

Analytical Bias Reduction for Small Samples in the US Consumer Price Index*

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Abstract

The U.S. Bureau of Labor Statistics (BLS) faces sample size constraints when computing its Consumer Price Index (CPI-U). The samples are not adequately large for the index to equal a true “fixed basket” price index. This study adjusts for this small sample bias by estimating the second order of a stochastic expansion of the index. Unlike increasing sample size, this adjustment is inexpensive because one uses the same data that is used to compute the CPI-U. From the beginning of 1999 to the end of 2003, I estimate that 63% of the difference between the BLS superlative index (CPI-C) and the CPI-U is the result of finite sample bias and the other 37% is commodity substitution bias.

Keywords: Stochastic Expansions, Laspeyres Price Index, Jevons Price Index

JEL Classification: C43, C13

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1 Introduction

Measuring inflation properly is an essential ingredient in refereeing US economic policy debates, in setting of optimal contracts, in guiding monetary policy, and in determining the financing needs of all levels of government. Therefore, accurate inflation measure is an important issue. Although, the U.S. Consumer Price Index (CPI-U) is perhaps the most widely used national inflation measure, it has many shortcomings as an inflation measure. This study looks at one source of bias in the CPI-U. To compute the CPI-U the Bureau of Labor Statistics (BLS) takes many separate and small samples throughout the nation's urban centers, and it is these small samples that induce an upward bias in the CPI-U.

The CPI-U is a two stage index. In the second stage, the "All-Items" CPI-U is computed as a weighted average of 8,018 sub indexes called price relatives or relatives. A relative is a price index for a group of commodities or services (called items) within a metropolitan area (called area).¹ The relatives are computed in the first stage. Examples of these item-areas are apples in Boston, rental housing in Seattle, etc. BLS is limited in the number of prices that it can collect for each item-area, and while it attempts to allocate the sample size among these item-areas optimally, the samples sizes that are used to compute a relative can remain as small as one. The relative is a non-linear moment of the collected prices, and while the relative itself asymptotically converges to its true value, BLS samples are not adequately large to achieve this consistency. Therefore, the CPI-U is a weighted average of relatives that suffer from finite sample bias.

¹Typically there are 38 geographic areas called "primary sampling units" (PSUs) and 211 items that generate $38 \times 211 = 8,018$ cells.

This study proposes a low cost method to improve the accuracy of the CPI-U by adjusting its relatives so that finite sample bias is reduced. This bias adjustment method uses the same data that is used to compute the CPI-U. It improves the accuracy of the relative by using the additional information about the second moments of the sample and the curvature of the function that computes the relative.

While this method improves the accuracy of the CPI, it is not as good a solution as the more costly alternative of increasing the item-area's sample size to the point where the relatives reach their asymptotic properties. The reason is that increasing sample size will not only reduce bias, but will eliminate the variance effects of sampling error. This low cost bias adjustment method in this study does not purge the effects that sampling error has on the variance in the same way that increasing the sample size does.

Finite sample bias in the relative has been widely reviewed. Before 1998, all CPI-U relatives were Laspeyres type indexes that were ratios of averages. Early studies used methods described by Cochran's textbook (1963) where he reviews the finite sample bias from ratio estimation. Examples of studies that use these methods to derive finite sample bias for price indexes are Kish, Namboordi, and Pillai (1962), McClelland and Reinsdorf (1997), and Greenlees (1998). After 1998, most of the CPI-U relatives changed from a Laspeyres form to a geometric mean or Jevons form. Unlike the Laspeyres type relatives, the finite sample bias of the geomean relative is systematically positive because of Jensen's Inequality. This study commenced after the BLS switch to the geomean relative, unlike

the earlier ones mentioned here. Since finite sample bias was no longer solely a problem of ratio estimation, this study uses the analytical bias reduction method outlined in Hahn and Newey (2004) and Rilstone et. al (1996). This method takes a stochastic second order expansion of the relative, and uses the second order term to approximate the bias.²

Because of the chaining of the index, finite sample bias in one month has a unit root and will compound with the finite sample bias in another month. When indexes are direct rather than chained, finite sample bias does not compound and thus has a smaller impact. Thus chaining the index can make the finite sample problem exponentially worse over time.

This study attempts to analytically reduce finite sample bias by using the second order term in stochastic expansion of the relative around its true parameter value to adjust the price relative. The basic intuition behind analytical bias adjustment is that one is using a second moment of prices and the curvature of the index function as additional information in the estimation of a price index. Stochastic expansion theory is the starting point for bootstrap theory that adjusts for bias and that improves confidence interval estimation with Edgeworth Expansions (See Hall 1992.) This idea is often used in other econometric problems such as weak instrumental variables (see Hahn and Hausman (2002)) or correcting for bias in non linear panel data models. (See Hahn and Newey (2004).) Analytical bias adjustment is not the only method of finite

²The idea of using asymptotic expansions to approximate finite sample bias originated with informal discussions that I had with John Greenlees. However, the derivation of the adjustments and the proofs of the propositions that establish the properties of my proposed adjustments are my own.

sample bias correction. Bradley (2001) does a simulated bootstrap correction only for food and home fuel items. However, if one wishes to do a simulated bootstrap correction on the entire set of samples, 8,018 bootstraps would need to be done each month. Even in today's environment of high speed chips, this is still computationally intractable.

BLS uses three different methods to compute relatives. The most widely used method is the geomean. In this study, I estimate that 96% of the finite sample bias adjustments in the CPI-U comes from the relatives computed by geomeans. The other methods used to compute relatives are the Laspeyres and the sixth root of a Laspeyres used for the rent and imputed rental equivalence. Although housing has the largest expenditure weight in the CPI-U, it contributes very little to the finite sample bias of the overall CPI, because the housing samples on an area basis are relatively large and there is surprisingly much less variance in rents than in the price of other goods and services. Food and apparel are the two groups that contribute the most to the finite sample bias of the CPI-U.³

There are several sources of bias in the CPI; finite sample bias is just one of them. Lebow and Rudd (2003) give the most up to date and comprehensive review of the various sources of bias or measurement error. It is important that finite sample bias in the CPI-U not be confused with commodity substitution bias. Much publicity has focused on the "Laspeyres type" form of the upper level CPI-U formula where the weights of the price relatives do not allow for commodity substitution across items when there is a change in the ratio of price

³Each month a randomly rotating set of outlets within an area place some of their food and apparel items on sale. This adds variance to the observed prices within the food and apparel items. These types of sales do not occur in housing rents.

relatives. Lebow and Rudd (2003) and Shapiro and Wilcox (1997) are examples of studies that measure commodity substitution bias by taking the difference between the CPI-U and a superlative price index.⁴ Bradley (2001) and this study show that this is not the correct measure of commodity substitution bias, since finite sample bias makes the CPI-U a biased estimator for a Laspeyres or “fixed basket” price index. In Bradley (2001) and in this study, there is a proof showing that some superlative indexes, such as Törnqvist, do not suffer from finite sample bias in the same way as a Laspeyres index does. Therefore, while the total bias that is measured by taking the difference between the current CPI-U and a superlative price index has not changed, this study shows that one cannot attribute all of this difference to commodity substitution bias. Additionally, the part that comes from finite sample bias can be reduced without incurring the cost of producing a timely superlative index, or increasing sample size.

One should not expect significant substitution opportunities among all the 8,018 item-areas. For instance, suppose that the price of bananas in Philadelphia increases. Philadelphia residents will not substitute any item sold in another city for bananas in Philadelphia. At best there are a few other items within food in Philadelphia that they can substitute in response to the price increase in bananas. If consumers do not substitute for items outside their geographic reach, then this means that if the price of any one of the 211 items increases then at most they only have substitution opportunities with less than 3% of the

⁴Lebow and Rudd (2003) do acknowledge that there is evidence of finite sample bias.

remaining cells.⁵ This study confirms the expectation that the true upper level form is “close to” a Laspeyres form. Most of the substitution activity perhaps should be taking place within the item-area cell. For example, if the price of a certain brand of cereal increases, consumers will usually substitute to another brand of cereal.

