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The Problem with Normalizing Preferences that Change in a Cost-of-Living Index

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Abstract

There are new proposals for prices indexes that attempt to correct for what they consider bias in standard indexes from changes in consumer preferences. But these proposals have a fundamental problem that changes in preferences between two periods cannot be identified by data on prices and quantities with only a normalization. This paper shows that the required normalization is not free, so that an arbitrary choice of normalization can yield any desired index result. In fact, a normalization using the Sato-Vartia weights yields a Sato-Vartia index, implying exactly zero bias.

1 Introduction

Multiple prices indexes that incorporate changing consumer preferences have been proposed. Such studies claim that standard cost-of-living indexes (COLIs) suffer from bias because they are defined for only one set of preferences. These studies include Redding & Weinstein (2020) (RW), Ueda, Watanabe, and Watanabe (2019), and Gabor-Toth & Vermeulen (2020). But these proposals have a fundamental problem that changes in preferences between two periods cannot be identified by data on prices and quantities. At any given time, the data can only identify relative preferences over goods in that time period, not how the entire standard of living (utility) compares between periods. Therefore, they use a normalization to fix the level. Using the RW framework, this paper shows that the required normalization is not free, so that an arbitrary choice among different normalizations will yield almost any desired index result.

For illustration, consider if there were only two goods, apples and oranges. In period s $<$ r, apples are preferred more, such that if the prices were

*I would like to thank Robert Martin and Peter Zadrozny for suggestions and comments. All estimates and analyses based on Nielsen data are by the author and not the data provider.

equal, the consumer would buy more apples, and vice versa in period r. The proposals that use only price and quantity data cannot equate preferences between the two periods without normalizing a set of preference parameters. Consider if consumers simply enjoy both goods proportionately more in the second period, so that consuming the same quantities gives a higher level of the standard of living. Then the cost of obtaining the standard of living of the first period would be lower, and there would be no way to rule out this possibility with the data because the observed shares would be the same. Therefore, the preferences are normalized to fix the level, such as by fixing some mean of the parameters, such as the mean of the desire for apples and oranges. Putting aside that this still doesn't identify the level change, any different weighting of that mean is still consistent with the data but could yield almost any relevant result. So if the desire for apples was averaged with double the desire for oranges and normalized to the same number, the observed shares of apples vs. oranges would be the same but the consumer would be better off in r relative to s for the same bundle. Since a COLI measures the change in cost of a standard of living, the COLI would be lower.

Section 2 presents the issue formally using the RW framework as the primary example, and the identification problem with the normalization, and shows alternative normalizations. Section 3 presents the calculated indexes implied by the alternative normalizations and compares them. Section 4 concludes.

2 Indexes with Changing Preferences

RW models consumer preferences with constant elasticity of substitution utility, which are homothetic, so the expenditure function can be characterized by the unit expenditure function[1]

$$P_t^* = \left[\Sigma_{k \in \Omega_{t-1,t}} \left(\frac{p_{kt}}{\phi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{1}$$

where $\Omega_{t-1,t}$ denotes the set of all goods common to both periods t-1 and t, p_{kt} is the price of good k in period t, σ is the elasticity of substitution between goods, and ϕ_{kt} is a preference parameter for good k in period t, which could be interpreted as good quality, appeal, as a demand shifter, or specification error or just a demand residual. While Martin (2020)[2] shows that how the common goods are defined in this framework changes the results, here I abstract from the choice of common goods.

[1] This is RW equation (1).
[2] This is shown in Appendix C of Martin (2020).

The COLI for CES preferences is therefore

$$COLI_{t-1,t} = \frac{P_t^*}{P_{t-1}^*} = \frac{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{\phi_{kt}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{k,t-1}}{\phi_{kt}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \tag{2}$$

