason is P_{CCV}, P_{LM}, and P_{BLM} essentially recover the taste parameters as residuals in the CES expenditure share equation. Quantification of taste impacts may therefore be sensitive to model fit. In fact, Martin (2019) uses a simulation study that suggests a neglected nesting structure causes negative biases in P_{CCV} and P_{LM}, positive bias in P_{BLM}, and negligible bias in P_{SV}, leading to positive bias in the taste change indicators $P_{CCV} - P_{SV}$ and $P_{BLM} - P_{LM}$.[39]

6 Application to CPI aggregation

The previous section, while making use of detailed transactions data, applies only to food and beverage products consumed at home, which constitute less than 10% of CPI-eligible expenditures (Bureau of Labor Statistics, 2020). To the extent possible, I now use CPI data

[37]Across the non-food products, where the data and model are perhaps less representative, differences among the CES indexes tend to be quite large. They imply department average $P_{SV} - P_{CCV}$ spreads of up to 18 percentage points and $P_{BLM} - P_{LM}$ spreads up to 43 percentage points. Results are available from the author by request.

[38]See, e.g., Diewert (1978).

[39]Neglected nesting structure means the true model is nested CES, while the researcher estimates the simple CES model.

to estimate what role changing tastes play in the calculation of price indexes over a broad consumption basket. Subject to the limitation described below, I find that year-over-year differences in CES indexes tend to be smaller and less persistent than those found in Section 5.

The basic unit of this analysis is the monthly elementary item-area index (e.g., Mens Suits in Pittsburgh), which is considerably more aggregated than the UPC-level data employed in Section 5. Such indexes are the inputs to both the CPI-U (which uses the Lowe formula for aggregation) and the C-CPI-U (which uses the Tornqvist). Currently, the BLS calculates elementary indexes for 243 item categories in 32 areas, for a total of 7,776 item-area indexes, though these dimensions have changed over time. Similar to Section 5, I consider an annual frequency of taste change by estimating a series of direct indexes where the base period for each is the same month during the prior year.[40] For comparison with BLS methodology, I also include the Tornqvist index, P_T.

A limitation of this analysis is that the elementary indexes are fixed. BLS lacks the frequency and detail of expenditure or quantity data to calculate indexes like P_{LM} and P_{BLM} at the elementary level.[41] Therefore, this exercise is only informative about category-level tastes (e.g., all ground beef) as opposed to variety-level tastes (e.g., 85% ground beef). Furthermore, the Lloyd-Moulton indexes require the elasticity of substitution σ. Estimation in the style of Feenstra (1994) requires $\sigma > 1$, which is not realistic for this application because it implies all varieties (or aggregates) are substitutes. Additionally, previous analysis of CPI elementary indexes, such as Klick (2018), have estimated elasticities less than one. RW also assume $\sigma > 1$, and for this reason, I do not present any estimates of P_{CCV} using CPI indexes. Given the importance of σ in the CES model's ability to separate price-related

[40] In contrast, the published C-CPI-U is a series of one-month indexes multiplied together. When calculating monthly chained versions of the CES indexes, I find monthly percent change differences to be quite small, but levels can drift apart somewhat over time for some values of σ. Results are available from the author upon request.

[41] Weights for elementary indexes are available at a lag of up to four years. As a result, most use either a weighted geometric mean or a modified Laspeyres formula instead of the Tornqvist (Bureau of Labor Statistics, 2018).

substitutions from taste-related substitutions, I present conditional COLI estimates for four different elasticities (0.6, 0.7, 0.8, and 0.9), and leave further inquiry into the correct choice of σ to future research.[42]

6.1 Results

Table 8 presents average 12-month percent changes of P_T, P_{SV}, P_{LM}, P_{BLM}, and P_{LMM} over the period from December 2000 to December 2017. Table 9 gives the average differences between pairs of indexes.[43] Figures 3 to 5 plot these indexes separately for the different values of σ chosen. For readability, the graphs have been split into three time periods.

While the official C-CPI-U is a series of chained month-over-month indexes, these results suggest that an alternative accounting for preferences would have a relatively modest average effect on year-over-year measurements. The Tornqvist, SV and LMM indexes tend to be very close on average, differing by less 0.03 percentage points in magnitude, again reflecting how functional form is less important when conditioning on an intermediate taste level. In fact, average differences among all of the indexes tend to be less than one tenth of one percentage point. The only exception is when $\sigma = 0.9$, P_{BLM} exceeds P_{LM} by 0.16 percentage points on average. Taking current preferences (i.e., P_{BLM}) as the most relevant reference point, then P_T overstates this conditional COLI by 0.057 percentage points (3.1%) under the assumption that $\sigma = 0.6$, while it understates it by 0.079 percentage points (4.0%) under the assumption that $\sigma = 0.09$. Overall, compared to individual variety tastes, the effect of category-level tastes appears relatively small.

Figures 3 to 5 reveal that despite the indexes tending to give similar answers on average, short term divergences can occur. From November 2008 to September 2009, for example,

[42] The initial and interim C-CPI-U use $\sigma = 0.6$, based on pooled, biennial regressions of logged, differenced shares on logged, differenced elementary indexes, in the style of Feenstra and Reinsdorf (2007). Using monthly shares and elementary indexes, I estimated $\sigma = 0.84$. Details are available upon request.

[43] Item structure changes in 2008, 2010, and 2013 reduce the number of overlapping item-areas for those years by 0.47%, 2.84%, and 14.81%, respectively. The averages in Table 9 are qualitatively the same when excluding these years. Results available from the author upon request. Indexes ending in 2018 were not calculated due to implementation of a new CPI area sample.

with σ set to 0.9, P_{BLM} exceeds P_{LM} by an average of 0.8 percentage points each month, with individual month differences ranging from 0.34 to 1.22 percentage points. In other periods, however, differences are quite small. For instance, the average difference from January to November 2007 is only 0.01 percentage point, with individual month's differences ranging from -0.10 to 0.09 percentage points. This is different from Section 5, where differences between P_{BLM} and P_{LM} indexes using Scantrack are found to be persistent over time.

As noted before, each formula uses the same elementary indexes, and so the most can be said is that tastes for broader item categories have relatively little impact on the the conditional COLI. Section 5 suggests a larger role of tastes at the individual item level, however.

7 Conclusion

The criticism that traditional price indexes assume constant preferences is not quite correct. It is true that the Fisher, Tornqvist, Sato-Vartia, and Quadratic Mean of Order r indexes are exact in models with constant tastes, but even when tastes change, these still estimate interesting conditional COLI. Furthermore, the pure taste change effects RW and others aim to capture are arguably out of scope for a consumer price index, and are measurable only under a very strong assumption. This paper's empirical analysis suggests the relative contribution of prices to such a cardinal index can be swamped by taste change effects, as RW's CCV index tends to imply cost-of-living deflation even as traditional price indexes show low-to-moderate inflation. If there is interest in a COLI that conditions on a specific period's taste vector, then this paper provides two novel possibilities for the CES case. Current data constraints for the CPI imply the BLS could only account for category-level tastes, which appear to have a relatively small impact on year-over-year inflation. Improvements to the simple CES model are likely possible, and so future research should include more general demand models to more precisely separate taste changes from price-related substitutions.