Because of finite sample bias, the CPI-U is greater than the true “fixed basket” price index; however, my proposed adjustment method reduces this bias using the same data that is used to compute the index in the first place. Thus, it is less expensive and easier to use this method to improve the CPI-U than it is to replace it with a timely superlative index where expenditure data must be updated every month.⁶ The best way to mitigate finite sample bias and regular sampling error variance is to increase sample size. But, this is also costly. If budgets are constrained, then it seems that analytical bias reduction is the “second best” alternative.

Section 2 briefly describes the properties of the price relative. It uses traditional first order expansion to show consistency. Next it uses stochastic second order expansions to approximate the finite sample bias. It is this approximation that adjusts the sample estimate of the relative. It then shows how these adjusted relatives can be used to generate a corrected upper level Laspeyres index. I show in this section the reason that one should not make the same adjustments to the relatives for either an upper level Törnqvist or geomean index.

⁵Since they can only substitute within the geographic area, this means that only $1/38 \approx 3\%$ of the 8,018 cells are available.

⁶In fact this is the reason that the final version of BLS superlative CPI-C is not a timely index. It would be prohibitively expensive to have a “real-time” expenditure update along with the current monthly collection of prices.

Finally, this section shows how to additively decompose both the CPI-U and the bias-adjusted CPI-U into the major commodity groups (i.e., food, medical care, housing, etc.). Section 3 describes a Monte Carlo simulation that verifies the properties established in Section 2. Section 4 gives a re-estimate of the CPI-U when the second order term is used to analytically correct for finite sample bias. Section 5 concludes.

2 Construction and Adjustment of the Price Relative

2.1 Basic Construction of the Relative and its Asymptotic Values

The formula used to construct the price relative in the CPI-U depends on the item. If it is believed that there are substitution opportunities within an item - such as breakfast cereal, or children’s apparel - a geometric mean index is computed. For other items, such as hospitals or home heating oil, where there seem to be very few substitution opportunities, a “Laspeyres type” index is used. The month to month price index for housing rent and the “rental equivalence” for homeowners is the sixth root of a Laspeyres index, where the denominator contains rents from the previous six months.

Let item-areas be indexed by i , the sample observations within an item-area be indexed by j , and the month-year by t . Denote the j^{th} collected price in item-area i and period t as p_{ijt} . For a sample of n_i , the geometric mean price

relative for item-area i is⁷

$$\widehat{R}_{Git} = \exp\left\{\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i\right\} \quad (1)$$

where n_i is the number of price quotes. \widehat{R}_{Git} is intended to be an estimate of the entire item-area index, $\text{plim}_{n_i \rightarrow N_i} \widehat{R}_{Git}$, where N_i is the total number of goods or services within the i^{th} item-area, then denote

$$\mu_{Git} = \text{plim}_{n_i \rightarrow N_i} \sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i = E\left[\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i\right]. \quad (2)$$

The finite sample bias of the geometric mean is

$$B_{Git} = E(\widehat{R}_{Git}) - \exp(\mu_{Git}). \quad (3)$$

Since $\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i$ is a linear average, its expectation equals μ_{Git} .

But from Jensen's Inequality, the finite sample bias will be positive if this sample average has non-zero variance.

⁷Typically, a geomean price index has the form $\exp(\sum_{j=1}^{n_i} w_{ij} \ln(p_{ijt}/p_{ijt-1}))$ where w_{ij} is the expenditure share of the j^{th} item in item-area i . For the CPI-U, the prices of the goods or services that are collected within an item-area are sampled with a probability of selection equal to that good or service's share of the total item-area's expenditure from a previous period. Even though a simple average is computed from this sample, on an expected value basis, this is an expenditure share weighted geomeans. To see this, let N_i be the population total. Then drawing n_i items will induce the following identity: $\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i = \sum_{j=1}^{N_i} I_{n_i}(j) \ln(p_{ijt}/p_{ijt-1})/n_i$, where $I_{n_i}(j)$ equals the number of draws of j in a sample of n_i items. But with the probability of selection equal to the expenditure share, it follows that $E(I_{n_i}(j))/n_i = w_{ij}$, and since the expenditure share is independent of the price, this implies that $E\left\{\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i\right\} = \sum_{j=1}^{N_i} w_{ij} \ln(p_{ijt}/p_{ijt-1})$.

The Laspeyres relative is computed as⁸

$$\widehat{R}_{Lit} = \frac{\sum_{j=1}^{n_i} p_{ijt}/n_i}{\sum_{j=1}^{n_i} p_{ijt-1}/n_i}. \quad (4)$$

If

$$\mu_{Lit} = \text{plim}_{n_i \rightarrow N_i} \widehat{R}_{Lit} = \frac{\text{plim}_{n_i \rightarrow N_i} \sum_{j=1}^{n_i} p_{ijt}/n_i}{\text{plim}_{n_i \rightarrow N_i} \sum_{j=1}^{n_i} p_{ijt-1}/n_i} = \frac{E(p_{ijt})}{E(p_{ijt-1})}, \quad (5)$$

then for a small n_i , $E(\widehat{R}_{Lit}) \neq \mu_{Lit}$. Unlike the geomean index, the Laspeyres relative is not globally convex, and as a result, unlike the geomean index, the finite sample bias, $B_{Lit} = E(\widehat{R}_{Lit}) - \mu_{Lit}$, of the Laspeyres is not always positive.

For the housing index, a sixth root of a six-month index is calculated. Letting \widehat{R}_{Hit} denote the housing relative, the month to month change is estimated as

$$\widehat{R}_{Hit} = \left(\frac{\sum_{j=1}^{n_i} p_{ijt}/n_i}{\sum_{j=1}^{n_i} p_{ijt-6}/n_i} \right)^{1/6}. \quad (6)$$

I denote its asymptotic value as

$$\mu_{Hit} = \left(\frac{\text{plim}_{n_i \rightarrow \infty} \sum_{j=1}^{n_i} p_{ijt}}{\text{plim}_{n_i \rightarrow \infty} \sum_{j=1}^{n_i} p_{ijt-6}} \right)^{1/6}, \quad (7)$$

and the finite sample bias is

$$B_{Hit} = E(\widehat{R}_{Hit}) - \mu_{Hit}.$$

This is a very unusual type of index. It is geometric average of a six month Laspeyres index. Since rents most often change on an annual basis, for each

⁸The textbook form of the Laspeyres index is $\frac{\sum_{j=1}^{n_i} p_{ijt} q_{ijt-1}}{\sum_{j=1}^{n_i} p_{ijt-1} q_{ijt-1}}$ where q_{ijt-1} is the quantity of j sold in item-area i in time $t-1$. As previously mentioned, goods and services are sampled with a probability of selection equal to expenditure share. Denote P_{ijs} as the collected price at time s . BLS scales P_{ijt} and P_{ijt-1} by a price from a previous period, $l < t-1$ so that $p_{ijt} = P_{ijt}/P_{ijl}$ and $p_{ijt-1} = P_{ijt-1}/P_{ijl}$. If there are N_i items in the entire item-area, then, with the probability proportional to expenditure share, one gets $E\left\{\sum_{j=1}^{n_i} p_{ijt}/n_i\right\} = E\left\{\sum_{j=1}^{N_i} I(j \text{ selected})(P_{ijt}/P_{ijl})/n_i\right\} = \sum_{j=1}^{N_i} q_{ij} P_{ijt}$ where q_{ij} is a fixed quantity measure.

observation this six month index has a higher probability of detecting a change than a month to month index.

2.2 First Order Expansion of the Relative

To establish the first order asymptotics of the relative (or \sqrt{n} consistency), I use a first order stochastic expansion. This method is the usual one used to establish the consistency and asymptotic normality of an estimator. (For example, Hansen (1982) uses this method to establish the asymptotic properties of the GMM estimator, and Amemiya's 1985 *Advanced Econometrics* uses this method to establish the asymptotic properties of the Maximum Likelihood Estimator.) Like Hansen (1982), throughout this paper, I assume that all price variances are bounded, for all i and s , $\{p_{ijs}\}_{j=1}^{n_i}$ is i.i.d. across j ⁹, and that N_i is "very large" so that n_i converging to N_i is equivalent to n_i converging to ∞ . For the geomean index I expand \hat{R}_{Git} around μ_{Git} to get¹⁰:

$$\hat{R}_{Git} - \exp(\mu_{Git}) = \exp(\mu_{Git}) \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right) + O_p(n_i^{-1}). \quad (8)$$

Since by assumption there exists a finite variance for $\ln(p_{ijt}/p_{ijt-1})$, which is denoted σ_{Git}^2 , and with the i.i.d. sample, one uses the Lindeberg-Levy Central Limit Theorem, to conclude that $\sqrt{n_i}(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git}) \xrightarrow{d} N(0, \sigma_{Git}^2)$. One can then conclude

$$\sqrt{n_i}(\hat{R}_{Git} - \exp(\mu_{Git})) \xrightarrow{d} N(0, \sigma_{Git}^2 \exp(2\mu_{Git})).$$

⁹ p_{ijs} and p_{ijt} can still be serially dependent, but there need to be independence among the draws within an item area and within a period of time.

