

**“Combining Hedonic with Multilateral Indexes
for Turnover and Chain Drift in Transactions
Data Consumer Price Indexes”**

Gregory Kurtzon

U.S. Bureau of Labor Statistics

Division of Price and Index Number Research

May 13th, 2024



Motivation

- There is a big movement across countries to use transactions data to make CPIs
- Has potential to measure elementary level substitution with current quantities
- Two major problems:
 - ▶ This data has many entering and exiting goods and it isn't possible to do usual substitution with universe
 - ▶ Using quantities concurrent with prices can cause chain drift (non-circularity)



Item Turnover

- Several countries make unit value indexes at a low aggregation
 - ▶ Average prices and sum of quantities
- This can be biasing
 - ▶ Quality improvements
 - ▶ Missing substitution within
 - ▶ Jensen's inequality

Chain Drift

- Defined here as non-circularity, if prices return to first values, index isn't 1
 - ▶ True COLI should
- Typically, downward chain drift is thought to be due to sales
 - ▶ Ivanic, Diewert & Fox (2011), Diewert & Fox (2022)
 - ▶ Quantity shoots up during a sale, and after price returns to pre-sale price, quantity is lower from stockpiling
 - ▶ TQ gives larger weight to downward relative than upward
- Other papers show other kinds of drift due to clearance sales and advertising
 - ▶ Melser & Webster (2021), Feenstra & Shapiro (2003)

- If we had a true measure of drift to measure other methods against, we wouldn't need them
- Long direct indexes are not a gold standard
 - ▶ Item turnover
 - ▶ Longer links have more dissimilar prices, worse Divisia approximations
 - ▶ Divisia (1925), Diewert (1976), Hill (2006), Kurtzon (2022)
- Often rely on informal 'reasonability test'
 - ▶ Extreme inflation rates

Contributions

- Bootstrapped fully hedonic indexes showing variance
- Combination of full hedonic indexes with multilateral method over all window lengths of a long period to show how multilaterals converge to circularity
- Chain drift reduction of hedonics
- Laspeyres-Paasche multilateral spreads converging as circularity is approached



Hedonics

- Hedonics redefine goods as characteristics to reduce turnover and avoid unit values
- Using Pakes (2003) method of running a hedonic regression every month and replacing all prices, missing and raw, with predictions
 - ▶ A dummy variable for every characteristic value
- Bias accumulates over time, variance averages out
- Bootstrapping to evaluate model, overfitting
- It can also reduce chain drift by smoothing sales prices
- Time product dummy, or time dummy characteristics models hold coefficients constant for long time



Hedonics Method

Let p_{ut} denote price of UPC i in month t , D_{uc} denote a dummy for characteristic c in UPC u , β_{uct} denote the coefficient on the dummy for UPC u for characteristic c in month t , and ε_{ut} denote error. The regression is

$$\ln p_{ut} = \sum_c \beta_{uct} D_{uc} + \varepsilon_{ut} \quad (1)$$

$$p_{ut} = \left(\prod_c \exp(\beta_{uct} D_{uc}) \right) \exp(\varepsilon_{ut}) \quad (2)$$

yielding a predicted price for u in month t of

$$\hat{p}_{ut} = \left(\prod_c \exp(\hat{\beta}_{uct} D_{uc}) \right) \exp(\hat{\varepsilon}_{ut}) \quad (3)$$

Putting into Törnqvist, denoting index relative for month t as R_t , the expenditure share of UPC u in month t as s_{ut} , then

$$R_t = \Pi_u \left(\frac{\left(\Pi_c \exp \left(\hat{\beta}_{uct} D_{uc} \right) \right) \exp \left(\hat{\varepsilon}_{ut} \right)}{\left(\Pi_c \exp \left(\hat{\beta}_{uc,t-1} D_{uc} \right) \right) \exp \left(\hat{\varepsilon}_{u,t-1} \right)} \right)^{\frac{s_{ut} + s_{u,t-1}}{2}} = \quad (4)$$

$$\Pi_u \left(\frac{\Pi_c \exp \left(\hat{\beta}_{uct} D_{uc} \right)}{\Pi_c \exp \left(\hat{\beta}_{uc,t-1} D_{uc} \right)} \right)^{\frac{s_{it} + s_{i,t-1}}{2}} \Pi_u \left(\frac{\exp \left(\hat{\varepsilon}_{ut} \right)}{\exp \left(\hat{\varepsilon}_{u,t-1} \right)} \right)^{\frac{s_{ut} + s_{u,t-1}}{2}} \quad (5)$$

Non-missing Only Imputation

- Only replaces actual observed prices with hedonic imputed prices, not entering or exited goods
- Purpose is to study the price smoothing effects of hedonics on sales and chain drift



CCDI Method

- Using Caves, Christensen, and Diewert (1983) CCDI index (RGEKS Törnqvist), makes unnormalized index levels as averages of all bilateral relatives in a rolling window of length T

$$\rho_t = \left[\prod_{\tau=1}^T \prod_i \left(\frac{p_{it}}{p_{i\tau}} \right)^{\frac{1}{2}(s_{i\tau} + s_{it})} \right]^{\frac{1}{T}}$$

- Spliced by Movement Splice, last period

$$\pi_{T+1}(T) = \left(\frac{\rho_T}{\rho_1} \right) \left(\frac{\rho_{T+1}^*}{\rho_T^*} \right)$$

- Looking at 24 different window length from 2 months (monthly chained) to entire period non-rolling window

Multiperiod Identity (MPID) Test

- A direct measure of chain drift from the definition

- ▶ Walsh (1901)

- $MPID$ test value = $P(p^1, p^2, q^1, q^2) P(p^2, p^3, q^2, q^3) \cdots P(p^{T-1}, p^T, q^{T-1}, q^T) P(p^T, p^1, q^T, q^1)$

- Same as ratio of chained to direct Törnqvist
- Last term could be a very long direct index

Data

- IRI academic data set scanner data
 - ▶ Universe of large grocery and drug store transactions from Jan. 2001-2011
 - Same data as Nielsen’s Scantrak but processed differently
 - ▶ 14 different goods categories, half of them food
 - Cold cereal, carbonated beverages, coffee, deodorant, diapers, facial tissue, laundry detergent, mayonnaise, peanut butter, paper towels, razors, salty snacks, toothbrushes, yogurt
 - ▶ Unit values for each UPC over a week
 - I aggregate to 4-week periods, 13 ‘months’ a year

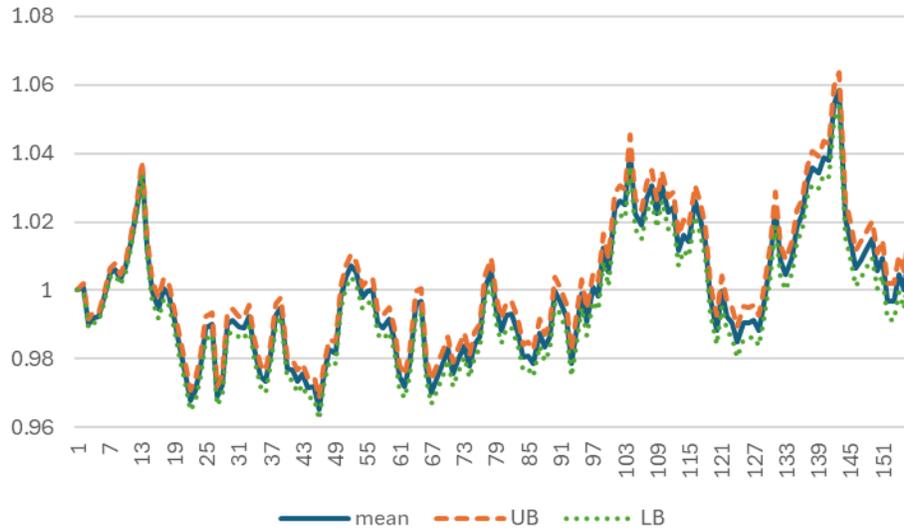
- Characteristic variables enough to identify the UPC
 - ▶ Mostly categorical
- Cold Cereal as example includes Store (outlet), brand, VOL_EQ, PRODUCT_TYPE, FLAVOR_SCENT, SUGAR_CONTENT, FIBER_INFO, TYPE_OF_GRAIN
 - ▶ 1,206,543 observations
 - ▶ About 4000 dummies
 - ▶ Usually around 300,000 UPCs per month
- About 1.5 million average across goods

Bootstrapping

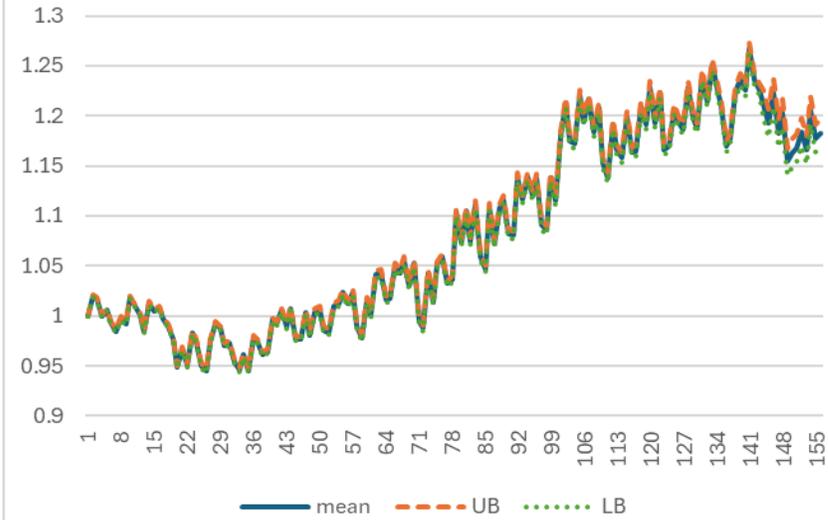
- 50 samples drawn with replacement with equal number of observations
- Variance across indexes and mean used by construct 2 standard deviation confidence intervals