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Tables

Table 1: Summary of Price Index Properties

Index	Formula	Model	Tastes	Struct. parameters?
Fisher	$\left(\frac{\sum_i p_{i1}q_{i0}}{\sum_i p_{i0}q_{i0}}\frac{\sum_i p_{i1}q_{i1}}{\sum_i p_{i0}q_{i1}}\right)^{\frac{1}{2}}$	Un-specif.	Fixed, intermed.	No
Tornqvist	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{.5(s_{i0}+s_{i1})}$	Translog	Fixed, geomean	No
Sato-Vartia	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{w_i}$	CES	Fixed, intermed.	No
CCV	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{\frac{1}{N}} \times \prod_i \left(\frac{s_{i1}}{s_{i0}}\right)^{\frac{1}{N(\sigma-1)}}$	CES	Vary, normalized	Yes
Lloyd-Moulton	$\left\{\sum_i s_{i0}\left(\frac{p_{i1}}{p_{i0}}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$	CES	Fixed, reference	Yes
Backwards Lloyd-Moulton	$\left\{\sum_i s_{i1}\left(\frac{p_{i0}}{p_{i1}}\right)^{1-\sigma}\right\}^{\frac{-1}{1-\sigma}}$	CES	Fixed, comparison	Yes

Table 2: Scantrack Food and Beverage Departments

Department	# PG	# UPC	Exp. Share
Alcoholic Beverages	4	46,656	0.073
Dairy	12	46,686	0.153
Deli	1	22,061	0.022
Dry Grocery	40	412,319	0.541
Fresh Meat	1	1,934	0.006
Fresh Produce	1	20,244	0.052
Frozen Foods	12	64,635	0.115
Packaged Meat	1	18,401	0.039
All	*72*	*632,936*	*1.000*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 3: Summary Statistics for $p_{it}/p_{i,t-4}$ by Department

	Obs	Mean	StDev	Skew	Kurt	Min	Max
Alcoholic Beverages	343,007	1.021	0.118	0.415	7.163	0.270	2.098
Dairy	383,519	1.037	0.137	0.908	6.577	0.469	2.256
Deli	128,081	1.025	0.117	0.322	6.578	0.500	1.635
Dry Grocery	2,941,985	1.036	0.141	0.777	9.977	0.210	2.782
Fresh Meat	12,162	1.028	0.117	0.767	6.633	0.557	1.759
Fresh Produce	113,895	1.033	0.165	0.844	6.490	0.458	1.992
Frozen Foods	454,187	1.027	0.125	0.253	6.360	0.281	1.807
Packaged Meat	148,512	1.025	0.106	0.516	5.623	0.625	1.618
All	*4,525,348*	*1.033*	*0.136*	*0.74*	*9.254*	*0.21*	*2.782*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 4: Summary of Elasticity of Substitution Estimates by Department

	# Prod. Gr.	P25	Med	P75
Alcoholic Beverages	4	5.96	7.06	8.63
Dairy	10	3.31	3.65	4.05
Deli	1	3.96	3.96	3.96
Dry Grocery	40	3.85	4.68	6.50
Fresh Meat	1	3.37	3.37	3.37
Fresh Produce	1	2.94	2.94	2.94
Frozen Foods	12	3.31	3.94	6.33
Packaged Meat	1	3.12	3.12	3.12
All	*70*	*3.39*	*4.32*	*6.29*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 5: Summary of CES Indexes by Department (percent change)

	CCV	SV	LMM	LM0	LM1
Alcoholic Beverages	0.0845	1.8585	1.8283	1.3839	2.2751
Dairy	1.3019	2.8165	2.7756	1.5423	4.0415
Deli	−2.4059	1.0556	1.0079	0.3502	1.6703
Dry Grocery	−3.4241	3.2381	2.8131	1.3882	4.3103
Fresh Meat	−2.0699	2.2678	2.2774	1.6551	2.9040
Fresh Produce	−2.0258	1.1468	1.1583	0.1446	2.1833
Frozen Foods	−9.6467	1.9408	1.8948	1.4150	2.3794
Packaged Meat	−1.0369	1.1688	1.1596	0.8894	1.4307
All	*−2.9549*	*2.6810*	*2.4373*	*1.3093*	*3.6107*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics weighted by product group expenditure.

Table 6: Summary of CES Index Differences (percentage points)

	SV − CCV	SV − LMM	SV − BLM	BLM − LM
P5	−1.3107	−0.1472	−3.2547	0.1450
P10	0.2764	−0.0748	−2.0360	0.3242
P25	2.1239	−0.0104	−1.1504	0.6187
Median	4.9995	0.0297	−0.5556	1.2058
P75	8.5198	0.1095	−0.2796	2.4390
P90	11.9109	0.3279	−0.1494	4.6537
P95	15.3193	1.0874	−0.0862	8.6224
Mean	5.6359	0.2437	−0.9297	2.3014

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics weighted by product group expenditure.

Table 7: Mean CES Index Differences by Department (percentage points)

	SV − CCV	SV − LMM	SV − BLM	BLM − LM
Alcoholic Beverages	1.7740	0.0302	−0.4167	0.8912
Dairy	1.5146	0.0409	−1.2250	2.4992
Deli	3.4615	0.0477	−0.6147	1.3201
Dry Grocery	6.6622	0.4250	−1.0722	2.9221
Fresh Meat	4.3377	−0.0096	−0.6363	1.2490
Fresh Produce	3.1727	−0.0114	−1.0365	2.0388
Frozen Foods	11.5876	0.0460	−0.4386	0.9644
Packaged Meat	2.2057	0.0092	−0.2619	0.5413
All	*5.6359*	*0.2437*	*−0.9297*	*2.3014*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics weighted by product group expenditure.

Table 8: CPI: Mean 12-mo. Indexes, 2000m12-2017m12 (perc. points)

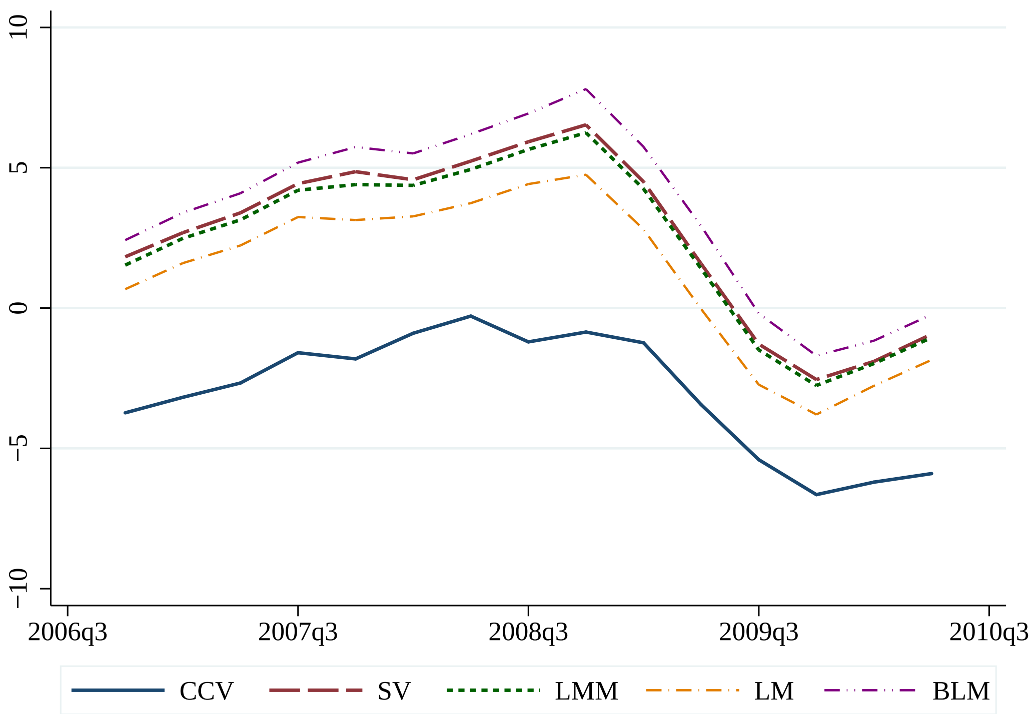
σ	Torn	SV	LMM	LM0	LM1
0.6	1.8834	1.8999	1.8676	1.9090	1.8263
0.7	1.8834	1.8999	1.8773	1.8733	1.8813
0.8	1.8834	1.8999	1.8815	1.8388	1.9243
0.9	1.8834	1.8999	1.8832	1.8045	1.9620

Table 9: CPI: Mean Differences in 12-mo. Indexes, 2000m12-2017m12 (perc. points)

σ	Torn. − SV	SV − LMM	SV − LM	SV − BLM	BLM − LM
0.6	−0.0165	0.0323	−0.0090	0.0736	−0.0827
0.7	−0.0165	0.0227	0.0266	0.0186	0.0080
0.8	−0.0165	0.0184	0.0611	−0.0244	0.0856
0.9	−0.0165	0.0168	0.0954	−0.0621	0.1575