¹⁰ Here $a_n = O_p(n^x)$ means that there are finite constants K , ε and N such that $\Pr(|n^{-x}a_n| > K) < \varepsilon$ for all $n > N$.

For the Laspeyres, I get the first order approximation

$$\begin{aligned}\widehat{R}_{Lit} - \mu_{Lit} &= \frac{1}{E(p_{ijt-1})} \left[\sum_{j=1}^{n_i} p_{ijt}/n_i - E(p_{ijt}) \right] \\ &\quad - \frac{E(p_{ijt})}{E(p_{ijt-1})^2} \left[\sum_{j=1}^{n_i} p_{ijt-1}/n_i - E(p_{ijt-1}) \right] + O_p(n_i^{-1}).\end{aligned}\tag{9}$$

Denoting $\sigma_{it-1}^2 = Var(p_{ijt-1})$, $\sigma_{it}^2 = Var(p_{ijt})$, and $\sigma_{it,t-1} = Cov(p_{ijt}, p_{ijt-1})$,

I get

$$\begin{aligned}\sqrt{n_i}(\widehat{R}_{Lit} - \mu_{Lit}) &\rightarrow N(0, \sigma_{Lit}^2) \\ \sigma_{Lit}^2 &= \frac{\sigma_{it}^2}{E(p_{ijt-1})^2} - 2 \frac{E(p_{ijt})\sigma_{it,t-1}}{E(p_{ijt-1})^3} + \frac{E(p_{ijt})^2 \sigma_{it-1}^2}{E(p_{ijt-1})^4}.\end{aligned}$$

Using the same approach for the housing Laspeyres, and denoting $\sigma_{it,t-6} = Cov(p_{ijt}, p_{ijt-6})$, I get the following $\sqrt{n_i}$ result:

$$\begin{aligned}\sqrt{n_i}(\widehat{R}_{Hit} - \mu_{Hit}) &\rightarrow N(0, \sigma_{Hit}^2) \\ \sigma_{Hit}^2 &= \frac{1}{36} \left[\frac{\sigma_{it}^2}{E(p_{ijt})^{5/3} E(p_{ijt-6})^{1/3}} - 2 \frac{E(p_{ijt})^{1/3} \sigma_{it,t-6}}{E(p_{ijt-6})^{4/3}} \right. \\ &\quad \left. + \frac{E(p_{ijt})^{1/3} \sigma_{it-6}^2}{E(p_{ijt-6})^{7/3}} \right].\end{aligned}$$

While first order asymptotic theory holds for estimated price indexes, in many cases n_i is small so that these asymptotic conditions are not attained. In the case of the first order expansion of (8), the small n_i induces the expected value of the $O_p(n^{-1})$ term to be strictly positive because the $\exp()$ function is strictly convex in its argument, and Jensen's Inequality applies. The Laspeyres and the Housing relatives are not strictly convex in their arguments and therefore the finite sample bias is not systematically positive or negative.

2.3 An Adjustment Using the Second Order Expansion of the Relative

The problem facing BLS is that the sample sizes for many item areas are smaller than 5 observations and therefore the asymptotics outlined in Section 2.2 do not hold. I propose an analytical bias correction of the relative by estimating the term in the second order expansion of the relative, and using this estimate to adjust the sample relative.

For the geometric mean relative the second order expansion of \hat{R}_{Git} around μ_{Git} is:

$$\begin{aligned} \hat{R}_{Git} - \exp(\mu_{Git}) &= \exp(\mu_{Git}) \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right) + \\ & (1/2) \exp(\mu_{Git}) \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right)^2 + O_p(n_i^{-3/2}). \end{aligned} \quad (10)$$

Passing the expectations operator through this expression gets:

$$\begin{aligned} B_{Git} &= E(\hat{R}_{Git} - \exp(\mu_{Git})) \\ &= .5 \exp(\mu_{Git}) E \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right)^2 + O(n_i^{-3/2}). \end{aligned} \quad (11)$$

Let $\sigma_{Git}^2 = E(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git})^2$, and $\hat{\mu}_{Git} = \sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i$.

I can get a sample estimate of σ_{Git}^2 from the existing data as

$$\hat{\sigma}_{Git}^2 = \frac{1}{n_i(n_i - 1)} \sum_{j=1}^{n_i} (\ln(p_{ijt}/p_{ijt-1}) - \hat{\mu}_{Git})^2. \quad (12)$$

The sample estimate of geometric mean bias correction is

$$\hat{B}_{Git} = (1/2) \hat{\sigma}_{Git}^2 \exp(\hat{\mu}_{Git}), \quad (13)$$

and the bias adjusted geometric mean relative is $\hat{R}_{Git} - \hat{B}_{Git}$. In the appendix,

I show

Proposition 1 *If i) $\ln(p_{ijt})$ is i.d.d. across j , and ii) $E(\ln(p_{ijt})^4) < \infty$, then there is a finite γ_{Git}^2 such that $n_i^{3/2}(\hat{B}_{Git} - B_{Git}) \rightarrow N(0, \gamma_{Git}^2)$ where B_{Git} is the true finite sample bias as defined in (11).¹¹*

For the geomean index, it follows that $\hat{B}_{Git} > 0$ for all $\hat{\sigma}_{Git}^2 > 0$. Proposition 1 says something about the precision of the sample estimate of the finite sample bias. It converges more rapidly to its true value, B_{Git} , than the sample relative \hat{R}_{Git} converges to $\exp(\mu_{Git})$. In this sense, \hat{B}_{Git} is a more precise estimator of B_{Git} , than \hat{R}_{Git} is for $\exp(\mu_{Git})$. This is to be expected since the true bias B_{Git} converges to zero more rapidly than \hat{R}_{Git} converges to its population value. To understand the full effect of the bias adjusted relative, $\hat{\hat{R}}_{Git} = \hat{R}_{Git} - \hat{B}_{Git}$, I make the following decomposition:

$$\begin{aligned} \sqrt{n_i}(\hat{\hat{R}}_{Git} - \exp(\mu_{Git})) &= \sqrt{n_i}(\hat{R}_{Git} - \exp(\mu_{Git})) - \sqrt{n_i}(\hat{B}_{Git} - B_{Git}) - \sqrt{n_i}B_{Git} \\ &= O_p(1) - O_p(n^{-1}) - O_p(n^{-1}). \end{aligned} \tag{14}$$

This shows that both $\hat{\hat{R}}_{Git}$ and \hat{R}_{Git} have the same limiting distribution, and this is what we want since the asymptotic distribution of the finite sample bias, B_{Git} , degenerates to zero. However, for “small” n_i $\hat{\hat{R}}_{Git}$ and \hat{R}_{Git} have different means because both $E(\hat{B}_{Git})$ and $E(B_{Git})$ are strictly greater than zero. Additionally, \hat{B}_{Git} is a more precise estimator of B_{Git} than \hat{R}_{Git} is for $\exp(\mu_{Git})$ in terms of n -convergence. Therefore, for finite samples $\hat{\hat{R}}_{Git}$ is a more precise estimator for $\exp(\mu_{Git})$ than \hat{R}_{Git} . The first term on the right hand side of (14) shows that adjusting the relative by \hat{B}_{Git} will not purge the variance in

¹¹Since we must prove a convergence of $\hat{\sigma}_{Git}^2$ to σ_{Git}^2 , I need the additional assumption of a bounded fourth moment in prices.

the relative that comes from sampling error. Increasing the sample size, n_i will both decrease the variance and the bias due to sampling error. This is what makes bias adjustment an inferior solution to increasing sample size.