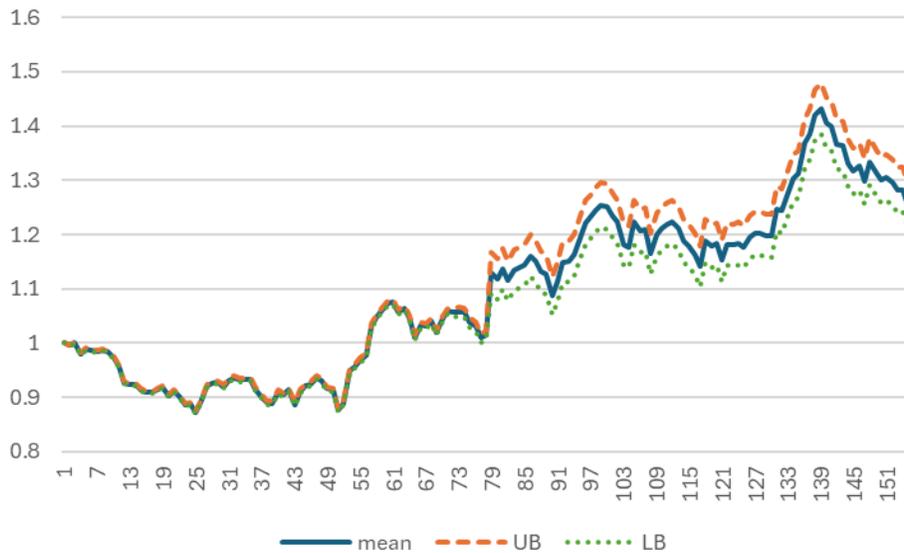
Bootstrapped Index of Coldcer Fullimp



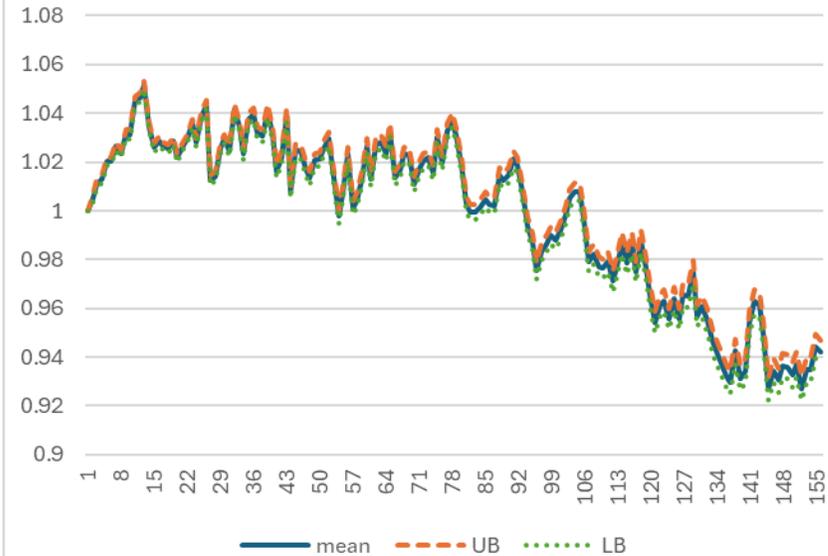
Bootstrapped Index of Carbbev Fullimp

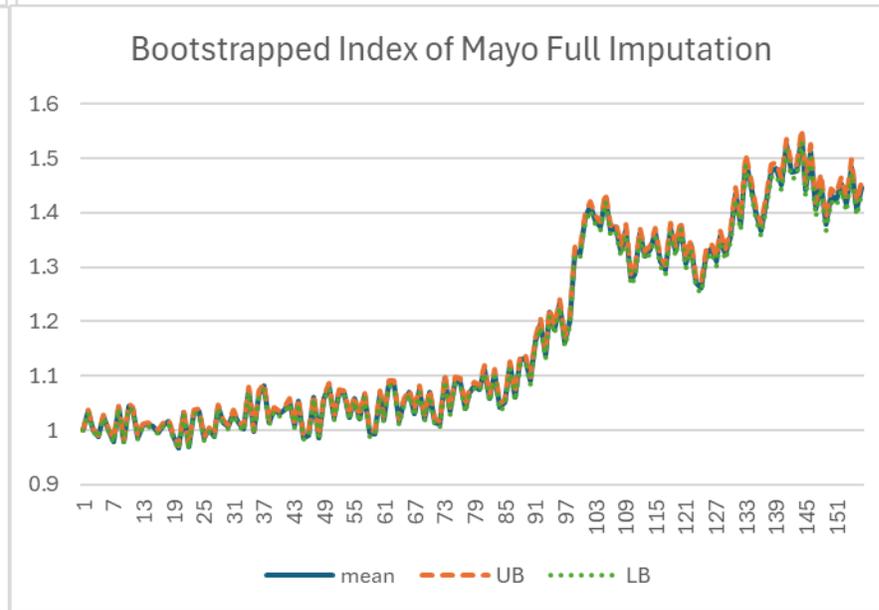
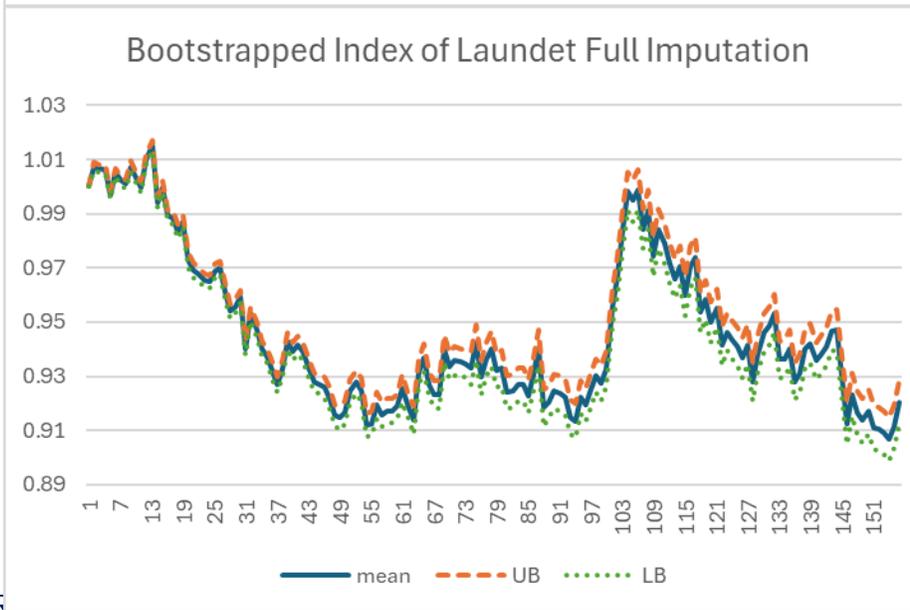
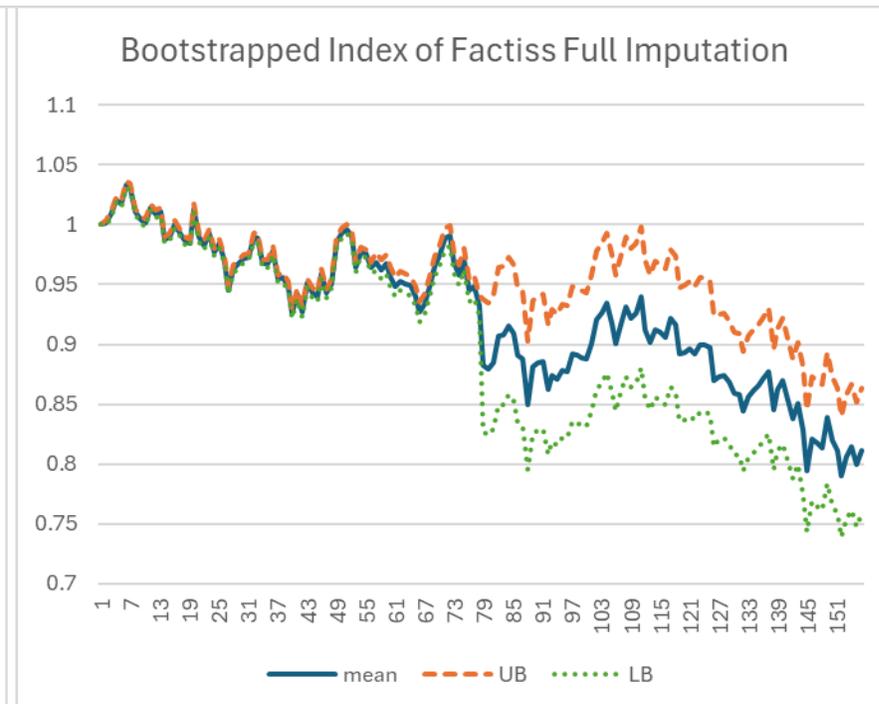
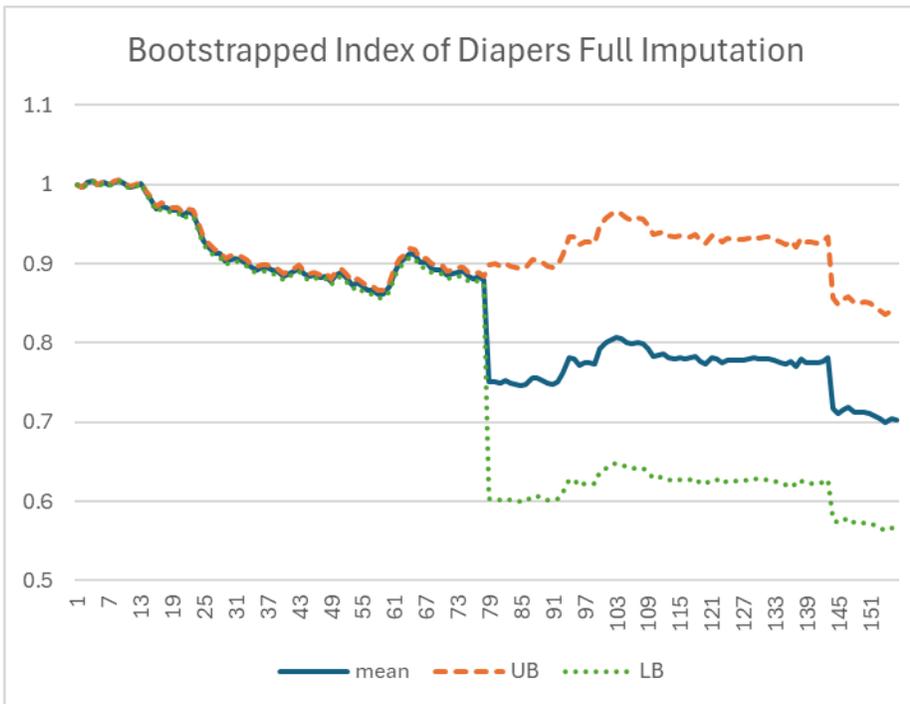