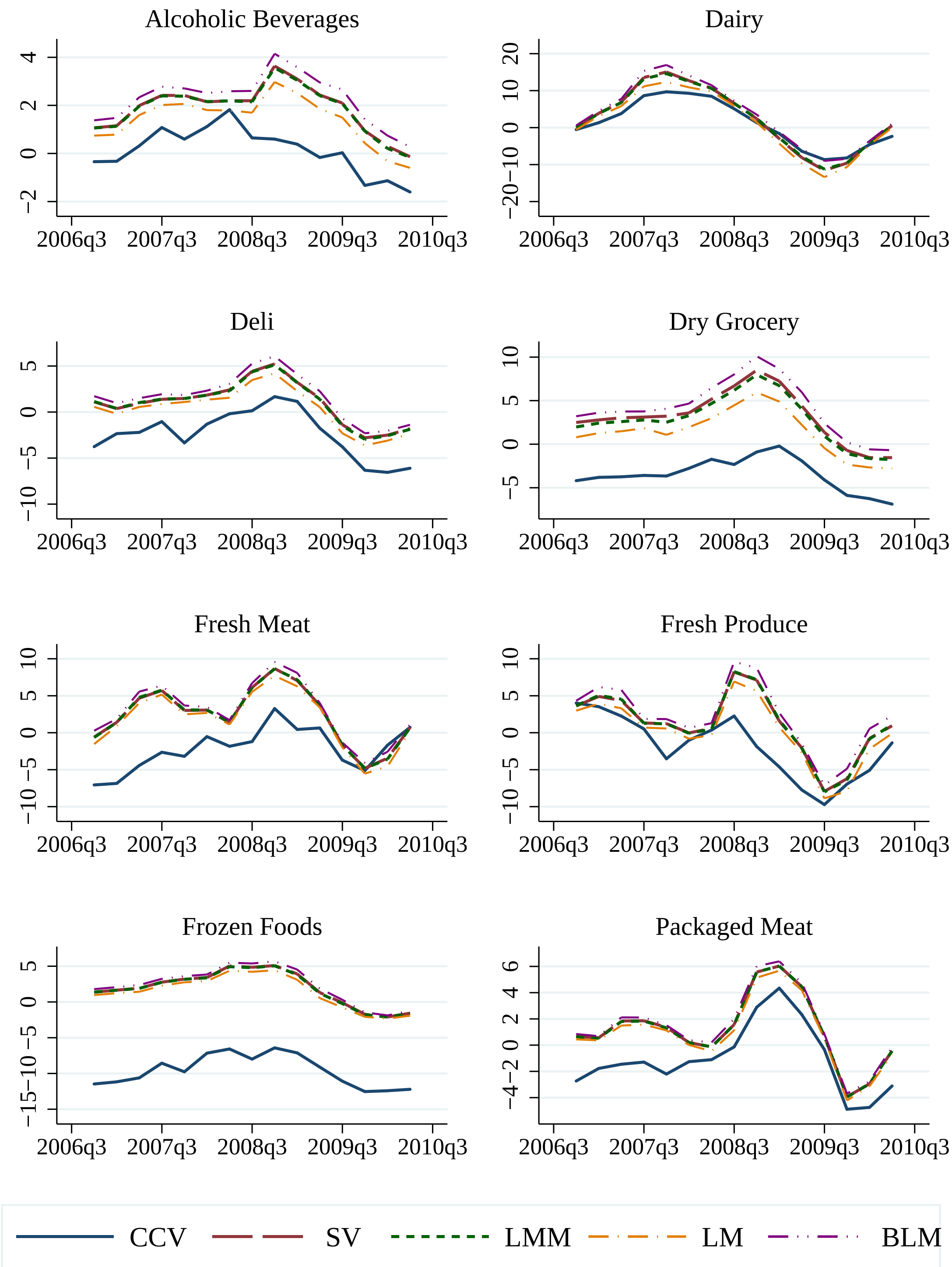
Figures

Figure 1: Scantrak CES Price Index Averages (% change versus year ago)



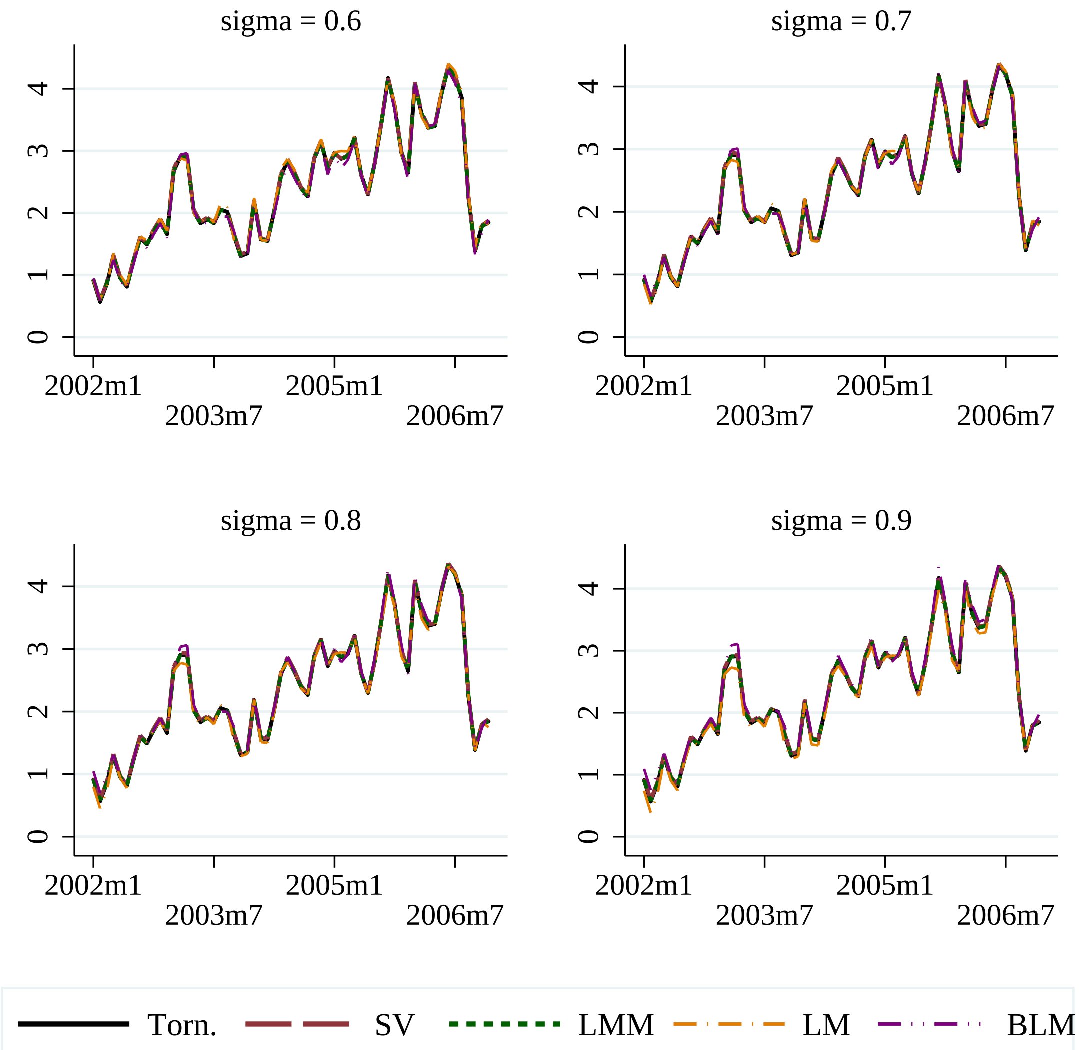
Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are expenditure-weighted averages of the four-quarter proportional changes implied by group-level indexes for food and beverage products. All but the SV indexes require estimated elasticities of substitution.