The proposed RW index is,

$$CCV_{t-1,t} = \frac{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{\phi_{kt}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{k,t-1}}{\phi_{k,t-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \tag{3}$$

where CCV denotes the price index for the goods that are common to both periods[3]. The Sato-Vartia (SV) index is an exact COLI for CES preferences[4], given by

$$\Phi_t^{*SV} = \Pi_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{p_{k,t-1}}\right)^{\omega_{kt}^*} \tag{4}$$

where ω_{kt}^* are the Sato-Vartia weights, $\omega_{kt}^* = \frac{\frac{s_{kt}^*-s_{k,t-1}^*}{\ln s_{kt}^*-\ln s_{k,t-1}^*}}{\Sigma_{k\in\Omega_{t-1,t}}\frac{s_{kt}^*-s_{k,t-1}^*}{\ln s_{kt}^*-\ln s_{k,t-1}^*}}$. The "taste shock bias" given by the CCV[5] is therefore

$$\ln\Phi_t^{*CCV} = \ln\Phi_t^{*SV} - \overbrace{\left[\Sigma_{k\in\Omega_{t-1,t}}\omega_{kt}^*\ln\frac{\phi_{kt}}{\phi_{k,t-1}}\right]}^{\text{taste-shock bias}} \tag{5}$$

.

The goods' shares in CES are[6]

$$s_{kt}^* = \frac{\left(\frac{p_{kt}}{\phi_{kt}}\right)^{1-\sigma}}{\Sigma_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{\phi_{kt}}\right)^{1-\sigma}} \tag{6}$$

which implies[7]

$$(P_t^*)^{1-\sigma} = \frac{1}{s_{kt}^*}\left(\frac{p_{kt}}{\phi_{kt}}\right)^{1-\sigma} \tag{7}$$

.

[3] This is equivalent to equation (9) in RW.

[4] If preferences change, the SV index is exact for preferences that are intermediate between the two periods. See Feenstra & Reinsdorf (2007).

[5] This is RW equation (13).

[6] This is RW equation (2).

[7] This is equivalent to RW equation (2).

Because all the ϕ_{kt} parameters could be divided by a constant and yield the same shares and index, only the ratios $\frac{\phi_{kt}}{\phi_{jt}}$ are identified. Therefore, for an index for periods t-1 to t, they are normalized by the unweighted geometric mean of all ϕ_{kt}, so that

$$\Pi_{k\in\Omega_{t-1,t}}\phi_{kt}^{\frac{1}{N_{t-1,t}}} \equiv 1 \tag{8}$$

, where $N_{t-1,t}$ denotes the total number of goods common between period t-1 and t.

Consider alternative normalizations with weights w_{kt}, all equally consistent with the data in (6),

$$\begin{aligned} \Pi_{k\in\Omega_{t-1,t}}\phi_{kt}^{w_{kt}} &\equiv 1 && (9)\\ \Sigma_{k\in\Omega_{t-1,t}}w_{kt} &= 1 && (10) \end{aligned}$$

. Taking the log of both sides of (7), time differencing and rearranging yields

$$(1-\sigma)\ln\frac{P_t^*}{P_{t-1}^*} = (1-\sigma)\ln\frac{p_{kt}}{p_{k,t-1}} - (1-\sigma)\ln\frac{\phi_{kt}}{\phi_{k,t-1}} - \ln\frac{s_{kt}^*}{s_{k,t-1}^*} \tag{11}$$

. Multiplying by w_{kt} and summing over common goods, yields

$$(1-\sigma)\ln\frac{P_t^*}{P_{t-1}^*} = (1-\sigma)\,\Sigma_{k\in\Omega_{t-1,t}}w_{kt}\ln\frac{p_{kt}}{p_{k,t-1}} - (1-\sigma)\,\Sigma_{k\in\Omega_{t-1,t}}w_{kt}\ln\frac{\phi_{kt}}{\phi_{k,t-1}} - \Sigma_{k\in\Omega_{t-1,t}}w_{kt}\ln\frac{s_{kt}^*}{s_{k,t-1}^*} \tag{12}$$

$\rightarrow$

$$\frac{P_t^*}{P_{t-1}^*} = \Pi_k\left(\frac{p_{kt}}{p_{k,t-1}}\right)^{w_{kt}}\left[\Pi_{k\in\Omega_{t-1,t}}\left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{w_{kt}}{\sigma-1}}\right] = \Pi_{k\in\Omega_{t-1,t}}\left(\frac{p_{kt}}{p_{k,t-1}}\left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{1}{\sigma-1}}\right)^{w_{kt}} \tag{13a}$$

For the RW normalization, $w_{kt} = \frac{1}{N_{t-1,t}}$,

$$\Phi_t^{*CCV} = \frac{P_t^*}{P_{t-1}^*} = \Pi_k\left(\frac{p_{kt}}{p_{k,t-1}}\right)^{\frac{1}{N_{t-1,t}}}\left[\Pi_{k\in\Omega_{t-1,t}}\left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{1}{N_{t-1,t}(\sigma-1)}}\right] \tag{14}$$

which is their CCV index[8].