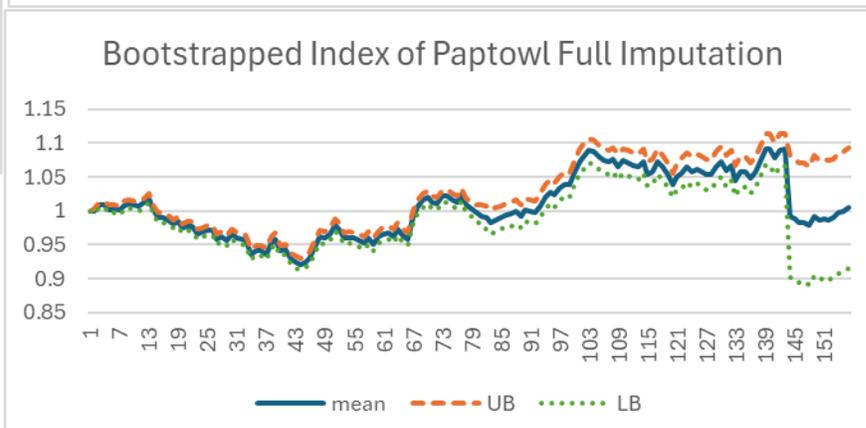
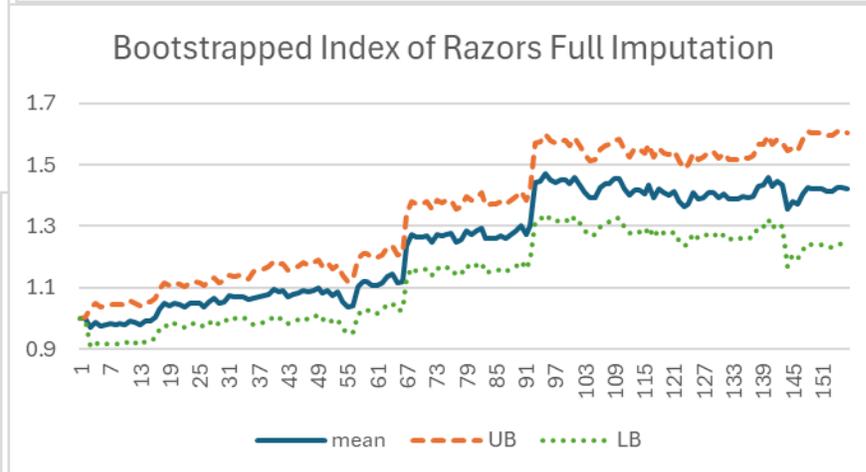
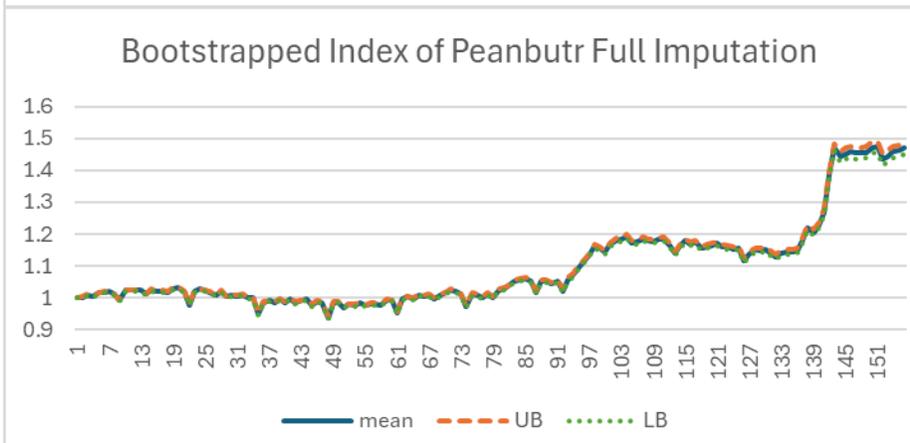
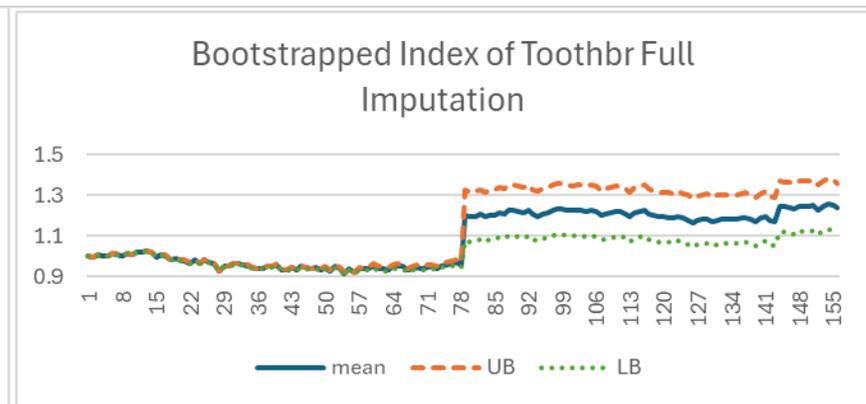
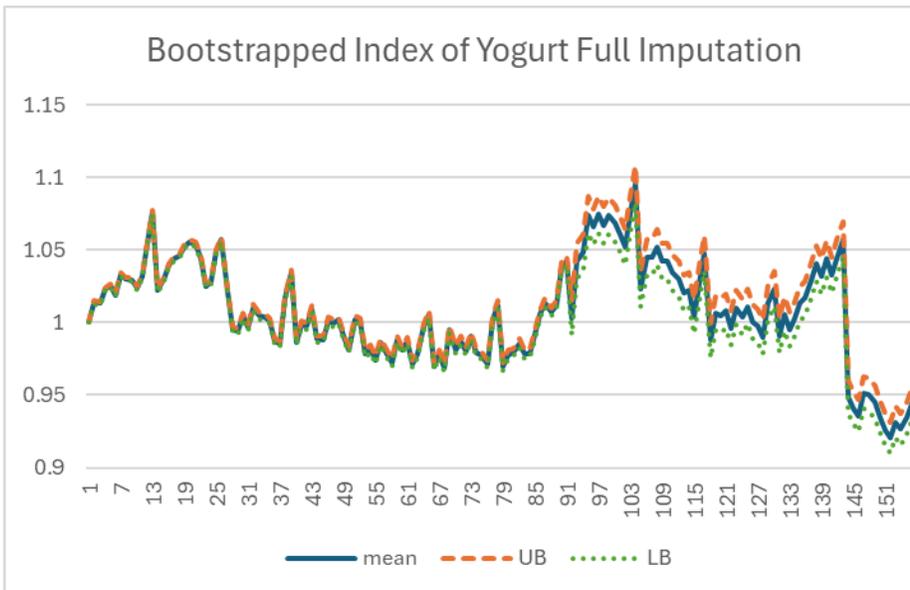
Bootstrapped Index of Coffee Full Imputation



Bootstrapped Index of Deod Full Imputation



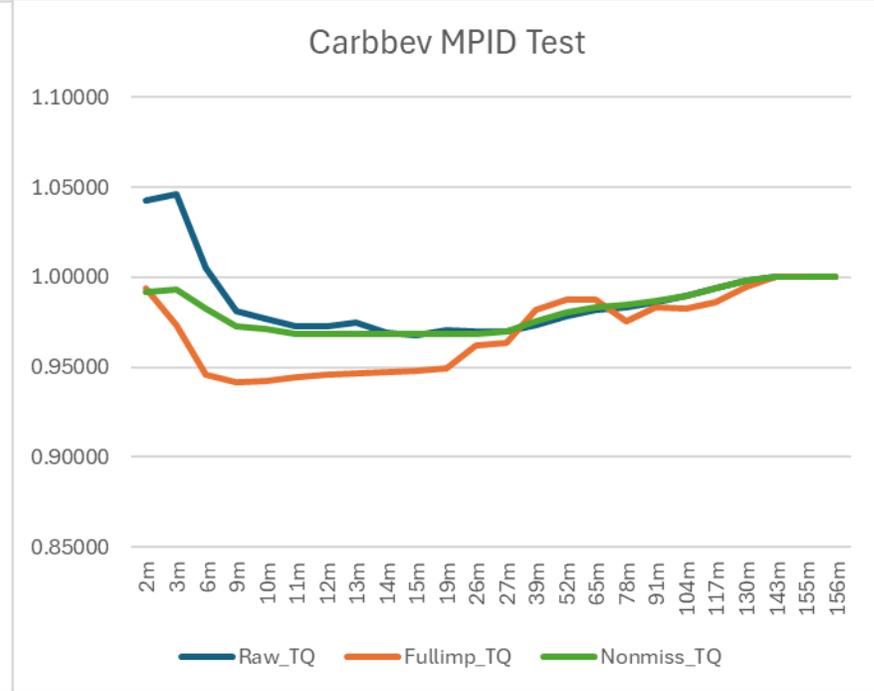
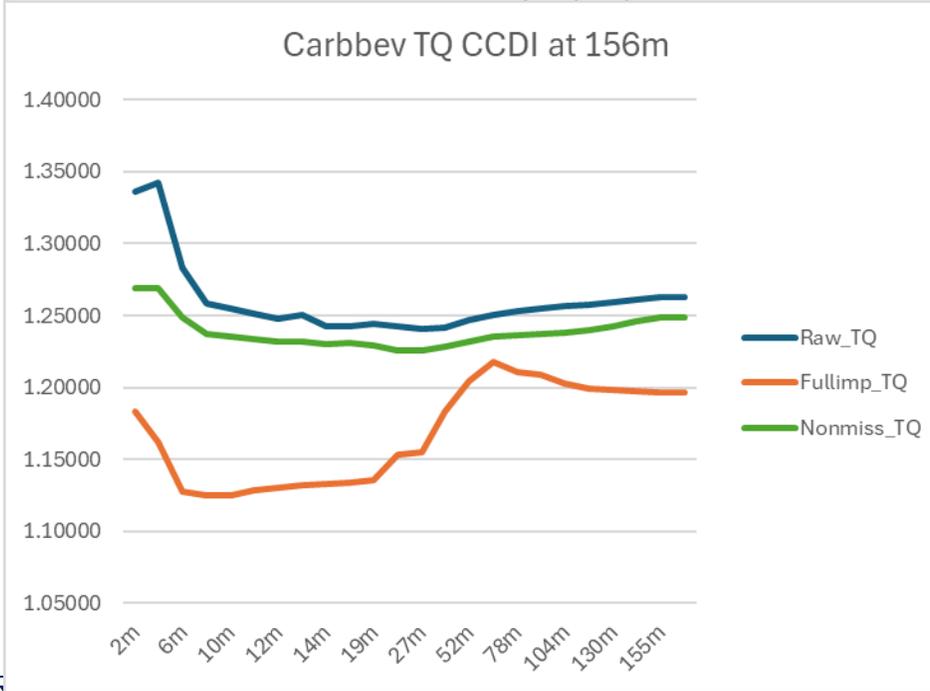
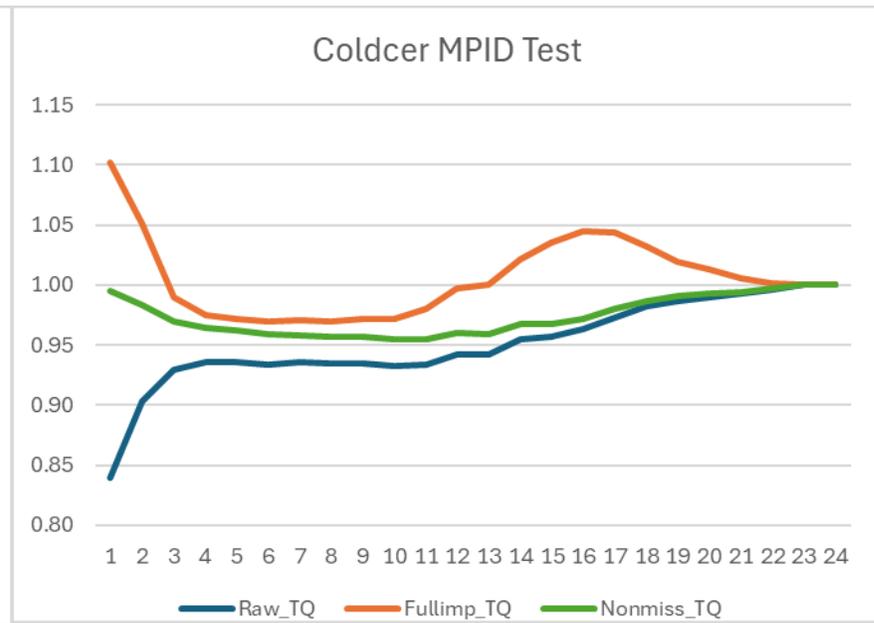
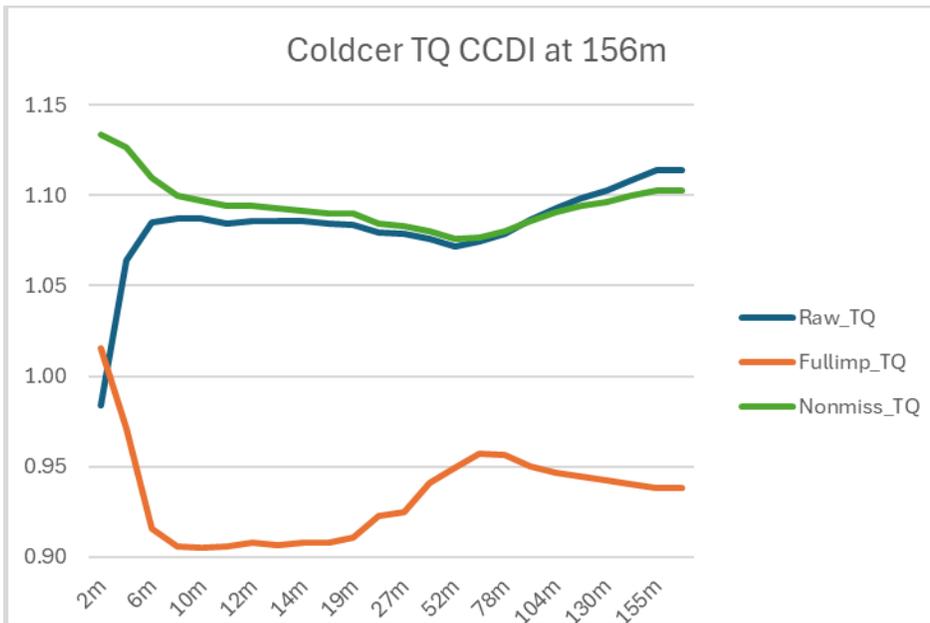