Figure 2: Scantrak CES Price Index Averages By Dept. (% change versus year ago)



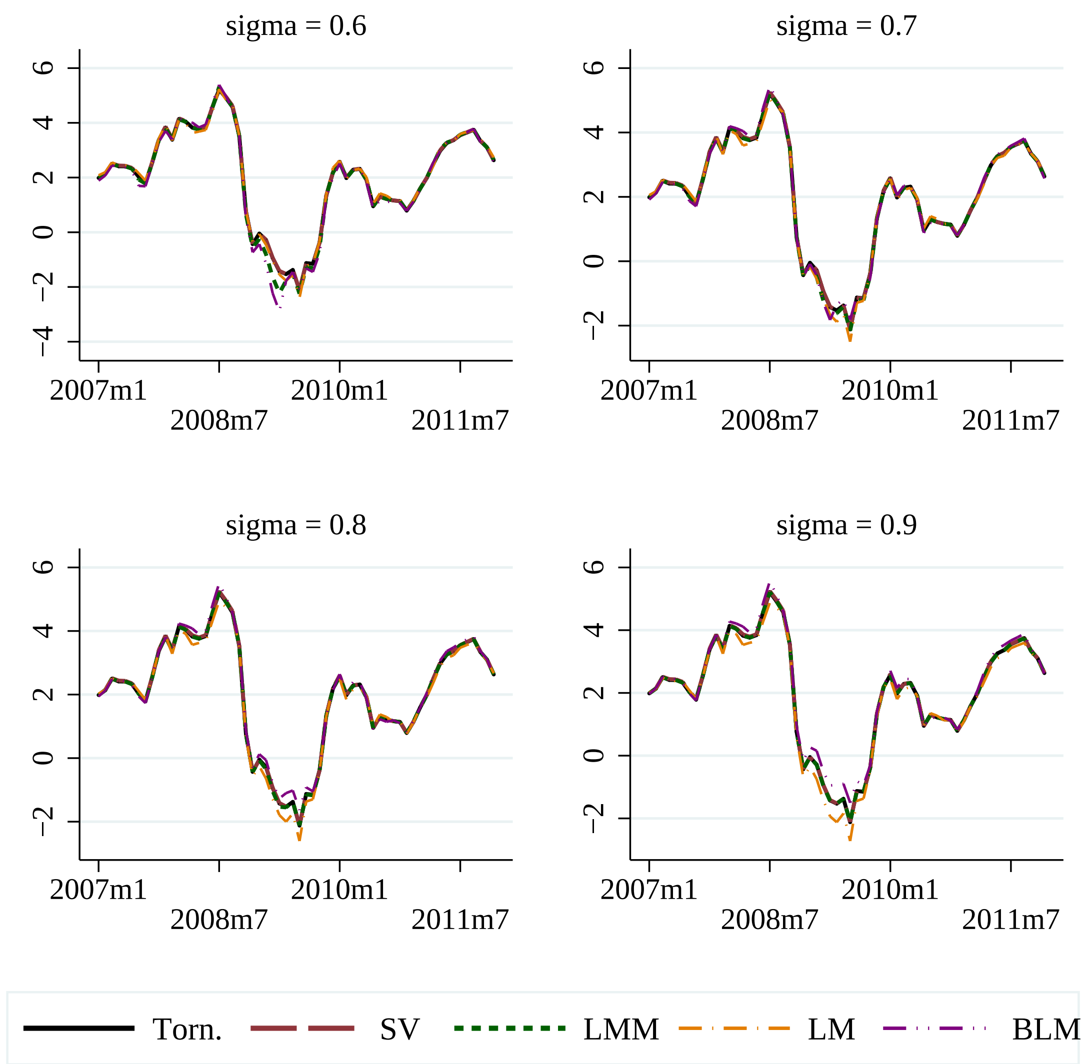
Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are expenditure-weighted averages of the four-quarter proportional changes implied by group-level indexes for food and beverage products. All but the SV indexes require estimated elasticities of substitution.

Figure 3: Comparison of 12-mo. CPI Aggregates, 2002m1-2006m12



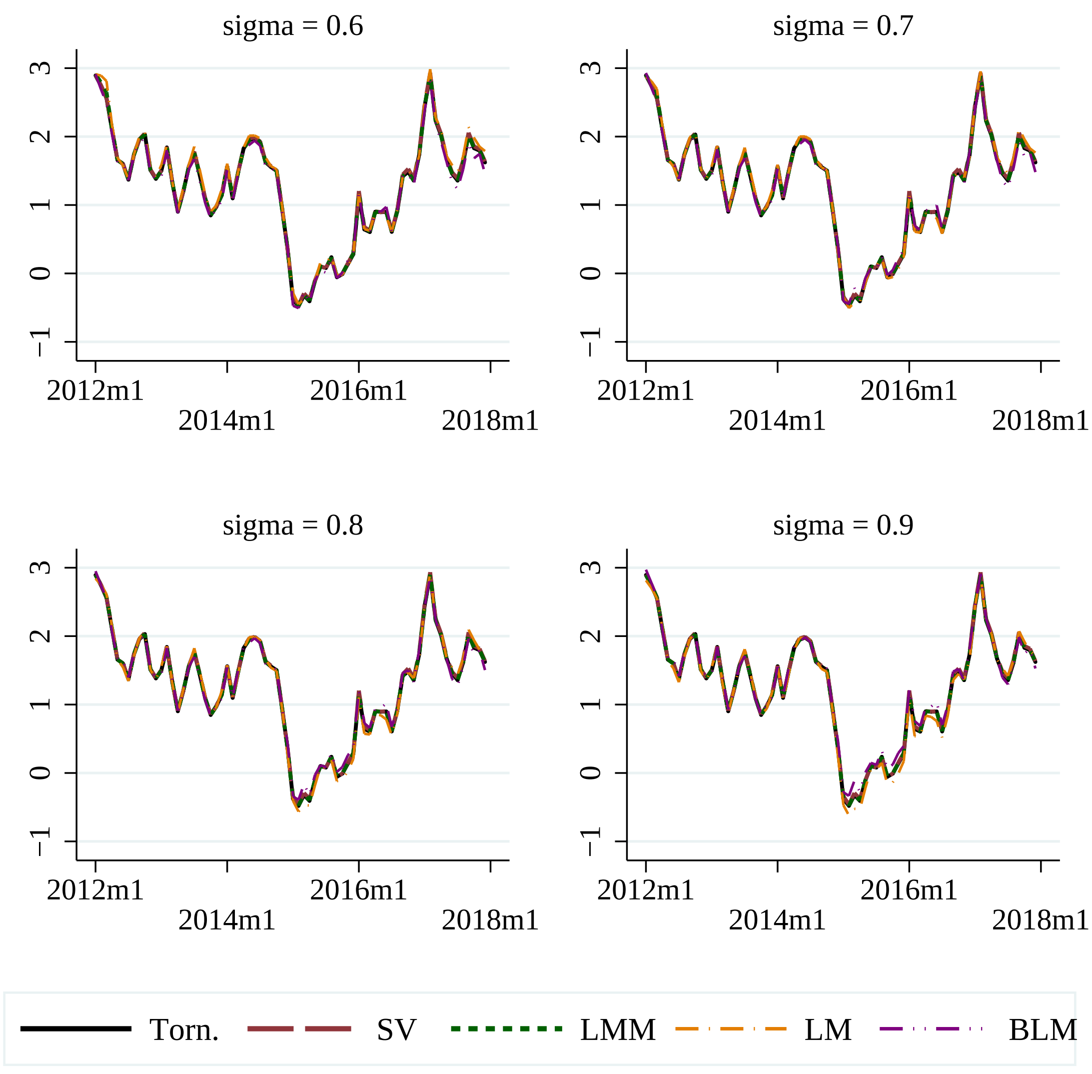
Note: Plots are twelve-month percent changes. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure 4: Comparison of 12-mo. CPI Aggregates, 2007m1-2011m12



Note: Plots are twelve-month percent changes. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure 5: Comparison of 12-mo. CPI Aggregates, 2012m1-2017m12



Note: Plots are twelve-month percent changes. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

A Traditional price indexes with retail scanner data

As in Section 5, I calculate four-quarter price indexes for each food and beverage product group in the Scantrack data using the Fisher, Tornqvist, Sato-Vartia, and Laspeyres formulas. All but the Laspeyres are variable-basket indexes, meaning they reflect consumer substitutions over time. While the Sato-Vartia reflects substitutions according to the CES expenditure function, the Fisher and Tornqvist are superlative, meaning they approximate an arbitrary homothetic expenditure function (Diewert, 1976). Figure A1 plots kernel density estimates of the differences between each index and the Fisher, while Table A1 lists mean differences and mean absolute differences.