However, an unweighted mean puts as much importance on a very widely consumed good as one with a trivial share. Consider the normalization with the SV weights instead, $w_{kt} = \omega_{kt}^*$,

$$\Pi_{k\in\Omega_{t-1,t}}\phi_{kt}^{\omega_{kt}^*} \equiv 1 \tag{15}$$

[8] This is RW equation (9) without the tilde notation.

. Then (13a) becomes

$$\frac{P_t^*}{P_{t-1}^*} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}}\right)^{\omega_{kt}^*} = \Phi_t^{*SV} \tag{16}$$

because

$$\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{\omega_{kt}^*}{(\sigma-1)}} \equiv 1 \tag{17}$$

. Therefore, taste-shock bias from (5) is exactly zero

$$\overbrace{\left[\Sigma_{k\in\Omega_{t-1,t}} \omega_{kt}^* \ln \frac{\phi_{kt}}{\phi_{k,t-1}}\right]}^{\text{taste-shock bias}} \equiv \ln \Phi_t^{*SV} - \ln \Phi_t^{*CCV} \equiv 0 \tag{18}$$

.

If the weights were last period's shares, $w_{kt} = s_{k,t-1}^*$, the index would be

$$\frac{P_t^*}{P_{t-1}^*} = G_{t-1,t} \left[\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{s_{k,t-1}^*}{\sigma-1}}\right] \tag{19}$$

where $G_{t-1,t} = \Pi_{k\in\Omega_{t-1,t}} \left(\frac{p_{kt}}{p_{k,t-1}}\right)^{s_{k,t-1}}$ is the geometric means index with base period (lag) shares.

Using current period shares, $w_{kt} = s_{k,t}^*$,

$$\frac{P_t^*}{P_{t-1}^*} = \Pi_k \left(\frac{p_{kt}}{p_{k,t-1}}\right)^{s_{k,t}^*} \left[\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{s_{k,t}^*}{(\sigma-1)}}\right] \tag{20}$$

. Using Törnqvist weights, $w_{kt} = \frac{1}{2}\left(s_{kt}^* + s_{k,t-1}^*\right)$,

$$\frac{P_t^*}{P_{t-1}^*} = TQ_{t-1,t} \left[\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s_{kt}^*}{s_{k,t-1}^*}\right)^{\frac{\frac{1}{2}\left(s_{kt}^*+s_{k,t-1}^*\right)}{\sigma-1}}\right] \tag{21}$$

where $TQ_{t-1,t}$ denotes the Törnqvist price index. In each case, (5) differs.[9]

[9] Zadrozny (2020) uses generalized CES preferences instead,which allow for non-homotheticity. However, the same normalization (9) is required, where the α_{it} parameters are analogous to the ϕ_{kt} RW parameters.

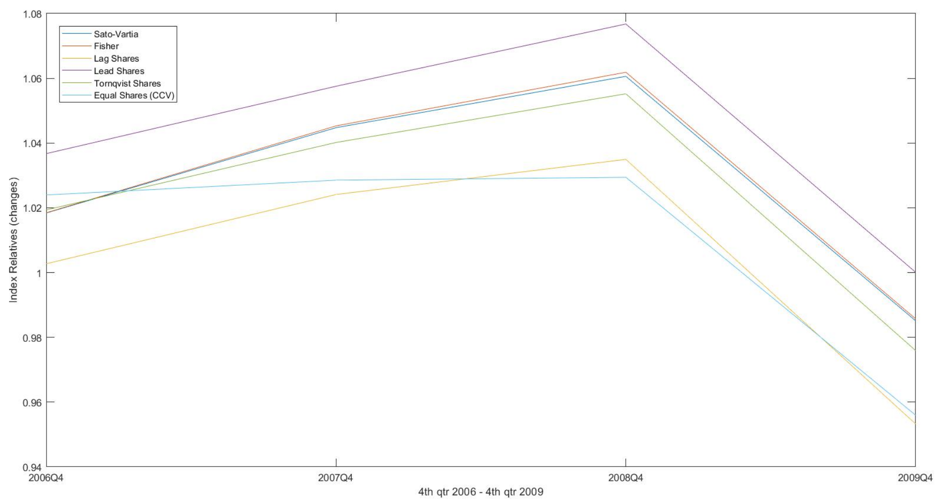
3 Comparison of Calculated Indexes with Alternate Preference Normalizations