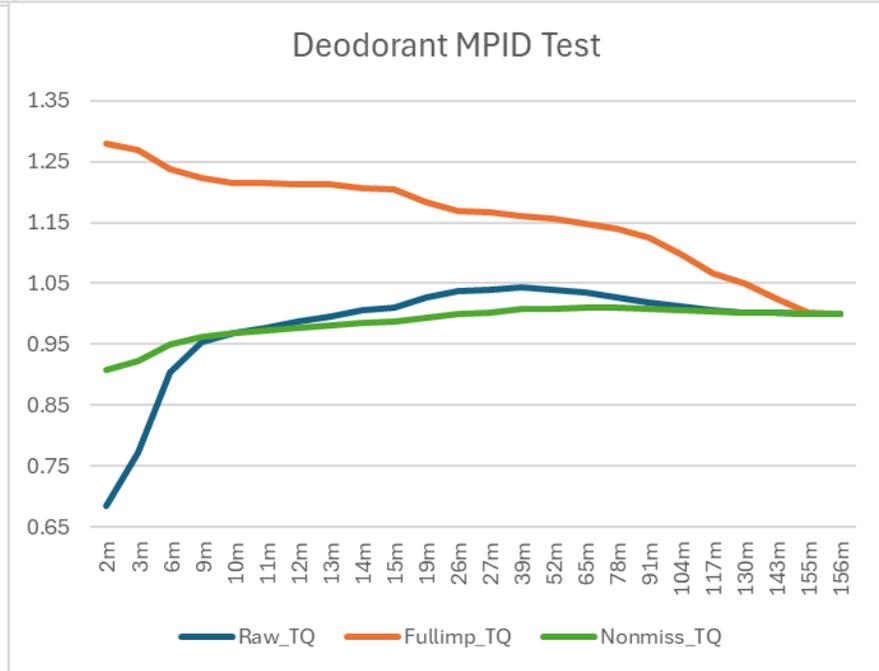
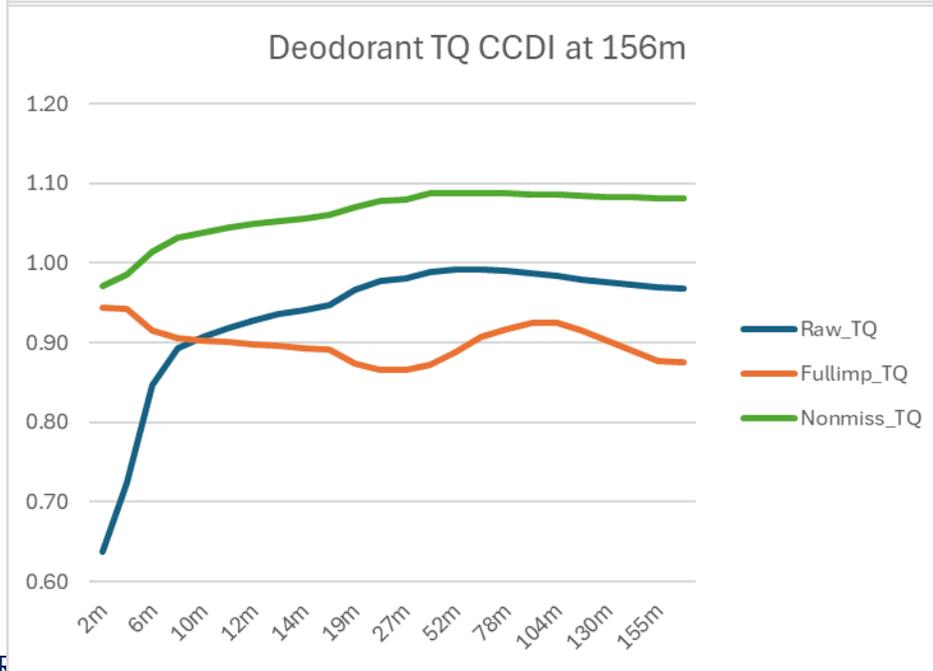
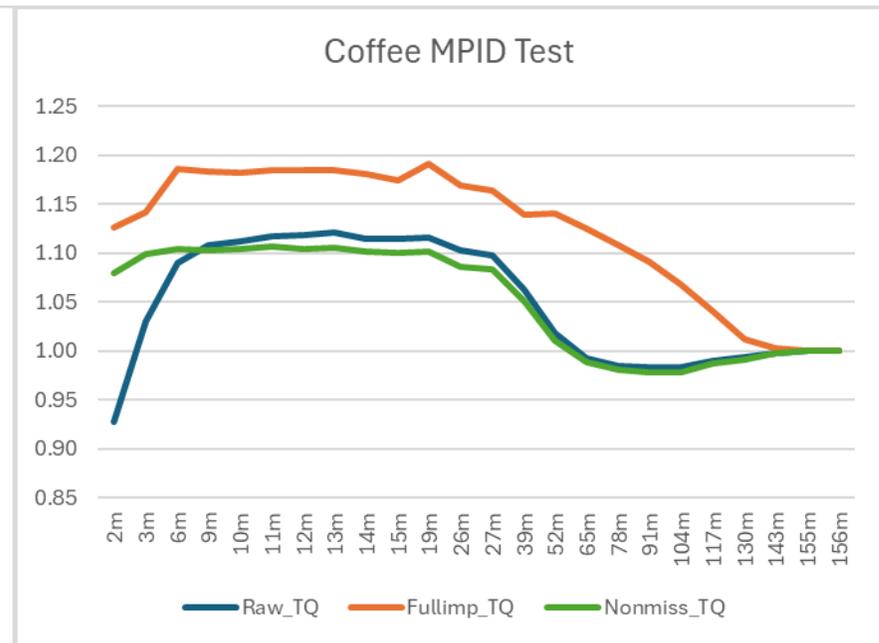
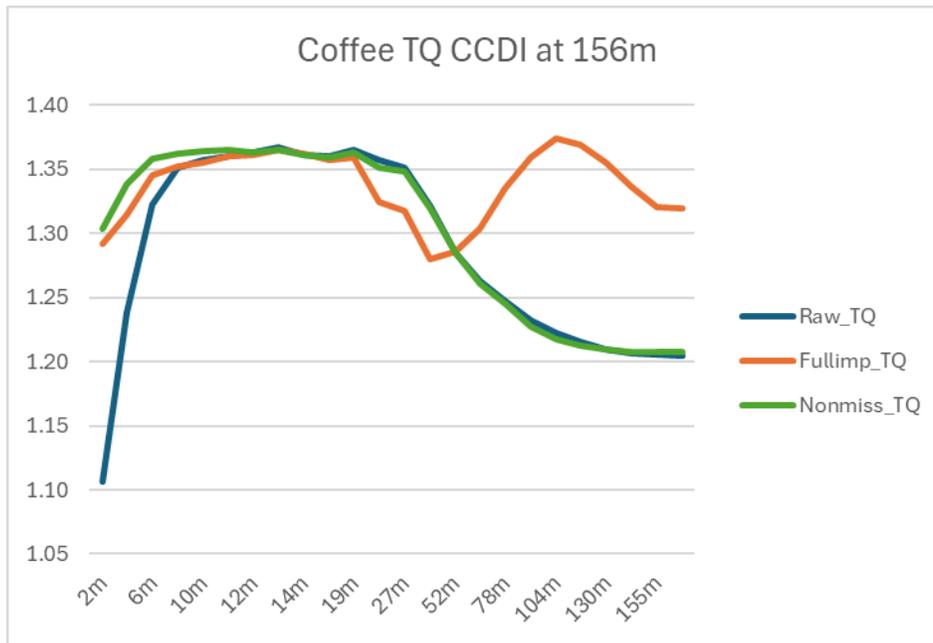