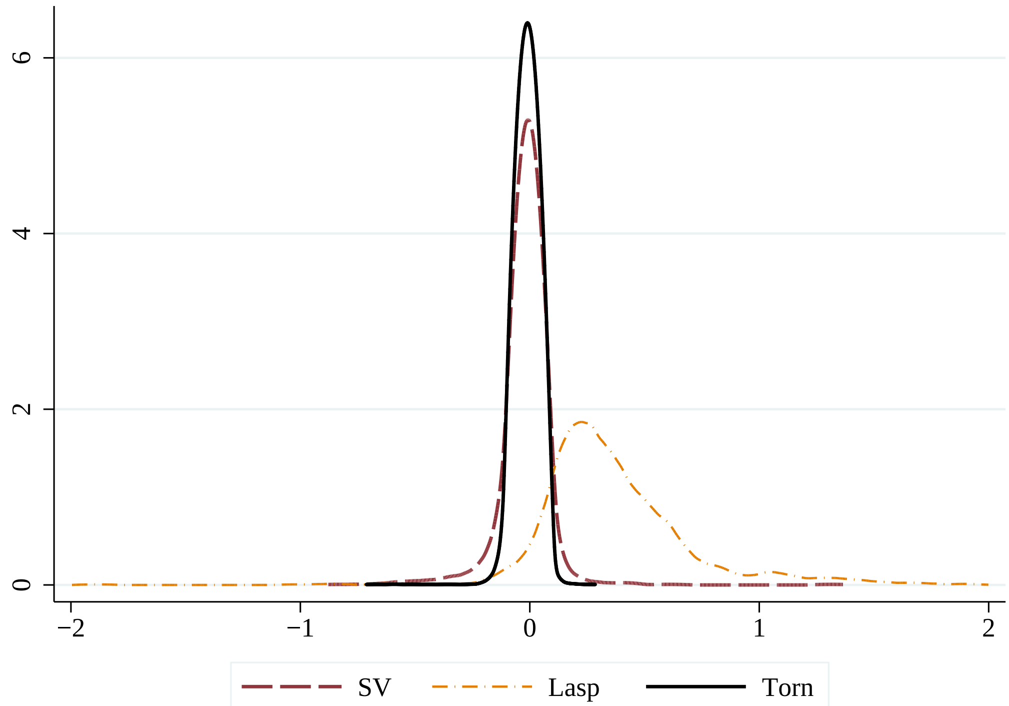
As many have found previously, there tends to be substantial agreement between the three traditional variable-basket indexes, with mean differences and mean absolute differences all less than one tenth of one percentage point in magnitude. In contrast, the Laspeyres indexes exceeds the Fisher index by about 0.4 percentage points, on average, which is on the order of estimates from Boskin, et. al. (1996) and others for lower-level substitution bias.

Table A1: Differences from a Fisher Index by Department (percentage points)

	Mean Difference			Mean Absolute Difference		
	Torn.	SV	Lasp.	Torn.	SV	Lasp.
Alcoholic Beverages	0.0005	0.0013	0.2093	0.0041	0.0101	0.2102
Dairy	−0.0019	−0.0021	0.2903	0.0071	0.0251	0.2916
Deli	−0.0029	0.0093	0.3257	0.0046	0.0252	0.3257
Dry Grocery	−0.0170	−0.0454	0.4043	0.0213	0.0683	0.4168
Fresh Meat	0.0031	0.0027	0.6404	0.0154	0.0276	0.6404
Fresh Produce	0.0097	0.0169	0.6203	0.0182	0.0428	0.6203
Frozen Foods	−0.0166	−0.0186	0.4255	0.0199	0.0416	0.4350
Packaged Meat	−0.0022	0.0012	0.4027	0.0046	0.0075	0.4027
All	*−0.0109*	*−.0257*	*0.3858*	*0.0165*	*0.0495*	*0.3940*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group differences are weighted by expenditure share.

Figure A1: Scanner Data: Differences from a Fisher Index



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Density estimates of index differences using Epanechnikov kernel with bandwidth of 0.05 percentage points. Index values are in percent change versus same quarter a year prior.

B Product turnover with CES preferences

The CES function is convenient for modeling the cost-of-living effects of entering and exiting varieties, originating with Feenstra (1994). I now consider that some varieties may be unavailable in one or more periods. Denote the set of varieties available in each period as $\mathcal{I}_0$ and $\mathcal{I}_1$, such that $\mathcal{I}_0 \cup \mathcal{I}_1 \subseteq \bar{\mathcal{I}}$. We assume the consumer has CES preferences over the superset of varieties $\bar{\mathcal{I}}$. Denote the sets of common varieties as $\mathcal{I}^C = \mathcal{I}_0 \cap \mathcal{I}_1$, exiting varieties as $\mathcal{I}^E = \mathcal{I}_0 \setminus \mathcal{I}^C$, and new varieties as $\mathcal{I}^N = \mathcal{I}_1 \setminus \mathcal{I}^C$.

An important feature of CES preferences is that optimal expenditure on a subset of varieties depends only on prices and taste parameters for varieties in that subset. Adjusting notation to account for an arbitrary set $\mathcal{I}$, Eq. 8 becomes (taking $\bar{u}$ to be one):

$$C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \left[\sum_{i\in\mathcal{I}}\left(\frac{p_i}{\varphi_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{18}$$

Eq. 9 becomes

$$s_i(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j\in\mathcal{I}}(p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I})]^{1-\sigma}} \tag{19}$$

As before, denote $s_{it} = \frac{p_{it}q_{it}}{\sum_{j\in\mathcal{I}_t} p_{jt}q_{jt}} = s_i(\boldsymbol{p}_t;\boldsymbol{\varphi}_t,\mathcal{I}_t)$, under the CES assumption. If a variety is unavailable, the COLI framework uses the reservation price implied by the model of preferences. Assuming $\sigma > 1$, the CES model implies infinite reservation prices.[44] Together, these properties imply $C(\boldsymbol{p};\boldsymbol{\varphi},\bar{\mathcal{I}}) = C(\boldsymbol{p};\boldsymbol{\varphi}_t,\mathcal{I}_t)$..

The class of conditional COLI is therefore given by

Definition B.1 *Conditional Cost-of-living Index with product turnover*

$$\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\bar{\mathcal{I}}) = \frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}_1)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}_0)} \tag{20}$$

Let $\mathcal{I}^* \subseteq \mathcal{I}^C$. Similar to Feenstra (1994), we can rewrite Eq. 20 as

$$\begin{aligned}\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\bar{\mathcal{I}}) &= \frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}^*)}\frac{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}^*)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}_0)}\frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}_1)}{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)}\\ &\equiv \Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}}\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}},\end{aligned} \tag{21}$$

where $\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)$ is the conditional COLI over the common set $\mathcal{I}^*$, and

$$\lambda_t(\boldsymbol{\varphi}) = \frac{\sum_{i\in\mathcal{I}^*}\left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}}{\sum_{i\in\mathcal{I}_t}\left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}},\ t = 0,1. \tag{22}$$

The term $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}}$ adjusts the COLI for the welfare loss from exiting products, while $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}}$ adjusts it for the welfare gain from new products.

[44]When $\sigma < 1$, consumption of all commodities is necessary for positive utility.

As before, $\boldsymbol{\varphi}_0$ and $\boldsymbol{\varphi}_1$ are interesting choices. From Feenstra (1994), $\lambda_t(\boldsymbol{\varphi}_t) = \frac{\sum_{i \in \mathcal{I}^*} p_{it} q_{it}}{\sum_{i \in \mathcal{I}_t} p_{it} q_{it}} \equiv \lambda_t$, which is the share of common varieties expenditure out of total expenditure occurring in period t. A challenge arises, however, with the terms $\lambda_s(\boldsymbol{\varphi}_t), s \neq t$. Intuitively, Eq. 19 implies the taste parameters for absent varieties are not identified—given an infinite price, expenditure shares are zero for any finite value of φ_{it}.