CCV type indexes with alternative normalizations were constructed to compare to the CCV and SV indexes. The data used was chosen to be as similar as possible to RW. However, RW use AC Nielsen's Homescan data, which is proprietary data of a household survey of purchases from grocery and drug stores in the U.S. Homescan data for the same time span as RW, 2004-2014, was not available for this study. Instead, I use BLS's five years of data of AC Nielsen's Scantrak data, which contains the universe of transactions data of all large grocery and drug store chains in the U.S., so this is not meant to replicate the exact results. The data contains weekly and monthly total quantities and unit value prices for goods defined at the detail of UPC, which generally identifies an exact good. Following RW, the data is aggregated to the quarterly level, and index changes from 4th quarter to 4th quarter were calculated, from 4th quarter 2005 - 4th quarter 2009. The data is divided into categories called product groups, which were aggregated using a Laspeyres formula with 4th quarter 2005 as the base period. The elasticities of substitution were estimated separately for each product group, using the methodology in Broda & Weinstein (2010), which in turn uses the Feenstra (1994) method, to be as consistent as possible with RW.

Figure 1[10] and the table below reports the index changes for the CCV, Fisher, Sato-Vartia share normalization (SV index), Fisher, Törnqvist share normalization, lagged share normalization, and current (lead) share normalization. The "taste change bias" is the difference between these indexes and the SV index. As shown above, the normalization using SV weights is exactly the SV index, with zero bias. The standard Fisher index is very close, and the Törnqvist share index is slightly below. As reported in RW, the equal shares index and lag shares index are greatly below the SV index, and except for the first year, similar to each other, implying upward bias in a SV index. However, the lead shares index is the opposite, being greatly higher and implying downward bias in the SV index. These differences are very significant since they compound over time. For example, the average difference between the lead share index and the SV is 1.5%/year, which over ten years would compound to a difference in index levels of 16.25%.

[10] These results are analogous to RW Figure V, and the lagged share normalization is what they refer to as the "CUPI - Initial Shares". RW include the Fisher index for comparison.

Figure 1: Index Relatives with Different Preference Normalizations, 4th qtr to 4th qtr

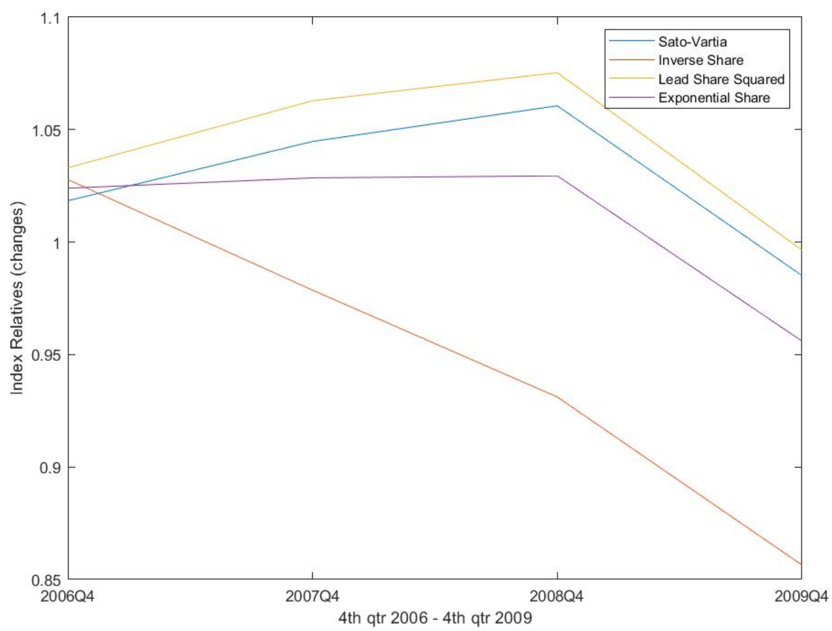


Index	Period			
	2006Q4	2007Q4	2008Q4	2009Q4
Sato-Vartia	1.0183	1.0447	1.06060	0.9851
Fisher	1.0184	1.0453	1.06186	0.9857
Lag Shares	1.0027	1.0241	1.03495	0.9532
Lead Shares	1.0367	1.0575	1.07676	1.0000
Törnqvist Shares	1.0193	1.0401	1.05521	0.9758
Equal Shares (CCV)	1.0239	1.0285	1.02943	0.9559
Inverse Share	1.0278	0.9786	0.93114	0.8564
Lead Share Squared	1.0330	1.0628	1.07532	0.9964
Exponential Share	1.0239	1.0285	1.02945	0.9559