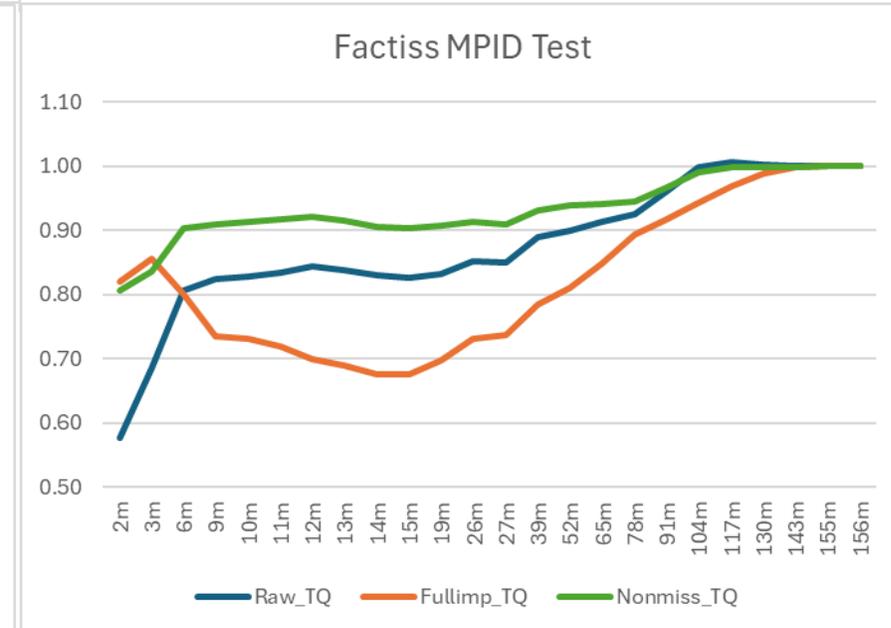
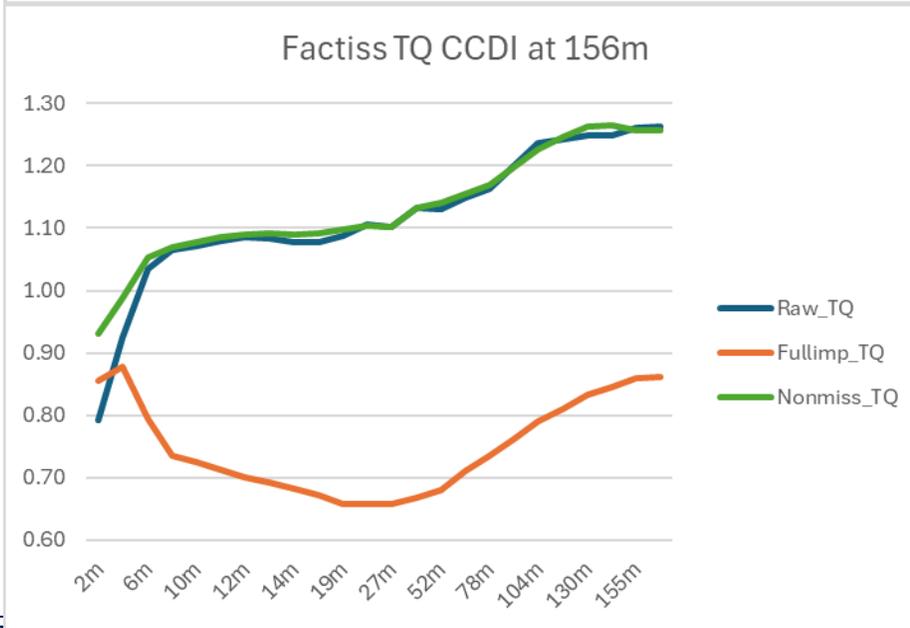
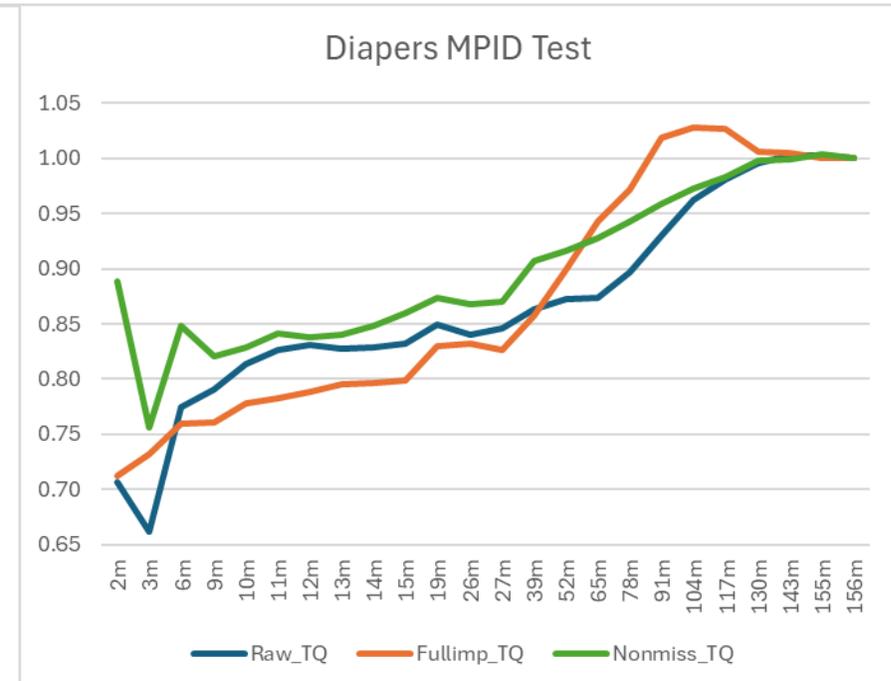
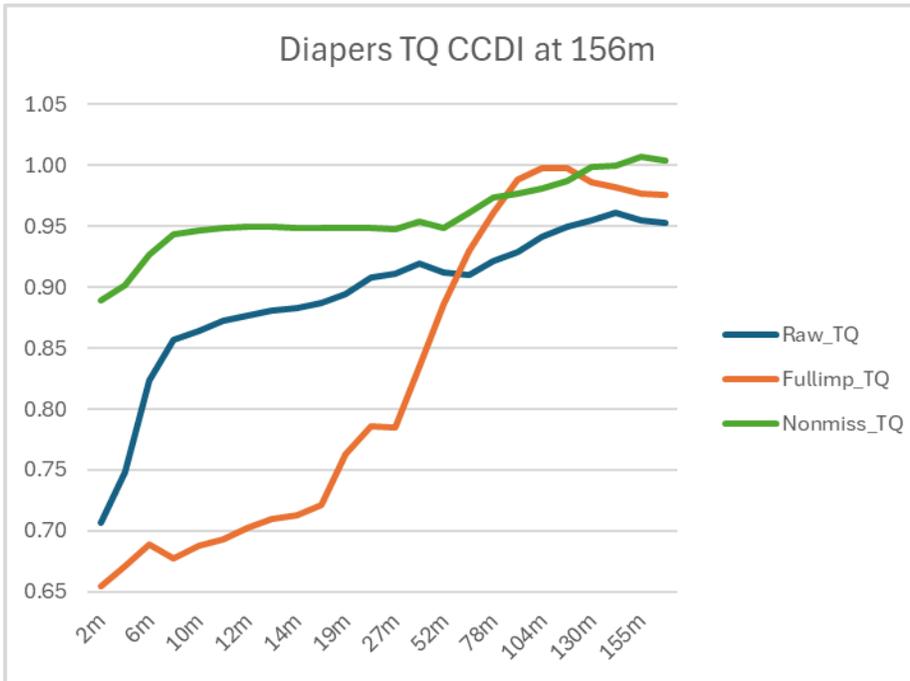
Bootstrapped Hedonic Results