For the Laspeyres relative I get the second order expansion

$$\begin{aligned} \widehat{R}_{Lit} - \mu_{Lit} = & \frac{1}{E(p_{ijt-1})} \left[\sum_{j=1}^{n_i} p_{ijt}/n_i - E(p_{ijt}) \right] - \frac{E(p_{ijt})}{E(p_{ijt-1})^2} \left[\sum_{j=1}^{n_i} p_{ijt-1}/n_i - E(p_{ijt-1}) \right] \\ & - \frac{\left[\sum_{j=1}^{n_i} p_{ijt}/n_i - E(p_{ijt}) \right] \left[\sum_{j=1}^{n_i} p_{ijt-1}/n_i - E(p_{ijt-1}) \right]}{E(p_{ijt-1})^2} \\ & + \frac{E(p_{ijt}) \left[\sum_{j=1}^{n_i} p_{ijt-1}/n_i - E(p_{ijt-1}) \right]^2}{E(p_{ijt-1})^3} + O_p(n_i^{-3/2}). \end{aligned}$$

Denoting $\sigma_{it-1}^2 = Var(p_{ijt-1})$, $\sigma_{it,t-1} = Cov(p_{ijt}, p_{ijt-1})$, and passing the expectations operator through this expression yields:

$$\begin{aligned} B_{Lit} &= E(\widehat{R}_{Lit} - \mu_{Lit}) = \frac{E(p_{ijt})\sigma_{it-1}^2/n_i}{E(p_{ijt-1})^3} - \frac{\sigma_{it,t-1}/n_i}{E(p_{ijt-1})^2} + O_p(n_i^{-3/2}) \approx \\ & \frac{E(p_{ijt})}{E(p_{ijt-1})} \left[cv(p_{ijt-1})^2 - \rho_{it,t-1} cv(p_{ijt}) cv(p_{ijt-1}) \right], \end{aligned} \quad (15)$$

where $\rho_{it,t-1}$ is the correlation between p_{ijt} and p_{ijt-1} , and $cv(p)$ is the coefficient of variation of the random variable p . Using sample moments I get estimates for σ_{it-1}^2 , $\sigma_{it,t-1}$, $E(p_{ijt-1})$, and $E(p_{ijt})$, where

$$\begin{aligned} \widehat{p}_{it} &= \sum_{j=1}^{n_i} p_{ijt}/n_i, \\ \widehat{p}_{it-1} &= \sum_{j=1}^{n_i} p_{ijt-1}/n_i, \\ \widehat{\sigma}_{it,t-1} &= \frac{\sum_{j=1}^{n_i} (p_{ijt} - \widehat{p}_{it})(p_{ijt-1} - \widehat{p}_{it-1})}{n_i - 1}, \end{aligned}$$

and

$$\widehat{\sigma}_{i,t-1} = \frac{\sum_{j=1}^{n_i} (p_{ijt-1} - \widehat{p}_{it-1})^2}{n_i - 1}.$$

Let the Laspeyres bias correction be approximated as:

$$\hat{B}_{Lit} = \frac{\hat{p}_{it}\hat{\sigma}_{it-1}^2/n_i}{\hat{p}_{it-1}^3} - \frac{\hat{\sigma}_{it,t-1}/n_i}{\hat{p}_{it}^2}. \quad (16)$$

The bias adjusted Laspeyres is then $\hat{R}_{Lit} - \hat{B}_{Lit}$. I conclude:

Proposition 2 *If i) p_{ijt} is i.d.d. across j , and ii) $E(p_{ijt}^4) < \infty$, there is a finite γ_{Lit}^2 such that $n_i^{3/2}(\hat{B}_{Lit} - B_{Lit}) \rightarrow N(0, \gamma_{Lit}^2)$.*

I can go through the same second order expansion for the housing relative and I get the following approximation of bias correction

$$\hat{B}_{Hit} = (1/2) \left(\frac{\hat{p}_{it}}{\hat{p}_{it-6}} \right)^{1/6} \left(\frac{7}{36} \frac{\hat{\sigma}_{it-6}^2/n_i}{\hat{p}_{it-6}^2} - \frac{2}{36} \frac{\hat{\sigma}_{it,t-6}/n_i}{\hat{p}_{it-6}\hat{p}_{it}} - \frac{5}{36} \frac{\hat{\sigma}_{it}^2/n_i}{\hat{p}_{it}^2} \right). \quad (17)$$

2.4 Effects on the All-Items Index

The month to month “All-Items” CPI-U based on the BLS samples is:

$$\hat{R}_t = \sum_{i \in G} \hat{R}_{Git} \hat{w}_{Git-1} + \sum_{i \in L} \hat{R}_{Lit} \hat{w}_{Lit-1} + \sum_{i \in H} \hat{R}_{Hit} \hat{w}_{Hit-1} \quad (18)$$

where $\sum_{i \in G}$, $\sum_{i \in L}$, and $\sum_{i \in H}$ denote the sum of Geomean, Laspeyres, and Housing relatives respectively, and \hat{w}_{Git-1} , \hat{w}_{Lit-1} , and \hat{w}_{Hit-1} are expenditure weights for period $t-1$.¹² Define the “true population” CPI-U as

$$R_t = \sum_{i \in G} R_{Git} \hat{w}_{Git-1} + \sum_{i \in L} R_{Lit} \hat{w}_{Lit-1} + \sum_{i \in H} R_{Hit} \hat{w}_{Hit-1} \quad (19)$$

where R_{Mit} is the asymptotic relative of method $M = \{G, L, H\}$. If all the relatives were geomean (i.e., $L = H = \emptyset$), then we get the unambiguous result

¹² A Laspeyres Index can be written in the form $\sum_i \frac{p_{it}}{p_{it-1}} w_{it-1}$ where w_{it-1} is the period $t-1$ expenditure share for good i . \hat{R}_{Git} , \hat{R}_{Lit} , and \hat{R}_{Hit} are proxies for $\frac{p_{it}}{p_{it-1}}$ and the $\hat{w}'s$ are estimates of w_{it-1} .

that the “All-Items” CPI-U has positive bias ($E(\hat{R}_t - R_t) > 0$). Let N be the total number of item-area cells; if all the relatives are computed by the geomeans and the variance of the sample means for each relative is greater than zero, we would get

$$\text{plim}_{N \rightarrow \infty} \hat{R}_t - R_t > 0. \quad (20)$$

However, since some of the relatives are not geomean we cannot get this unambiguous result. The bias adjusted “All-Items” index is:

$$\hat{\hat{R}}_t = \sum_{i \in G} (\hat{R}_{Git} - \hat{B}_{Git}) \hat{w}_{Git-1} + \sum_{i \in L} (\hat{R}_{Lit} - \hat{B}_{Lit}) \hat{w}_{Lit-1} + \sum_{i \in H} (\hat{R}_{Hit} - \hat{B}_{Hit}) \hat{w}_{Hit-1}. \quad (21)$$

In the previous section, I showed that while both \hat{R}_{Mit} and $\hat{\hat{R}}_{Mit} = \hat{R}_{Mit} - \hat{B}_{Mit}$ have the same asymptotic properties, for finite samples $E(\hat{\hat{R}}_{Mit})$ is a more precise estimator for R_{Mit} .¹³

One might conclude that one needs to make the same bias adjustment “plug in” for a superlative index such as a Törnqvist. But this is not correct.¹⁴ To see this, I start with a simplified Törnqvist functional form

$$T_t = \prod_{k=1}^N (R_{kt})^{w_{kt}}, \quad (22)$$

where R_{kt} is the true price relative between periods t and $t - 1$ for item-area k , and w_{kt} is its expenditure share weight which is a simple average of expenditure shares in period t and $t - 1$. Suppose that T_t is estimated by

$$\hat{T}_t = \prod_{k=1}^N (\hat{R}_{kt})^{w_{kt}} \quad (23)$$

¹³Notice that it is $E(\hat{\hat{R}}_{Mit})$ that is more precise. $\hat{\hat{R}}_{Mit}$ itself is calculated with sample moments and therefore, is subject to sampling error.