The situation is helped somewhat by the fact that $\lambda_t(\boldsymbol{\varphi}) \in [0, 1]$, and so $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}} \geq 1$ and $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}} \leq 1$.[45] This implies the following bounds:

$$\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}} \geq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}}) \tag{23}$$

$$\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_1^{\frac{1}{\sigma-1}} \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}}), \tag{24}$$

where $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ and $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ are Lloyd-Moulton style indexes which include only adjustments for either exit or entry, not both.

Given these bounds, it is possible to apply the method of proof in Konüs (1924) and Diewert (2001) (Proposition 8) to show that there exists an intermediate taste vector $\check{\boldsymbol{\varphi}}$ such that either $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ or $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$. Of course, these bounds may not be particularly tight, and a symmetric average (e.g., a geometric mean) might not be attractive because the missing factors $\lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$ and $\lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ are not likely to be of comparable magnitudes. For instance, Feenstra (1994), Broda and Weinstein (2010), and others have found that $\lambda_1^{\frac{1}{\sigma-1}}$ dominates $\lambda_0^{\frac{1}{1-\sigma}}$, resulting in a net downward adjustment to the COLI. Consequently, $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ might be a lot closer to its target than $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$.

It seems reasonable that the welfare loss from exiting varieties is larger if conditioning on reference period preferences, and the welfare gain from new varieties is larger if conditioning on comparison period preferences, as market exit decisions may themselves be tied to tastes.

[45] The term $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}} \leq 1$ is also greater than zero.

This motivates the following assumption:

Assumption B.1 *Non-continuing varieties are valued more in the period in which they are available.*

$$\lambda_t(\boldsymbol{\varphi}_t) \leq \lambda_t(\boldsymbol{\varphi}_s), t = 0, 1;\ s \neq t$$

The implies $\lambda_0^{\frac{1}{1-\sigma}} \geq \lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ and $\lambda_1^{\frac{1}{\sigma-1}} \leq \lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$. Assumption B.1 would be true, for example, if taste parameters for common varieties were constant while taste parameters were lower for varieties when they were absent from the market.

Suppose we use Feenstra's adjustment $\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}$ on both conditional COLI. Define

$$P_{FLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) = P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}} \tag{25}$$

and

$$P_{FBLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) = P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}. \tag{26}$$

Under assumption B.1, we have:

$$P_{FLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \tag{27}$$

and

$$\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}}) \leq P_{FBLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}). \tag{28}$$

Of course, neither of these bounds may be tight enough to be useful, but previous research has found that the effect new varieties tends to dominate that of disappearing varieties in Feenstra-style CES indexes (Broda and Weinstein, 2010). Indeed, using the Nielsen Retail Scanner data (Table B1), I find the adjustment for exiting varieties is relatively small on average for many departments, ranging from 0.03 percentage points for Alcoholic Beverages to 1.43 percentage points for Deli. The adjustments for new varieties is between two and twelve times larger in magnitude, ranging from -0.38 percentage points for Alcoholic Beverages to −4.12 percentage points for Deli.[46] As a result, the bounds on $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}})$ appear tighter than the bounds on $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}})$. Figure B1 plots the average P_{BLM} index over common varieties versus the upper and lower bounds for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}})$. Overall, the average difference in bounds is 0.34 percentage points for food products and 0.26 percentage points for non-food products. As the graph indicates, this margin is small relative to the net adjustment for product turnover. Consequently, a geometric mean of the observable bounds seems reasonable to estimate the COLI conditional on comparison period tastes.[47] Note, such an index will imply a smaller welfare effect from exiting varieties, leading to a larger net adjustment or "new goods bias" for the common varieties index.

Table B1: Mean Product Turnover Adjustments and Bounds by Department

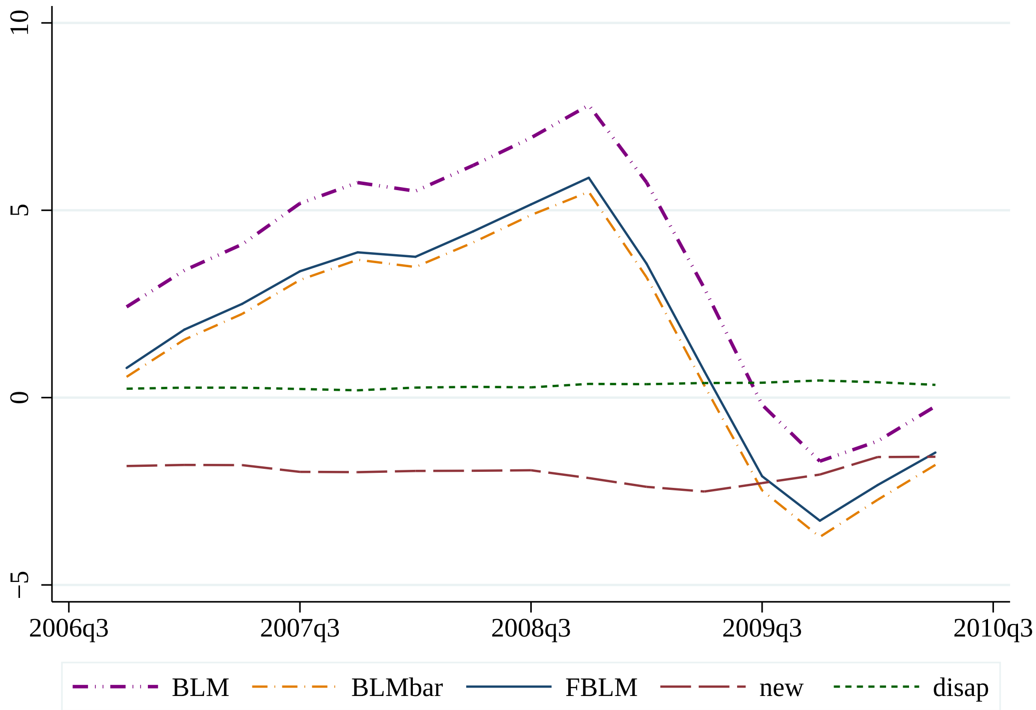
	Adj. New	Adj. Disapp.	LMbar − FLM	FBLM − BLMbar
Alcoholic Beverages	−0.3824	0.0332	0.3858	0.0336
Dairy	−1.4611	0.3869	1.4921	0.3797
Deli	−4.1236	1.4308	4.1879	1.3852
Dry Grocery	−1.9752	0.2197	2.0076	0.2235
Fresh Meat	−2.2178	1.0607	2.2682	1.0673
Fresh Produce	−2.0288	0.5090	2.0362	0.5073
Frozen Foods	−3.4847	0.4003	3.5433	0.3886
Packaged Meat	−1.8188	0.6936	1.8458	0.6886
All	*−1.9961*	*0.3169*	*2.0284*	*0.3153*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group differences are weighted by expenditure share. BLM is the index over common varieties, BLMbar uses the Feenstra adjustment for new varieties, and FBLM uses the Feenstra adjustments for new and exiting varieties.

[46] These adjustment magnitudes are significantly larger than what was reported in Broda and Weinstein (2010) using consumer scanner data over 1994-2003. As my estimates of σ using the retail data are lower, the larger adjustments are to be expected.

[47] In the style of Konüs (1924) and Diewert (2001), one can show P_{FBLM} and P_{FLM} bound a COLI evaluated at an intermediate taste level, though with retail scanner data, I found these bounds to also be wide for most departments.

Figure B1: Scanner Data BLM Indexes with Product Turnover (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes. BLM is the index over common varieties, BLMbar uses the Feenstra adjustment for new varieties, and FBLM uses the Feenstra adjustments for new and exiting varieties.