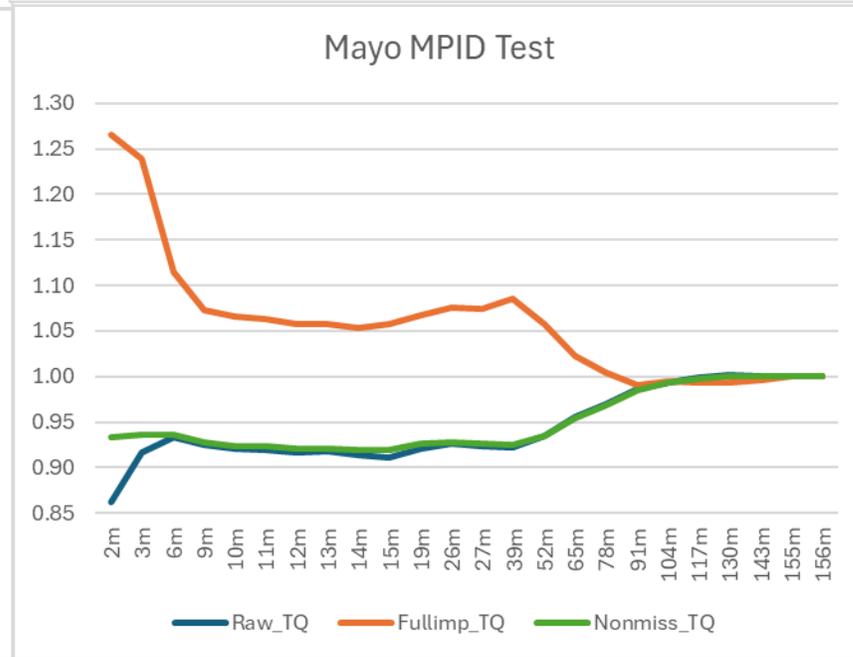
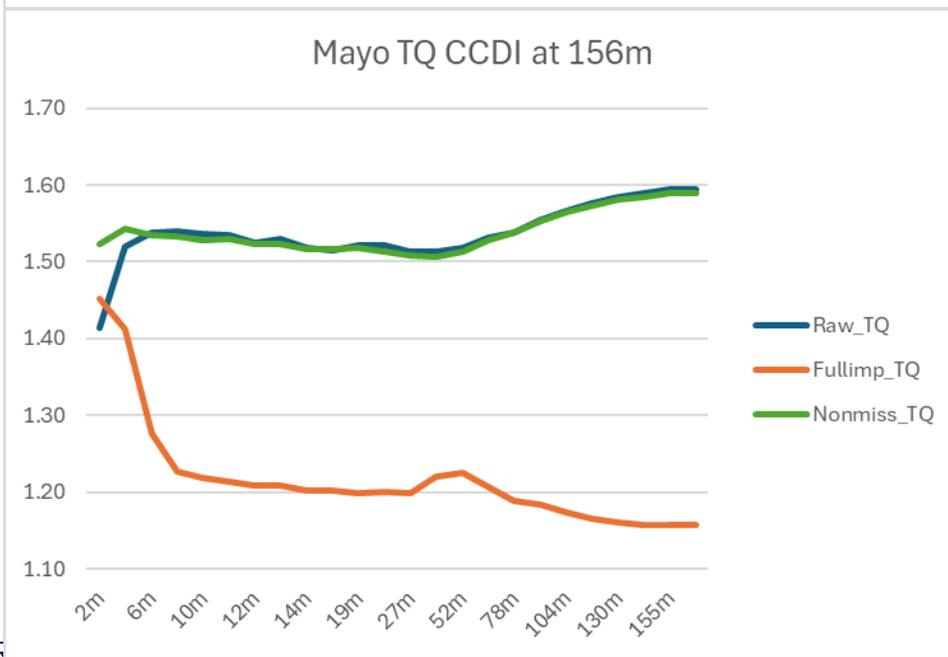
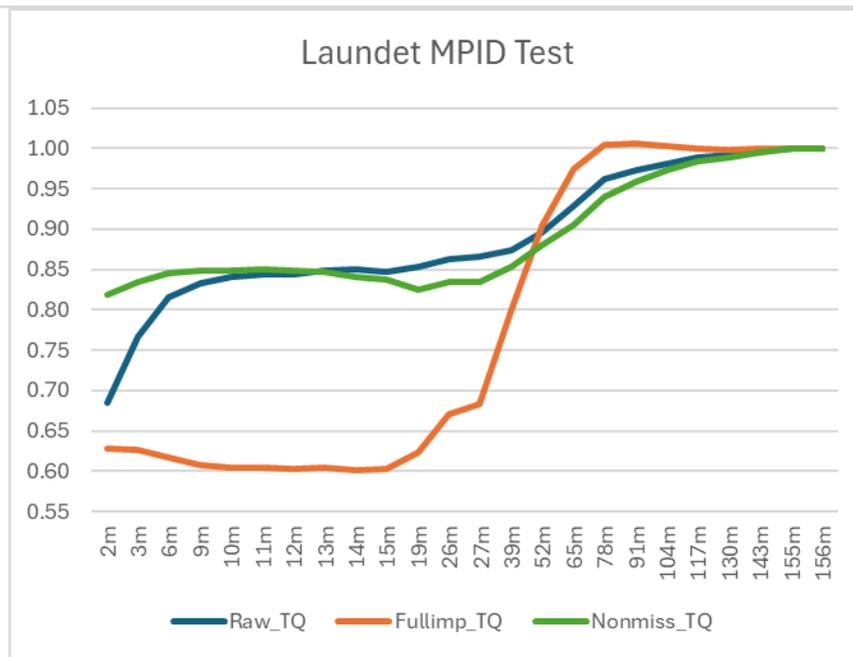
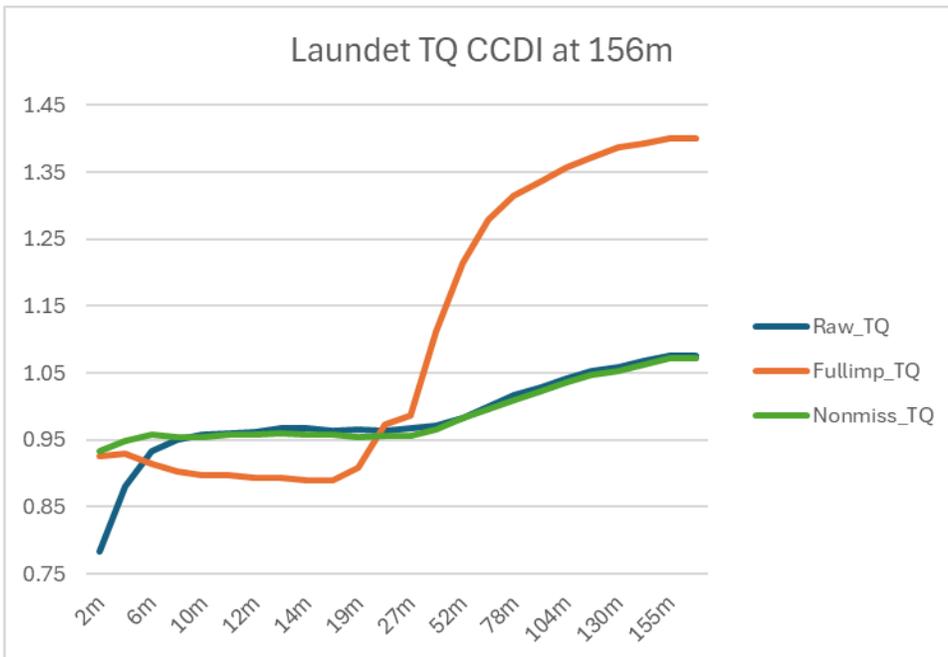
- Usually, variance very low
- A small number of months cause indexes to diverge
 - ▶ Often same months across goods
 - ▶ Usually January

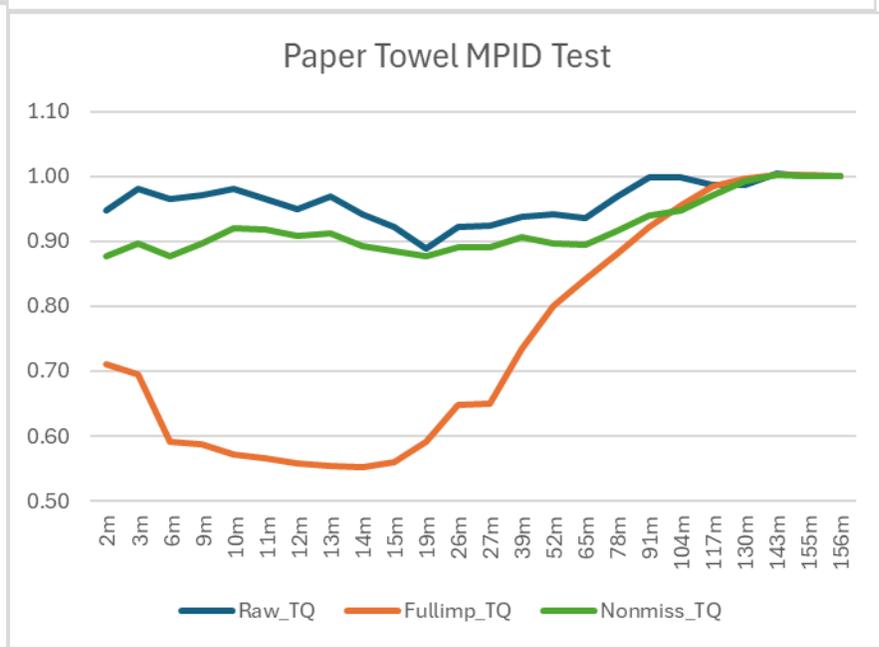
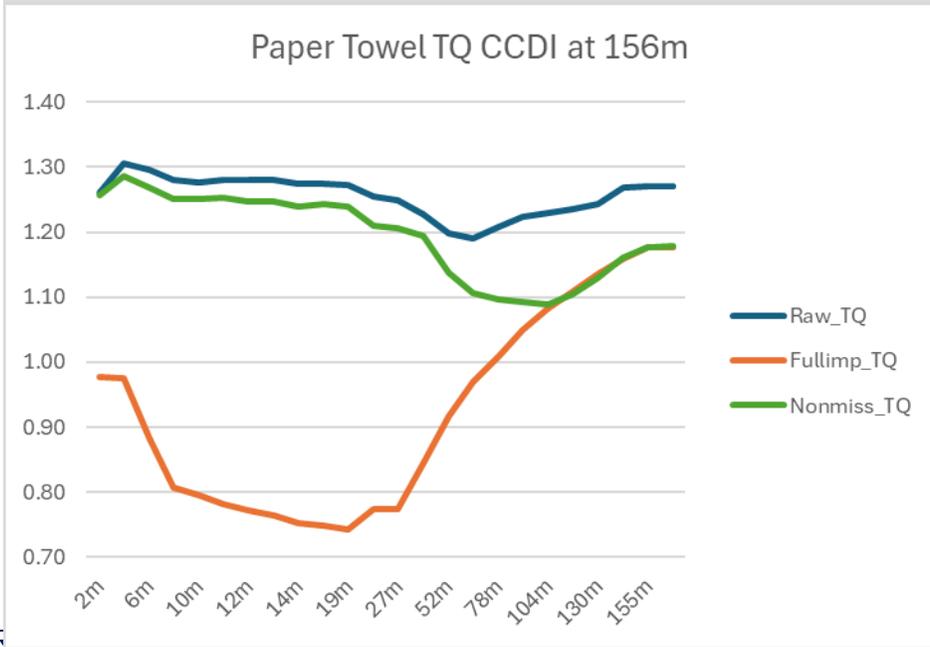
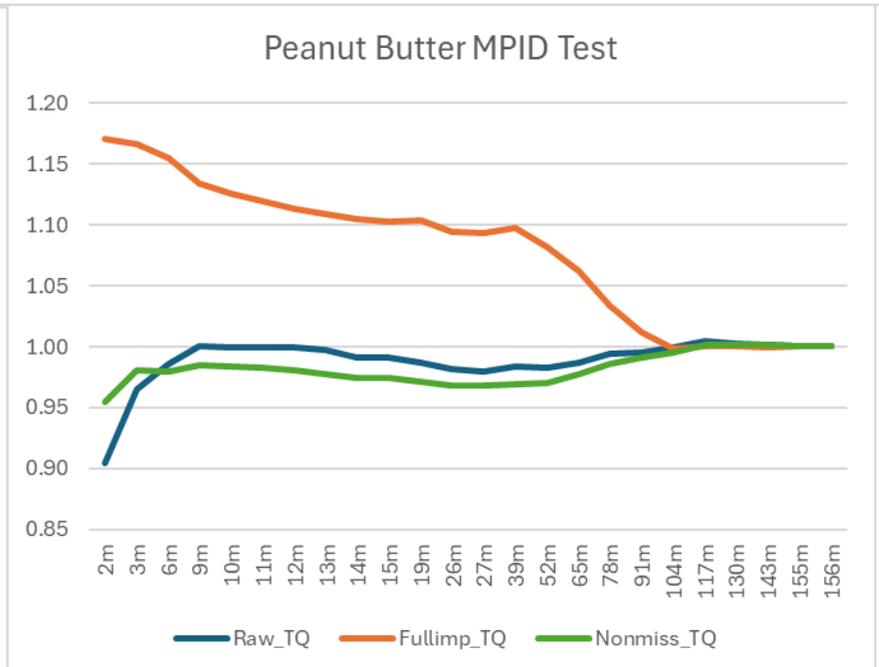
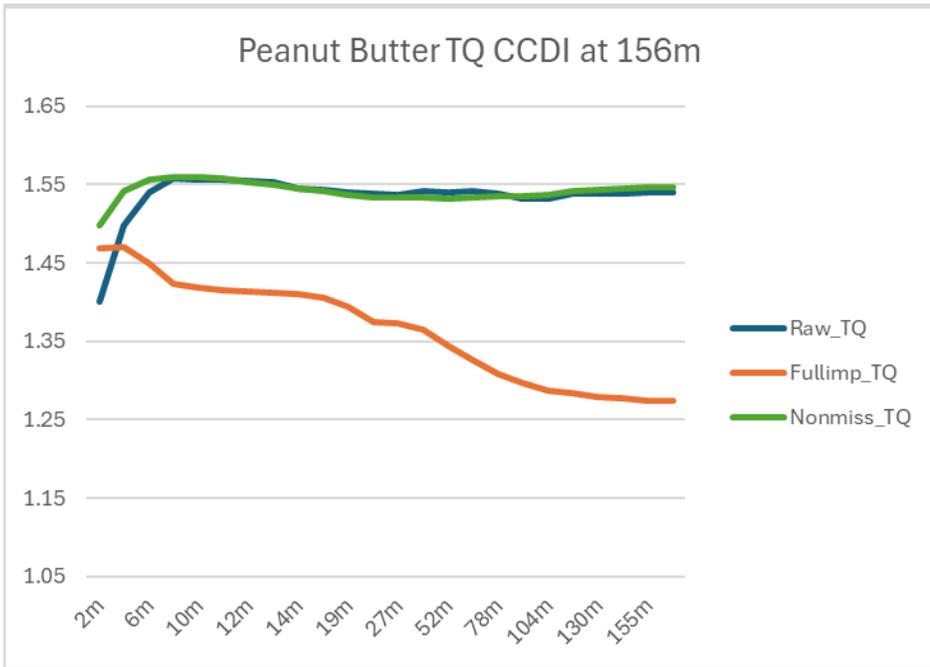