¹⁴This is proved in great detail in Bradley (2001), I describe the results again here.

and for simplicity assume that all \hat{R}_{kt} are estimated by geomean indexes with a fixed sample size n that is sufficiently small so that $Var(\hat{R}_{kt}) > 0$, for all k . Then

$$E[\ln(\hat{R}_{kt}) - \ln(R_{kt})] = 0. \quad (24)$$

even though $E[\hat{R}_{kt} - R_{kt}] > 0$. \hat{T}_t can be rewritten as

$$\hat{T}_t = \exp\left\{\sum_{k=1}^N w_{kt} \ln(\hat{R}_{kt})\right\}$$

Since $w_{kt} = O_p(N^{-1})$ and $\ln(\hat{R}_{kt}) = O_p(n^{-1})$, and given condition (24), I get that for a fixed n ,

$$\text{plim}_{N \rightarrow \infty} \left\{ \sum_{k=1}^N w_{kt} \ln(\hat{R}_{kt}) \right\} = E\left\{ \sum_{k=1}^N w_{kt} \ln(\hat{R}_{kt}) \right\} = \sum_{k=1}^N w_{kt} \ln(R_{kt}). \quad (25)$$

I use the following lemma:

Lemma 3 *If $\text{plim}_{N \rightarrow \infty} \hat{\theta} = \theta$, then $\text{plim}_{N \rightarrow \infty} \exp(\hat{\theta}) = \exp(\theta)$. (The proof for a more general function can be found in Amemiya, 1985, pages 112-113.)*

Letting $\hat{\theta} = \sum_{k=1}^N w_{kt} \ln(\hat{R}_{kt})$ and $\theta = \sum_{k=1}^N w_{kt} \ln(R_{kt})$, I conclude that

$$\text{plim}_{N \rightarrow \infty} \hat{T}_t - T_t = 0, \quad (26)$$

even though for the Laspeyres index $\text{plim}_{N \rightarrow \infty} (\hat{R}_t - R_t) > 0$ when n is fixed and all the relatives are geomean. BLS does publish a ‘‘Törnqvist Type’’ index which is labeled the CPI-C. For the CPI-U and the CPI-C, N is 8,018. This is an adequately large size to assume that the asymptotic properties of a Törnqvist are satisfied. However, notice that if I ‘‘plug in’’ $\hat{R}_{kt} - \hat{B}_{kt}$ into a Törnqvist then

$$E[\ln(\hat{R}_{kt} - \hat{B}_{kt}) - \ln(R_{kt})] < 0.$$

and (24) and (25) no longer hold, and therefore the condition for Lemma 3 no longer holds. Plugging in a bias adjustment to the Törnqvist will make it downwardly biased, and this bias will persist as $N \rightarrow \infty$. Notice that the CPI-U cannot be written as a continuous function of $\sum_{k=1}^N w_{kt} \ln(\hat{R}_{kt})$, therefore the condition of Lemma 3 does not apply to the CPI-U.

The CPI-C is a Törnqvist type index, but some of the relatives are not geomeans. However, in the empirical section of this paper, I show that 96% of the finite sample bias in the “All-items” CPI-U can be attributed to the geomean indexes. Thus, plugging bias adjustments into the CPI-C will most likely induce a negative bias.¹⁵

2.5 The Additive Decomposition of the Indexes and Bias Adjustments

The “all-items” CPI-U is the upper chained index:

$$\hat{I}_t = \left(\sum_{i \in G} \hat{R}_{Git} \hat{w}_{Git-1} + \sum_{i \in L} \hat{R}_{Lit} \hat{w}_{Lit-1} + \sum_{i \in H} \hat{R}_{Hit} \hat{w}_{Hit-1} \right) \hat{I}_{t-1}. \quad (27)$$

After computing the bias adjustment, I calculate the bias-adjusted CPI-U index as

$$\hat{\hat{I}}_t = \left(\sum_{i \in G} (\hat{R}_{Git} - \hat{B}_{Git}) \hat{w}_{Git-1} + \sum_{i \in L} (\hat{R}_{Lit} - \hat{B}_{Lit}) \hat{w}_{Lit-1} + \sum_{i \in H} (\hat{R}_{Hit} - \hat{B}_{Hit}) \hat{w}_{Hit-1} \right) \hat{\hat{I}}_{t-1}. \quad (28)$$

¹⁵If the weights in the Törnqvist are constant over time, then it is a geometric mean index, which implies that separability conditions allow the index to be decomposed into groups of item-areas. For a set of item-areas S , the group index is $\hat{T}_{St} = \exp\{\sum_{k \in S} (w_k \ln \hat{R}_{kt}) / \sum_{k \in S} w_k\}$. Let N_S be the number of item-areas in S . For a fixed n , $\text{plim}_{N_S \rightarrow \infty} (\hat{T}_{St} - T_{St})$; however, for some groups such as apparel and food N_s is still not sufficiently large to achieve its asymptotic properties. So far, there is no evidence that the expenditure weights are constant. Therefore, the Törnqvist cannot be decomposed into sub-indexes. If different groups have difference variances in prices and different sample sizes, then they will be at different stages of their asymptotic convergence. Since $N_s < N$, it may still be necessary to bias adjust some of the groups.

For both \widehat{I}_t and $\widehat{\bar{I}}_t$, I set the index at $t = \text{December, 1998}$ equal to 1. The final period denoted as T is December, 2003. Notice in (28) that \widehat{B}_{Git} , \widehat{B}_{Hit} , and \widehat{B}_{Lit} have unit roots that keep them in the index indefinitely, and they compound.

From here on, I drop the G, L , and H subscripts so that the relative and the bias correction are now \widehat{R}_{it} and \widehat{B}_{it} . There are 8 major groups in the “All Items” CPI-U. Each group is a set of similar items. For example, the food group includes the banana, meat, cereal, and dairy items. Here is the listing of the 8 BLS groups with the designated letter in parentheses denoting the set of all items within the group:

- Food and beverages (F)
- Housing (H)
- Apparel (A)
- Transportation (T)
- Medical care (M)
- Recreation (R)
- Education and communication (E)
- Other goods and services (G)

Each major group contains many items. For instance food contains, apples, cereal, coffee, etc. I denote $\mathbf{G} = \{F, H, A, T, M, R, E, G\}$ as the set of groups. Since different groups have different price distributions and sample sizes, it is useful to disaggregate both the indexes and the bias adjustments to determine how each group contributes to these statistics. Since the indexes in (27) and (28) are chained, additive decomposition of the “All-Items” CPI-U bias adjustment into the 8 groups is not straightforward. To find the percent contribution of each

group to the average monthly indexes and bias adjustment over the T periods,

I need the following proposition proven in the appendix:

Proposition 4 *Monthly average growth respectively in the CPI-U and the bias adjusted CPI-U is $(\widehat{I}_T)^{1/T} - 1$ and $(\widehat{\widehat{I}}_T)^{1/T} - 1$. The following identity holds*

$$\widehat{I}_T^{1/T} - \widehat{\widehat{I}}_T^{1/T} = \sum_{t=1}^T \left[\left(\frac{\widehat{R}_t m(\widehat{R}_t, \widehat{I}_T^{1/T})}{\sum_{t=1}^T m(\widehat{R}_t, \widehat{I}_T^{1/T})} - \frac{\widehat{\widehat{R}}_t m(\widehat{\widehat{R}}_t, \widehat{\widehat{I}}_T^{1/T})}{\sum_{t=1}^T m(\widehat{\widehat{R}}_t, \widehat{\widehat{I}}_T^{1/T})} \right) \left(\frac{\sum_i \widehat{w}_{it-1} \widehat{B}_{it}}{\widehat{R}_t - \widehat{\widehat{R}}_t} \right) \right].$$

where $m(a, b) = (\ln(a) - \ln(b))/(a - b)$, $\widehat{R}_t = \left(\sum_i \widehat{R}_{it} \widehat{w}_{it-1} \right)$, and $\widehat{\widehat{R}}_t = \left(\sum_i (\widehat{R}_{it} - \widehat{B}_{it}) \widehat{w}_{it-1} \right)$.