Additional normalizations are shown in Figure 2 and the table, including weighting by the inverse share, squared lead share, or and exponential of the share, for weights of $w_{kt} = \frac{\frac{1}{\omega^*_{kt}}}{\Sigma_k \frac{1}{\omega^*_{kt}}}$, $\frac{s^{*2}_{kt}}{\Sigma_k s^{*2}_{kt}}$, and $\frac{\exp(1+s^*_{kt})}{\Sigma_k \exp\left(1+s^*_{kt}\right)}$ respectively. The inverse and exponential share weighted indexes imply upward bias in the SV of greatly different magnitude, while the lead share squared index implies the opposite.

More generally, a weighted mean always lies between the minimum and maximum values, depending on the weights. The index is a weighted geometric

Figure 2: Additional Index Relatives with Different Preference Normalizations, 4th qtr to 4th qtr



mean of

$$x_{kt} = \frac{p_{kt}}{p_{k,t-1}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{1}{\sigma-1}} \tag{22}$$

. Therefore,

$$x^{\min}_{kt} \leq \frac{P^*_t}{P^*_{t-1}} \leq x^{\max}_{kt}$$

. The highest x_{kt} over good and time, $x^{\max}_{kt}$, is about 4800%, while the lowest x_{kt}, $x^{\min}_{kt}$, is about 86% deflation. Therefore, to yield the desired index I^{choice}_t, pick a weighting on the maximum x_{kt}, $w^{\max}_{kt}$ such that

$$I^{choice}_t = \left(x^{\min}_{kt}\right)^{w^{\max}_{tk}} \left(x^{\min}_{kt}\right)^{1-w^{\max}_{tk}} \tag{23}$$

and place a trivial weight on all other x_{kt} values,

$$w_{kt} = \begin{array}{ccc} w^{\max}_{kt} & if & k \text{ is max} \\ 1 - w^{\max}_{kt} & if & k \text{ is min} \\ \varepsilon & otherwise & \end{array} \tag{24}$$

. Thus, if a statistical agency wanted to incorporate changing preferences by this method, they could always claim they were doing it by choosing a set of weights that yielded exactly the same indexes that they produced before!

A simple example shows how different weights affect the index. "Taste-Shock Bias" is only positive for the equal weighting normalization if product sales shares diverge, as can be seen from (14) when the geometric mean of the shares rises between periods. The problem with the equal weighting normalization is that smaller shares have a proportionately higher relative change. Suppose there are only two goods, with shares = .8 and .2, which then change to .9 and .1, with $\sigma = 4$. The second term in the r.h.s. of $(13a)$ becomes

$$\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{w_{kt}}{\sigma-1}} = \left(\frac{.9}{.8}\right)^{\frac{.5}{3}} \left(\frac{.1}{.2}\right)^{\frac{.5}{3}} = (1.0198)\,(0.89090) = 0.90856 < 1 \tag{25}$$

. Higher weighting on higher share goods could reverse this, such as $w_{kt} = s^*_{kt}$, so that

$$\Pi_{k\in\Omega_{t-1,t}} \left(\frac{s^*_{kt}}{s^*_{k,t-1}} \right)^{\frac{w_{kt}}{\sigma-1}} = \left(\frac{.9}{.8}\right)^{\frac{.9}{3}} \left(\frac{.1}{.2}\right)^{\frac{.1}{3}} = (1.0360)\,(.97716) = 1.0123 > 1 \tag{26}$$

. The weight on the higher share goods therefore determines whether this term moves the index upwards or downwards. Depending on how share ratios are correlated with prices ratios (determined by the elasticity), this determines how the index compares to a SV index.

4 Conclusions/Discussion

Since any index could be rationalized by the choice of a particular set of weights, the decision to adjust for changing preferences by normalizing them is empty. However, it is troubling that what is being measured, the cost of a certain standard of living for certain preferences, may become irrelevant if consumer preferences do change significantly. With additional data, it may be possible to connect preferences between two periods by measuring the changes in preferences, thus making a larger preference mapping that includes both periods. But prices and quantities of goods are not sufficient.

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