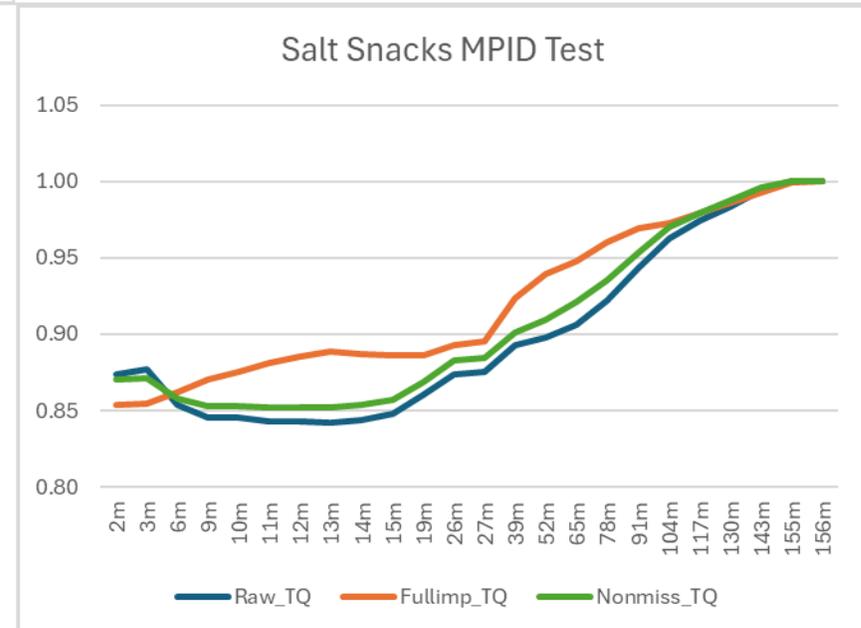
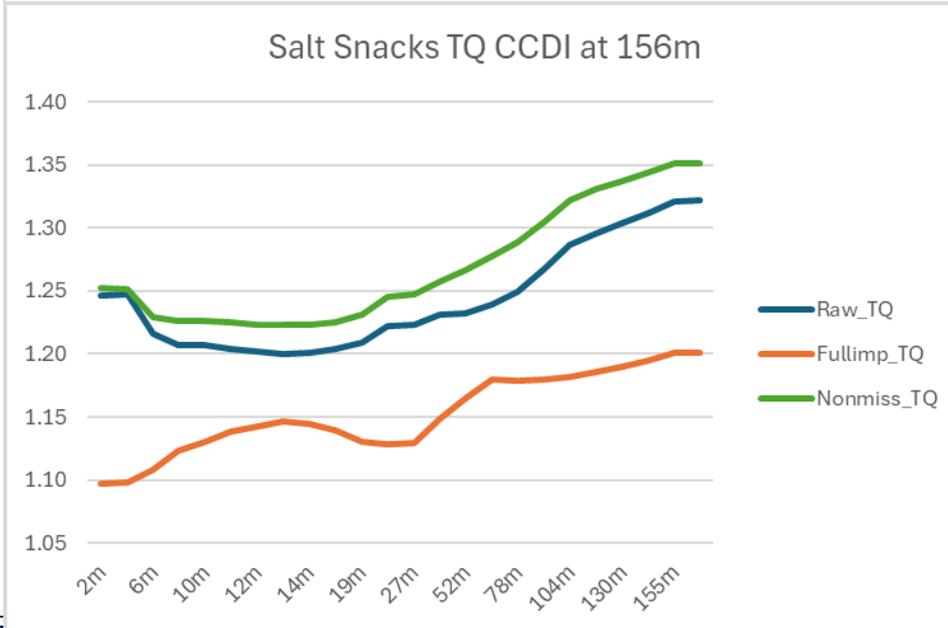
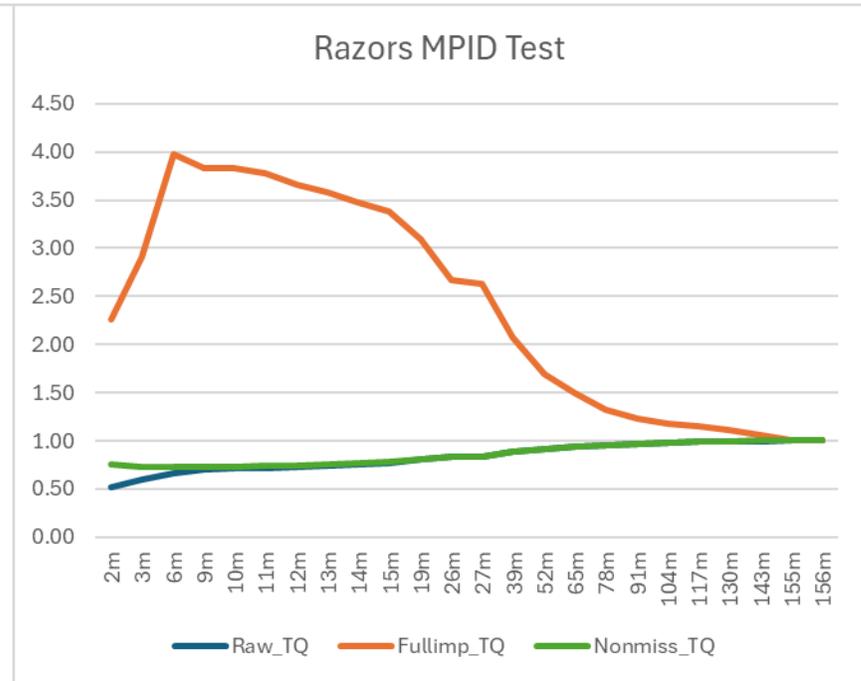
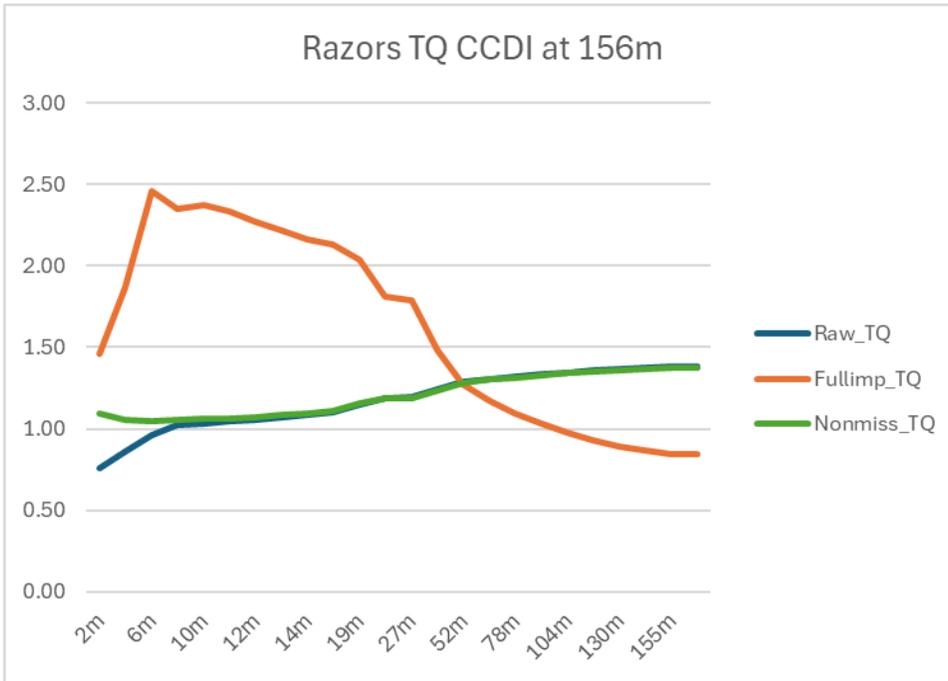