As a corollary, I can get

Corollary 5 *The contribution to $\widehat{I}_T^{1/T} - \widehat{\widehat{I}}_T^{1/T}$ by a group $J \in \mathbf{G}$ is*

$$\sum_{t=1}^T \left[\left(\frac{\widehat{R}_t m(\widehat{R}_t, \widehat{I}_T^{1/T})}{\sum_{t=1}^T m(\widehat{R}_t, \widehat{I}_T^{1/T})} - \frac{\widehat{\widehat{R}}_t m(\widehat{\widehat{R}}_t, \widehat{\widehat{I}}_T^{1/T})}{\sum_{t=1}^T m(\widehat{\widehat{R}}_t, \widehat{\widehat{I}}_T^{1/T})} \right) \left(\frac{\sum_{i \in J} \widehat{w}_{it-1} \widehat{B}_{it}}{\widehat{R}_t - \widehat{\widehat{R}}_t} \right) \right].$$

This allows me to additively decompose the total bias adjustment, $\widehat{I}_T^{1/T} - \widehat{\widehat{I}}_T^{1/T}$, into the conttibution that is made by each group..

3 Monte Carlo Simulation

Section 2 shows that for the fixed-basket CPI-U, the bias adjusted relative is a more precise estimator of the true relative. To correct for finite sample bias, sample estimates of higher moments are used as proxies of their population counterparts. The small sample sizes that induce bias will also introduce sampling error in the moment estimators and this in turn will add sampling error

to bias adjustments. Is it possible that sampling error makes the second order approximation of a bias adjustment too imprecise? Propositions 1 and 2 claim the opposite. To both verify the properties established in Section 2 and to address the issue of sampling error effects on the bias adjust, I conduct a Monte Carlo Experiment.

The geomean, the Laspeyres, and the Housing relatives have respectively 71%, 11%, and 18% of the expenditure weight in the “All-Items” CPI-U. I take a random samples of prices for twelve time periods ($t = 1, 2, \dots, 12$). Historically the average sample sizes for geometric mean, Laspeyres, and Housing relatives are 10, 12, and 40, respectively. Therefore, in each of the twelve time periods, I draw one sample of size 10 to compute a geomean relative, another of size 12 to compute a Laspeyres relative, and a final one of size 40 to compute a Housing relative. I sample log prices from a $N(.002463233t, 0.0025)$ distribution so that the compounded twelve month price growth is 3%. The true population relative for each period is then $\exp(.002463233)$. I repeat this process 2,000 times and this repetition is indexed as r . Let $\hat{R}_{M,t,r}$ be the relative computed from the M^{th} method ($M = \text{geomean, Laspeyres, Housing}$), and let $\hat{B}_{M,t,r}$ be its corresponding bias adjustment computed from the sample moments.

Table 1a reports the average of $\hat{R}_{M,t,r} - \exp(.002463233)$, $\hat{B}_{M,t,r}$, and $\hat{\hat{R}}_{M,t,r} - \exp(.002463233)$ in percentage terms ($\hat{\hat{R}}_{M,t,r} = \hat{R}_{M,t,r} - \hat{B}_{M,t,r}$). Even though each of the methods face the same data generating process, the geomean has the largest bias. This cannot be explained entirely by sample size because the sample size of the Laspeyres has only two more observations, but the average

bias is 35% less than the geomean. For the Housing index, there is almost no bias. Therefore, the difference in functional form between the geomean and the Laspeyres plays a role in finite sample bias. This gives evidence that when BLS changed from a Laspeyres relative to a geomean, finite sample bias became a greater problem.

The geomean bias adjustment reduces the bias by 83%. The Laspeyres slightly overadjusts, and the Housing correction is ineffective. Because there was so little bias in the Housing relative, I decided to re-investigate the Housing adjustment by conducting another simulation where the sample size was 10 and the variance of the log of prices was increased to .64. I show the results in the row entitled “Alternative Housing.” Here the bias correction does perform better, but not as well as the geomean adjustment.

To investigate the impact of sampling error, Table 1b shows the performance of the estimated standard deviations of the log price for the geomeans and the price for both the Laspeyres and Housing. Since log price is drawn from a $N(.002463233t, 0.0025)$, the true standard deviation of the log of prices is $\sqrt{0.0025} = .05$ and for prices it is $\{\exp(.00492t + 0.0050) - \exp(.00492t + 0.0025)\}^{1/2}$. Table 1b reports the results for the differences between the simulated standard deviations and the true standard deviations. As expected, the column labelled “Mean Difference” shows that the sample estimate is unbiased; however, the last three columns show that sampling error exists. In other words, on average we get the population standard deviation; however, for a particular sample there will be some error. For the geomean, over 95% of the draws produce stan-

dard deviations in the interval (.045,.055) where the true standard deviation is .05, and although this does not allow for perfect bias adjustment, the interval is narrow enough to induce an improvement. Additionally, this gives evidence that the expected bias adjusted relative is more a precise estimator of the true relative than the unadjusted relative.

Finally, for each of the 2,000 repetitions, I compute for each time period an “All-Items” index by expenditure weighted sum where the geomeans relative is given a 71% weight, the Laspeyres is given an 11% weight, and the Housing a 18% weight. I do the same for the bias adjusted relative. I then compute a “yearly” index from the twelve month “All-items” index. The average difference between the unadjusted yearly index and the true index is .34%, while the average difference between the bias adjusted index and the true index is .09%.

This experiment provides evidence that bias adjustment based on sample estimates of second order approximations does not completely remove the bias. Sampling error is still a problem. However, this bias adjustment does greatly lower the bias. It is important to note that the sampling error from the small samples will continue to be a problem under bias adjustment. This should confirm that the best alternative to mitigate finite sample bias is to increase sample size. If this is not feasible then finite sample bias adjustment based on higher sample moments is a “second best” alternative.

4 Results for the CPI-U

I compute both the CPI-U (27) and the bias adjusted CPI-U (28). Table 2 contains the annual and cumulative results over the 60 month period from December 1998 to December 2003. I also list the results for BLS's recently published "Törnqvist type" index, the CPI-C. The column labelled "(CPI-U - CPI-C)" gives the difference between the published CPI-U and the "Törnqvist type" index. This difference fluctuates positively with the underlying inflation rate and ranges from 0.28% to 0.79%. The next columns decompose this difference between the difference of the CPI-U and the bias adjusted index, and the difference between the bias adjusted index and the CPI-C. The first difference can be attributable to finite sample bias and the second difference may be attributable to commodity substitution bias. Over the five year period, on average 62.5% of the difference between the published CPI-U and the CPI-C can be attributed to finite sample bias and the rest to commodity substitution bias. This table also lists correlations between the bias adjustment and key sample variables. Since the geomean bias adjustment is a function of (a) the sample variance of the difference in log prices and (b) sample size, I show the correlation for those key sample variables. The Laspeyres and Housing bias adjustments are based on the sample variance of prices - not the variance of the difference of log prices. Therefore, I list those correlations. Notice that for the Laspeyres and Housing adjustments the correlations with the variances of the base period prices are larger than the current period variances. Additionally, the correlations between the adjustments and the variances are weaker for the Laspeyres. If one looks

at the bias adjustments in (16) and (17), this is to be expected. The reason is that $\hat{\sigma}_{it-1}^2$ and \hat{p}_{it-1}^3 are highly correlated. Thus a large $\hat{\sigma}_{it-1}^2$ is offset by a large \hat{p}_{it-1}^3 .

Table 3 gives the additive decompositions based on the method in Section 2.5. Table 3a lists the group contribution respectively to the CPI-U, the bias adjusted CPI, and the bias adjustment. Notice that the group contribution to each of the indexes is the same. This occurs because there is very little difference generally between the unadjusted and the adjusted relative. The round off error hides the slight difference. Although food and apparel contribute only 20.5% to the indexes, they contribute 65.8% to the bias. The reason is that both food and apparel have unusually volatile prices partly due to the frequency of sales. Table 3b lists the contribution of the different methods.¹⁶ From the result of the Monte Carlo simulation, it is not surprising to find that 96% of the bias comes from the relatives based on the Geomean method. Combining this result with the theoretical results in Section 2.4 gives evidence that “plugging in” bias adjusted relatives into the CPI-C would induce additional bias. Table 3c breaks down the “All-Items” index into the Core and Non Core parts. The Core includes all items except non-alcohol related food and beverages and energy items such as motor and heating fuel. The reason for the Core index is to remove items that are highly volatile over time but not necessarily volatile within an item-area. However, the volatility here is volatility over time rather than within an item area. The variances of the sample means for log prices for energy items

¹⁶Please note that the Housing Group index is different from the Housing Relative method. The Housing group includes home heating fuel, cleaning supplies, and various services. These items are not computed with the Housing method.

are smaller because their sample sizes are larger. The average sample size for an energy item-area is 28, almost three times the sample size of a typical geomean index. While energy prices fluctuate widely over time, there is relatively little price variation within time. Since the food part of the Non Core excludes alcoholic beverages, there is a slight difference between the food results in Table 3a and Table 3c.