C Scale and normalization of tastes

The following derives results similar to those found in Kurtzon (2019). Start from the unconditional COLI from Eq. 3.5 which is based on the CES expenditure function from Eq. 8. Without loss of generality, we can write $\varphi_{it} = \tilde{\varphi}_t \ddot{\varphi}_{it}$, $\ddot{\varphi}_{it} \equiv \varphi_{it}/\tilde{\varphi}_t$, where division by $\tilde{\varphi}_t$ represents a normalization of the taste parameters (e.g., RW restrict the unweighted geometric means to be a constant). The true unconditional COLI is then:

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) = \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1} \frac{\left[\sum_{i\in\mathcal{I}} \left(\frac{p_{i1}}{\ddot{\varphi}_{i1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\sum_{i\in\mathcal{I}} \left(\frac{p_{i0}}{\ddot{\varphi}_{i0}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}$$
$$= \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1} \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1), \tag{29}$$

Ignoring potential estimation error in σ, RW's CCV index is exact for $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$. The challenge is expenditure *shares* are invariant to the scale of tastes (i.e., $\tilde{\varphi}_t$) while the minimized unit expenditure *level* is not. The estimable unconditional COLI, $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$ then differs from the true unconditional COLI $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ by the factor $\tilde{\varphi}_0/\tilde{\varphi}_1$, which is unidentified (RW assume it to be equal to one). Therefore, the normalization is not "free" in the sense of being inconsequential to the estimand of interest. Even the direction of the true cardinal cost-of-living change is not identified by $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$.

C.1 Alternative normalizations

For $i = 1, \ldots, N$, let $\ddot{\varphi}_{it} = \varphi_{it}/\tilde{\varphi}_t$ and $\bar{\varphi}_{it} = \varphi_{it}/\hat{\varphi}_t$ be two potential normalizations. For example, $\tilde{\varphi}_t$ could be the unweighted geometric mean and $\hat{\varphi}_t$ a weighted geometric mean (as considered in Kurtzon (2019)), or $\tilde{\varphi}_t$ could be the geometric mean over varieties common to both the reference and comparison periods, while $\hat{\varphi}_t$ could be the geometric mean over all varieties observed in period t. It follows that

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1) = \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \bar{\boldsymbol{\varphi}}_0, \bar{\boldsymbol{\varphi}}_1) \frac{\tilde{\varphi}_t}{\tilde{\varphi}_{t-1}} \frac{\hat{\varphi}_{t-1}}{\hat{\varphi}_t} \tag{30}$$

As a consequence, the CES cardinal index is not invariant to the normalization chosen. There are an unlimited number of possible normalizations; in fact, Kurtzon (2019) shows that there exists a normalization such that $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$ such that taste-shock bias is identically zero.

This is salient for interpretation of RW's results. Similar to Eq. 21 The CES unconditional COLI that allows product turnover can be decomposed:

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \bar{\mathcal{I}}) = \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}^*)} \frac{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}_0)} \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}_1)}{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}^*)}$$
$$\equiv \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*) \lambda_0^{\frac{1}{1-\sigma}} \lambda_1^{\frac{1}{\sigma-1}}, \tag{31}$$

where the λ_t terms are defined in Appendix B and $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*)$ is the unconditional COLI for the common set $\mathcal{I}^*$. This decomposition holds for any $\mathcal{I}^* \subseteq \mathcal{I}^C = \mathcal{I}_0 \cap \mathcal{I}_1$. In their empirical application, RW use this argument to restrict $\mathcal{I}^*$ to include only products that had a lifespan of six years that were not within three quarters of birth or death in periods 0 or 1. In contrast, Redding and Weinstein (2018) uses the full set $\mathcal{I}^C$.

Using the smaller set, the taste-shock bias estimate in RW is much lower in magnitude (around 0.4 percentage points per year), than that reported in Redding and Weinstein (2018) (around 2-4 percentage points per year). This could be because the bias associated with the index over the more restrictive set of varieties happens to be smaller. However, changing the set $\mathcal{I}^*$ amounts to also changing the normalization of the taste parameters using RW's method, as the geometric mean that is assumed to be time-constant is now taken over a different set of varieties. While the product turnover adjustments are invariant to the scale of the taste parameters, Eq. 30 implies the common varieties index is not.

The connection between normalizations of the φ_{it} and RW's CCV index formula in Eq. 14 can be seen by evaluating Eq. 9 at $\boldsymbol{p}_t, \boldsymbol{\varphi}_t$, dividing both sides by their geometric means, and rearranging. This yields:

$$\frac{\varphi_{it}}{\tilde{\varphi}_t} = \tilde{p}_t^{1-\sigma} \left(\frac{s_{it}}{\tilde{s}_t} \right). \tag{32}$$

Setting $\tilde{\varphi}_t$ equal to a constant in periods 0 and 1, one can plug Eq. 32 into Eq. 8 and Definition 3.5 to get Eq. 14. By alternative normalization, I mean that one may just as readily back out the normalized φ_{it} by using expenditure shares and geometric means pertaining to the full set $\mathcal{I}^C$ even if estimating the unconditional COLI over the subset $\mathcal{I}^*$.

I attempt to replicate RW's use of a more restricted set of varieties. Since I have only about five years of data, my restricted set includes only those products in the dataset for each period between 2005Q3 and 2010Q2. To avoid products that may have been within three quarters of birth or death, I focus only on the period from 2007Q3 to 2009Q3. The restricted sets end up having about half as many UPC's as the full sets of common varieties. Using the more restricted sets of common varieties and the CCV formula in Eq. 14 (i.e., normalizing based on the geometric mean across the narrower set of varieties), I also find lower estimates of taste-shock bias, as shown in Table C1 (compare to Table 7). For food products, the average taste-shock bias ranges from -0.17 percentage points per year for Deli, to 3.24 percentage points per year for Fresh Meat. Across all food and beverage products, the average taste-shock bias is 1.2 percentage points per year, which is higher than RW, but less than one fourth of what it was using the full set of common UPCs. Figure C1 plots the index averages. The patterns of comparisons between LM and BLM indexes for food and nonfood products largely follow what was observed in Figure 1 using the full set of common varieties.

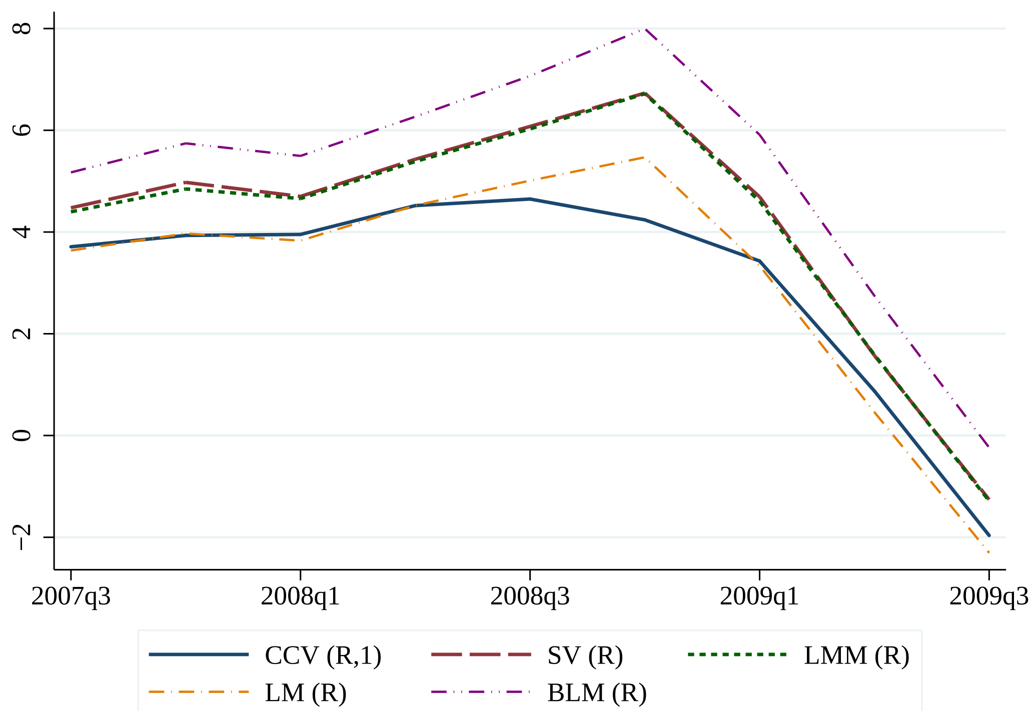
I also estimate the unconditional COLI across the restricted set of common varieties, but using the same normalization of the taste parameters as in the CCV indexes presented in Section 5–using the geometric mean across the full common varieties set $\mathcal{I}^C$. In Figure C2, I compare the averages for of all these possible CCV indexes, along with the SV indexes for both sets of common varieties. SV(R), CCV(R, 1) and CCV(R, 2) cover the restricted set of varieties. CCV and CCV(R,2) normalize using the geometric mean across all common varieties, while CCV(R, 1) normalizes using the geometric mean across the restricted set of varieties. The results indicate a striking influence of the choice of normalization on the CCV index. Comparison of SV and SV(R), or CCV and CCV(R, 2) suggest that there is little difference in the average price changes across the two sets of varieties. However, the wide gap between CCV(R, 2) and CCV(R, 1) suggests that the choice of normalization is playing a large role in the SV to CCV comparison.