There is a large difference between a group's contribution to the index and its contribution to the bias. The reason is that the bias adjustment varies widely by group. If one looks at Table 4, the bias adjustment for apparel is the largest while its share of the index is small. On the other hand, the bias adjustment for housing is smaller than average while its share of the index is large. The food group is the largest contributor to the bias adjustment. Although its average bias adjustment is less than apparel's, it has a higher expenditure share. Both the apparel and food group relatives are computed with geomean indexes. Table 5 gives a yearly breakdown by Core and Non Core Items. This shows the high "over time" volatility of energy, which contrasts with the relatively small "within time" variability exhibited in Table 3c.

It is important to remember that the adjusted indexes from this historical data do not completely eliminate finite sample bias. They are imperfect adjustments that bring us "closer" to the true "fixed basket" index. One should not read Table 2 and conclude "The annual finite sample bias for the CPI-U is 0.27% on average." This is too strong a conclusion, and the factors that influence finite sample bias can change in the future. However, we do have two major

results. First, the currently published CPI-U is an upwardly biased estimate of the true “fixed basket” index. Second, if budget constraints do not allow BLS to increase item-area sample sizes adequately enough for the asymptotic properties to be realized, then the bias adjustments in this study represent a “second best” solution.

5 Conclusions

The currently published CPI-U is an upwardly biased estimate of a “fixed basket” price index. Therefore the difference between the CPI-U and a superlative index cannot be entirely attributed to commodity substitution bias, as previous studies have done. The CPI-U’s finite sample bias can be reduced by using the same price quotes that are used to initially generate the index. Therefore unlike correcting for commodity substitution bias, no additional data is needed.

On a year to year basis, it is not possible to predict the reduction in the CPI-U if it is adjusted for finite sample bias, but in the five years of this study, the CPI-U is reduced by .27% on average. However, the bias adjustments are unpredictable, since they are based on the variance of prices within a cell, and these variances change unpredictably over time.

Analytical bias reduction is not the only method to adjust for finite sample bias but it requires less computation than bootstrapping, and it is less expensive than expanding sample sizes. However, if analytical bias reduction were implemented the final relatives that are used in the CPI-U would differ from those used in the CPI-C.

Appendix

Proof of Proposition 1

Proof of Proposition 1: From (10) the following holds

$$\begin{aligned}\widehat{R}_{Git} - \exp(\mu_{Git}) &= \exp(\mu_{Git}) \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right) - \\ &= (1/2) \exp(\mu_{Git}) \left(\sum_{j=1}^{n_i} \ln(p_{ijt}/p_{ijt-1})/n_i - \mu_{Git} \right)^2 = O_p(n_i^{-3/2}).\end{aligned}$$

Passing through the expectations operator, I get

$$B_{Git} = E[\widehat{R}_{Git} - \exp(\mu_{Git})] = (1/2) \exp(\mu_{Git}) \sigma_{Git}^2 / n_i + O_p(n_i^{-3/2}).$$

I wish to show that

$$\widehat{B}_{Git} - B_{Git} = O_p(n_i^{-3/2}),$$

where $\widehat{B}_{Git} = (1/2) \widehat{\sigma}_{Git}^2 \exp(\widehat{\mu}_{Git})$, and thus showing that $n_i^{3/2}(\widehat{B}_{Git} - B_{Git}) \xrightarrow{d} N(0, \gamma_{Git}^2)$. By the triangle inequality

$$|\widehat{B}_{Git} - B_{Git}| \leq |\widehat{B}_{Git} - (1/2) \exp(\mu_{Git}) \sigma_{Git}^2 / n_i| + |O(n_i^{-3/2})|.$$

I need to only show that $|\widehat{B}_{Git} - \exp(\mu_{Git}) \sigma_{Git}^2 / n_i|$ is $O_p(n_i^{-3/2})$.

$$\begin{aligned}\widehat{B}_{Git} - \exp(\mu_{Git}) \sigma_{Git}^2 / n_i &= \\ &= (1/2) \exp(\widehat{\mu}_{Git}) \widehat{\sigma}_{Git}^2 - (1/2) \exp(\mu_{Git}) \sigma_{Git}^2 / n_i = \\ &= \frac{1}{2n_i(n_i - 1)} \sum_{j=1}^{n_i} (\ln(p_{ijt}/p_{ijt-1}) - \widehat{\mu}_{Git})^2 \exp(\widehat{\mu}_{Git}) - (1/2) \exp(\mu_{Git}) \sigma_{Git}^2 / n_i = \\ &= \frac{1}{2n_i} \left\{ \frac{\sum_{j=1}^{n_i} (\ln(p_{ijt}/p_{ijt-1}) - \widehat{\mu}_{Git})^2}{n_i - 1} \exp(\widehat{\mu}_{Git}) - \exp(\mu_{Git}) \sigma_{Git}^2 \right\} \\ &= \frac{1}{2n_i} A(n_i).\end{aligned}$$

I need to show that $A(n_i) = O_p(n_i^{1/2})$. I do this by taking a first order expansion of $A(n_i)$ around μ_{Git} and σ_{Git}^2 .

$$A(n_i) = \exp(\mu_{Git})(\hat{\sigma}_{Git}^2 - \sigma_{Git}^2) + \sigma_{Git}^2 \exp(\mu_{Git})(\hat{\mu}_{Git} - \mu_{Git}) + O_p(n^{-1}).$$

$E(A_n) = O(n^{-1})$. Using the condition (i) that $|E(\ln(p_{ijt}^4))| < \infty$, $(\hat{\sigma}_{Git}^2 - \sigma_{Git}^2) = O_p(n_i^{-1/2})$ and $(\hat{\mu}_{Git} - \mu_{Git}) = O_p(n_i^{-1/2})$. This establishes that $|A(n_i)| = O_p(n_i^{1/2})$. Since $\text{var}(\hat{\sigma}_{Git}^2)$ and $\text{var}(\hat{\mu}_{Git})$ are bounded then $\text{var}(n^{3/2}(\hat{B}_{Git} - B_{Git}))$ is bounded and since p_{Git} is i.i.d., I can use the Levy-Lindberg Central Limit Theorem to conclude that there is a γ_{Git}^2 such that

$$n^{3/2}(\hat{B}_{Git} - B_{Git})) \xrightarrow{d} N(0, \gamma_{Git}^2).$$

The proof of propositions 2 and 3 follow the same process.

Proof Proposition 4

From Diewert, Ehemann, and Reinsdorf (2000), it is shown that the index

$$P_G = \prod_{i=1}^n (p_{it}/p_{it-1})^{w_i}$$

has the additive decomposition

$$P_G = \frac{\sum_{i=1}^n w_i p_{it}/m(p_{it}, P_G p_{it-1})}{\sum_{i=1}^n w_i p_{it-1}/m(p_{it}, P_G p_{it-1})}.$$

I can then use this to additively decompose both $\hat{I}_T^{1/T}$ and $\hat{\hat{I}}_T^{1/T}$ into the contribution of each month. Since

$$\hat{I}_T^{1/T} = \prod_{t=1}^T (\hat{R}_t)^{1/T}$$

and

$$\hat{\hat{I}}_T^{1/T} = \prod_{t=1}^T (\hat{\hat{R}}_t)^{1/T},$$

I can additively decompose the months as

$$\hat{I}_T^{1/T} = \frac{\sum_{t=1}^T (1/T) \hat{R}_t / m(\hat{R}_t, \hat{I}_T^{1/T})}{\sum_{i=1}^n (1/T) / m(\hat{R}_t, \hat{I}_T^{1/T})} = \frac{\sum_{t=1}^T \hat{R}_t / m(\hat{R}_t, \hat{I}_T^{1/T})}{\sum_{i=1}^n 1 / m(\hat{R}_t, \hat{I}_T^{1/T})}$$

and

$$\hat{\hat{I}}_t^{1/T} = \frac{\sum_{t=1}^T (1/T) \hat{\hat{R}}_t / m(\hat{\hat{R}}_t, \hat{\hat{I}}_T^{1/T})}{\sum_{i=1}^n (1/T) / m(\hat{\hat{R}}_t, \hat{\hat{I}}_T^{1/T})} = \frac{\sum_{t=1}^T \hat{\hat{R}}_t / m(\hat{\hat{R}}_t, \hat{\hat{I}}_T^{1/T})}{\sum_{i=1}^n 1 / m(\hat{\hat{R}}_t, \hat{\hat{I}}_T^{1/T})}.$$

The within month contribution of each item in the unadjusted and adjusted index is just $\hat{w}_{it-1} \hat{R}_{it}$ and $\hat{w}_{it-1} \hat{\hat{R}}_{it}$. Since $\hat{B}_{it} = \hat{R}_{it} - \hat{\hat{R}}_{it}$, I get the desired result.

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Table 1a
Simulation Results - Finite Sample Bias,
the Adjustment and Bias for Adjusted Relative

Relative Type Index (Weight)	Sample Size	Average Bias of Initial Relative	Average Adjustment	Average Bias of Adjusted Relative
Geomeans (71%)	10	0.0300%	0.0250%	0.0050%
Laspeyres (11%)	12	0.0195%	0.0209%	-0.0014%
Housing (18%)	40	0.0008%	0.0002%	0.0006%
Alternative Housing	10	0.2300%	0.1730%	0.0570%

Table 1b
Simulated Difference between the Sample Standard
Deviations and the True Standard Deviations

Relative Type	Mean Difference	Standard Deviation of Difference	Minimum Difference	Maximum Difference
Geomeans	-6.74E-05	0.0023601	-0.00494566	0.020122428
Laspeyres	2.22E-06	0.0011421	-0.00250337	0.008251568
Housing	-8.06E-07	0.000601	-0.00177075	0.003574577

Table 2
Summary Statistics for
the CPI-U, the Bias Adjusted CPI-U, and the
CPI-C

Period from Dec-Dec	CPI-U	Bias Adjusted CPI-U	CPI-C	(CPI-U)- (CPI-C)	(CPI-U)- (Bias Adjusted CPI- U)	(Bias Adjusted CPI-U) -CPI-C	Source of Difference between CPI-U and CPI-C	
							% Difference from Finite Sample Bias	% Difference from Commodity Substitution Bias
1998-1999	2.663%	2.335%	2.139%	0.524%	0.329%	0.195%	62.7%	37.3%
1999-2000	3.390%	3.090%	2.600%	0.790%	0.300%	0.490%	38.0%	62.0%
2000-2001	1.548%	1.300%	1.267%	0.280%	0.248%	0.033%	88.3%	11.7%
2001-2002	2.400%	2.107%	2.021%	0.379%	0.294%	0.086%	77.4%	22.6%
2002-2003	1.870%	1.585%	1.509%	0.360%	0.285%	0.075%	79.0%	21.0%
Entire 5 year Period	12.433%	10.851%	9.902%	2.531%	1.582%	0.949%	62.504%	37.496%

Correlations wirh the Bias Adjustments

Factor	Geomean Relative
Var(lnP _t -lnP _{t-1})	92.6%
n	-4.7%

Factor	Laspeyres Relative
Var(p _t)	2.1%
Var(p _{t-1})	3.8%
n	-2.3%

Factor	Housing Relative
Var(p _t)	-3.0%
Var(p _{t-6})	0.0%
n	-1.9%

Table 3a
Contributions to Index Growth and Bias Adjustment

Group Name	Group Contribution CPI-U	Group Contribution Adjusted CPI-U	Group Contribution to Bias Adjustment
Apparel	4.5%	4.5%	25.6%
Education	5.5%	5.5%	3.1%
Food	16.0%	16.0%	40.2%
Housing	40.4%	40.4%	15.7%
Medical	5.8%	5.8%	1.2%
Recreation	6.0%	6.0%	6.2%
Transportation	17.3%	17.3%	4.4%
Other	4.6%	4.6%	3.5%

Table 3b
Method Type Contributions to Index Growth and Bias Adjustment

Relative Type	Type Contribution CPI-U	Type Contribution Adjusted CPI-U	Type Contribution to Bias Adjustment
G	71.1%	71.1%	96.5%
L	17.7%	17.7%	2.5%
H	11.3%	11.3%	1.0%

Table 3c
Core Type Contributions to Index Growth and Bias Adjustment

Relative Type	Type Contribution CPI-U	Type Contribution Adjusted CPI-U	Type Contribution to Bias Adjustment
Core	77.8%	77.6%	59.7%
Energy	7.1%	7.1%	0.8%
Food*	15.3%	15.3%	39.8%

* Excludes alcoholic beverages

Table 4
Annual Group Detail

1999				2002			
Group	Annual Growth CPI-U	Bias Adjusted	Difference	Group	Annual Growth CPI-U	Bias Adjusted	Difference
Apparel	-0.47%	-2.08%	1.61%	Apparel	-1.76%	-3.57%	1.80%
Education	1.58%	1.03%	0.55%	Education	2.13%	1.99%	0.14%
Food	2.00%	1.24%	0.76%	Food	1.51%	0.80%	0.72%
Other	5.08%	4.83%	0.25%	Other	3.30%	3.10%	0.20%
Housing	2.17%	2.08%	0.09%	Housing	2.35%	2.24%	0.11%
Medical	3.66%	3.57%	0.10%	Medical	5.04%	5.00%	0.05%
Recreation	0.79%	0.34%	0.45%	Recreation	1.11%	0.75%	0.35%
Transportation	5.39%	5.30%	0.09%	Transportation	3.82%	3.77%	0.05%
2000				2003			
Group	Annual Growth CPI-U	Bias Adjusted	Difference	Group	Annual Growth CPI-U	Bias Adjusted	Difference
Apparel	-1.70%	-3.23%	1.53%	Apparel	-2.05%	-3.87%	1.82%
Education	1.32%	1.24%	0.08%	Education	1.55%	1.47%	0.09%
Food	2.73%	1.90%	0.83%	Food	3.50%	2.76%	0.74%
Other	4.17%	3.90%	0.27%	Other	1.48%	1.31%	0.17%
Housing	4.28%	4.19%	0.09%	Housing	2.23%	2.10%	0.12%
Medical	4.18%	4.14%	0.05%	Medical	3.69%	3.62%	0.07%
Recreation	1.67%	1.42%	0.25%	Recreation	1.09%	0.87%	0.22%
Transportation	4.08%	3.99%	0.09%	Transportation	0.35%	0.28%	0.07%
2001							
Group	Annual Growth CPI-U	Bias Adjusted	Difference				
Apparel	-3.26%	-4.66%	1.40%				
Education	3.20%	3.13%	0.06%				
Food	2.77%	2.08%	0.70%				
Other	4.52%	4.29%	0.23%				
Housing	2.91%	2.83%	0.08%				
Medical	4.73%	4.67%	0.05%				
Recreation	1.57%	1.32%	0.25%				
Transportation	-3.82%	-3.88%	0.07%				

Table 5
Annual Detail by
Core and Non Core Index Type

1999			
Type	Annual Growth CPI-U	Bias Adjusted	Difference
Core	1.936%	1.677%	0.260%
Energy	13.400%	13.372%	0.028%
Food	1.971%	1.188%	0.783%

2002			
Type	Annual Growth CPI-U	Bias Adjusted	Difference
Core	1.915%	1.691%	0.224%
Energy	10.720%	10.692%	0.028%
Food	1.497%	0.759%	0.739%

2000			
Type	Annual Growth CPI-U	Bias Adjusted	Difference
Core	2.543%	2.339%	0.204%
Energy	14.215%	14.173%	0.043%
Food	2.741%	1.884%	0.857%

2003			
Type	Annual Growth CPI-U	Bias Adjusted	Difference
Core	1.110%	0.894%	0.216%
Energy	6.914%	6.881%	0.033%
Food	3.580%	2.811%	0.769%

2001			
Type	Annual Growth CPI-U	Bias Adjusted	Difference
Core	2.754%	2.572%	0.181%
Energy	-13.055%	-13.093%	0.038%
Food	2.828%	2.110%	0.718%