Table C1: Mean Differences of CES Indexes Over More Restrictive Common Goods Sets (percentage points)

	SV – CCV	SV – LMM	SV – BLM	BLM – LM
Alcoholic Beverages	0.4516	0.0014	−0.3923	0.7859
Dairy	0.1392	0.0207	−1.4827	2.9646
Deli	−0.1797	0.0449	−0.6407	1.3662
Dry Grocery	1.7494	0.0846	−1.1414	2.4202
Fresh Meat	3.2415	−0.0351	−0.6033	1.1329
Fresh Produce	0.8757	0.0072	−0.8923	1.7901
Frozen Foods	1.3658	0.0299	−0.4703	0.9961
Packaged Meat	−0.4275	−0.0018	−0.3004	0.5962
All	*1.1965*	*0.0532*	*−1.0035*	*2.0886*

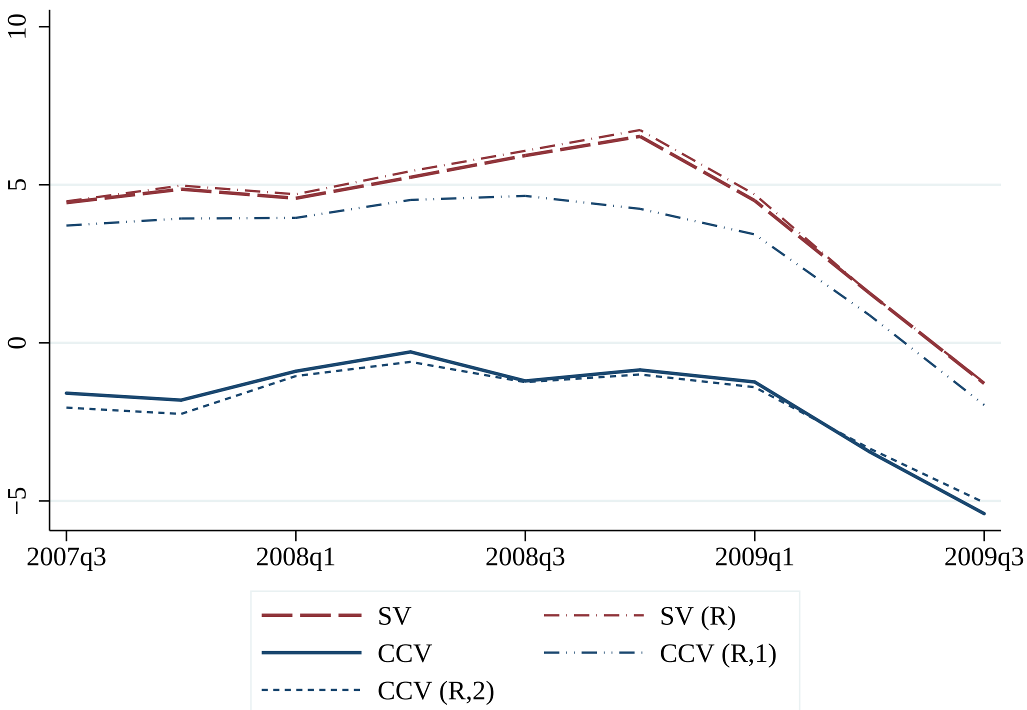
Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group differences are weighted by expenditure share. Indexes include varieties observed in all quarters between 2005Q3 and 2010Q2.

Figure C1: Scanner Data CES Price Indexes Over More Restrictive Common Goods Sets (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes. All but the SV index require the estimated elasticity of substitution. Indexes include varieties observed in all quarters between 2005Q3 and 2010Q2.

Figure C2: Scanner Data SV and CCV Index Comparison (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes. SV and CCV cover all common varieties. SV (R), CCV (R, 1) and CCV (R, 2) cover a restricted set of varieties with a lifespan of 2005Q3-2010Q2. CCV and CCV (R,2) assume that the taste parameter geomean across all common varieties is equal to one in each period. CCV (R, 1) assumes the taste parameter geomean across the restricted set of varieties is equal to one in each period.

D Homothetic Translog

This section shows how to derive conditional COLI for the homothetic translog model. To match RW's parameterization of tastes, the representative agent's minimized unit expenditure function is given in the following definition.

Assumption D.1 *Homothetic translog expenditure function*

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln \alpha_0 + \sum_{i\in\mathcal{I}} \alpha_i \ln\left(\frac{p_i}{\varphi_i}\right) + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_i}{\varphi_i}\right) \ln\left(\frac{p_j}{\varphi_j}\right),\ t = 0, 1. \tag{33}$$

where the restriction $\gamma_{ij} = \gamma_{ji}$ is made without loss of generality.

After some algebra, we can rewrite Eq. 33 as

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln\left[a_0(\boldsymbol{\varphi})\right] + \sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) \ln p_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_i \ln p_j, \tag{34}$$

where $\ln\left[a_0(\boldsymbol{\varphi})\right] = \ln \alpha_0 - \sum_{i\in\mathcal{I}} \alpha_i \ln \varphi_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \ln \varphi_i \ln \varphi_j$ and $a_i(\boldsymbol{\varphi}) = \alpha_i - \sum_{j\in\mathcal{I}} \gamma_{ij} \ln \varphi_j$. From Diewert (1976), homogeneity and symmetry then imply the restrictions $\sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) = 1$ and $\sum_{j\in\mathcal{I}} \gamma_{ij} = 0$.

Eq. 34 reveals two salient points. First, the time variation in $\boldsymbol{\varphi}$ affects the parameter on the first order $\ln p$ terms only, and so the Caves, Christensen, and Diewert (1982) result on the Tornqvist index applies. Second, the $\ln\left[a_0(\boldsymbol{\varphi})\right]$ term captures the pure effect of tastes on unit expenditure, but cancels from the ordinal index that holds tastes fixed.

Under Assumption D.1, the $a_i(\boldsymbol{\varphi}_0)$ and $a_i(\boldsymbol{\varphi}_1)$ are recoverable up to estimates of the γ_{ij}. To see this, the expenditure share equation for variety i is given by:

$$\begin{aligned} s_i(\boldsymbol{p};\boldsymbol{\varphi}) =& \alpha_i + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_j}{\varphi_j}\right) \\ =& a_i(\boldsymbol{\varphi}) + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_j. \end{aligned} \tag{35}$$

This implies the following counterfactual expenditure shares do not depend on the α_i.

$$s_i(\boldsymbol{p_1};\boldsymbol{\varphi}_0) = s_{i0} + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right), \tag{36}$$

$$s_i(\boldsymbol{p_0};\boldsymbol{\varphi}_1) = s_{i1} - \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right). \tag{37}$$

Denote $s_{it} = s_i(\boldsymbol{p_t};\boldsymbol{\varphi}_t)$ the observed expenditure share, $t = 0, 1$, $s^*_{i1} = s_i(\boldsymbol{p_1};\boldsymbol{\varphi}_0)$ and $s^*_{i0} = s_i(\boldsymbol{p_0};\boldsymbol{\varphi}_1)$. Define the following Tornqvist style price indexes.

Definition D.1 *Tornqvist Price Index*

$$\ln P_T = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s_{i0} + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{38}$$

Definition D.2 *Reference taste Tornqvist index*

$$\ln P_{T0} = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s_{i0} + s^*_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{39}$$

Definition D.3 *Comparison period taste Tornqvist index*

$$\ln P_{T1} = \sum_{i\in\mathcal{I}} \frac{1}{2}\left(s^*_{i0} + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{40}$$

Proposition 3 *Under Assumption D.1, $P_{T0} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ and $P_{T1} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$.*

The proof follows from substitution of Eq. 35 into Eq. 34.