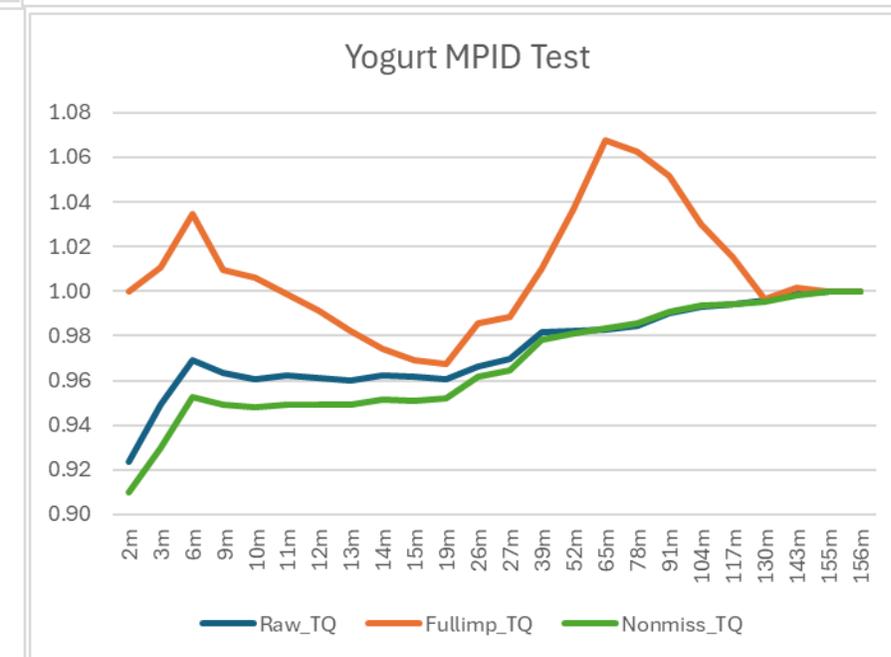
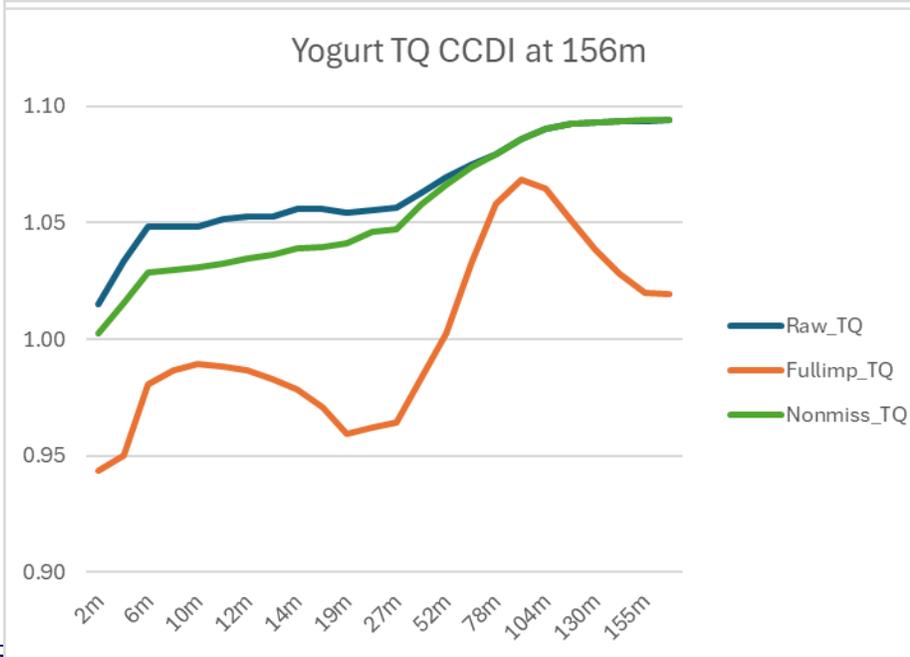
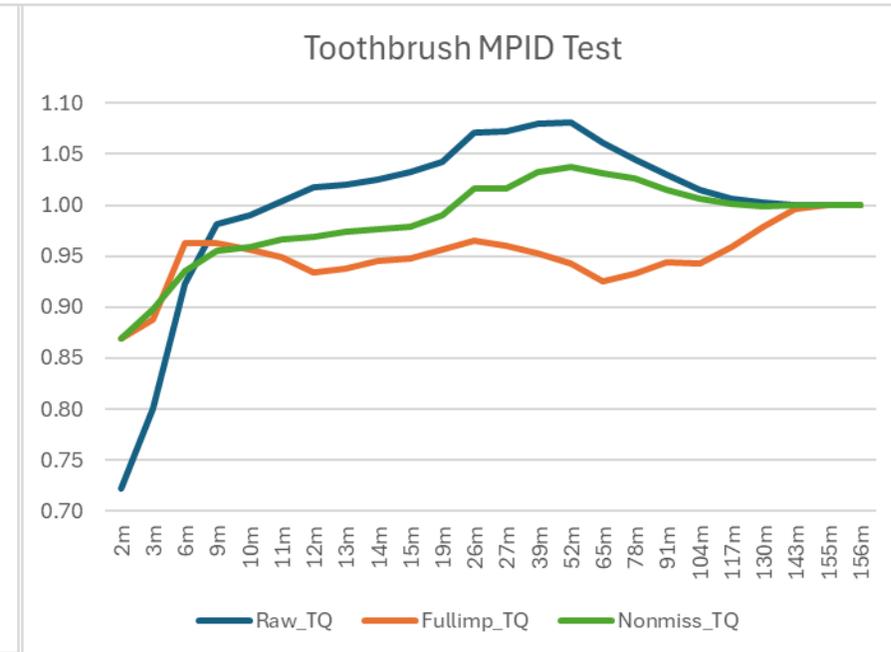
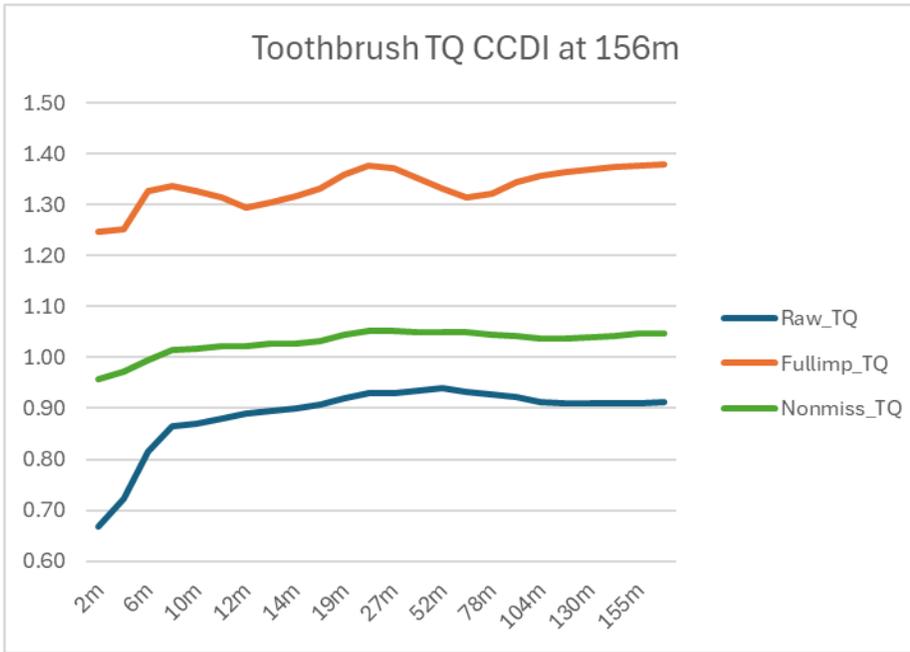












CCDI, Imputation, and MPID Results

- Levels show major differences from hedonics
- MPID tests as measure of drift
 - ▶ Big difference for short windows, but not necessarily better
 - ▶ Longer windows can make drift worse
 - ▶ Often only converges to one with very long windows, over 2 years
- Non-missing only hedonic improves drift the most, better for all but paper towels and yogurt
 - ▶ Eliminates it in monthly chained index for cold cereal and carbonated beverages

■ Full imputation Indexes

- ▶ Makes large difference in levels
- ▶ Usually as 'reasonable' or more than raw
- ▶ 2-month levels closer to full window, 156 months
- ▶ Often opposite drift pattern
- ▶ Rolling window just as likely to worsen drift as improve it



- At least some of the drift reduction of the non-missing only and full imputation indexes can be explained by the smoothing of sales prices with imputations

Correlations Between Raw and Predicted Prices

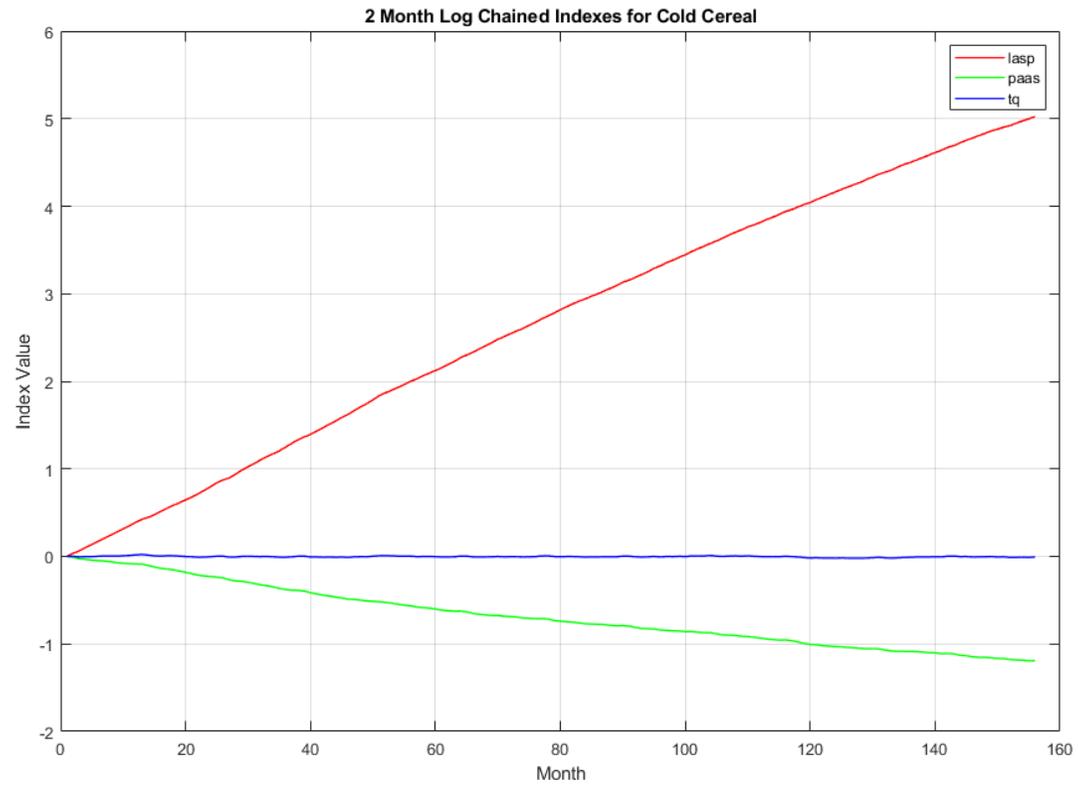
<u>Item</u>	<u>Correlation</u>
cold cereal	.3204
mayonnaise	.4676
coffee	.3915
carbonated beverages	.3004
peanut butter	.4531
razors	.4152
deodorant	.2837
facial tissue	.3861
laundry detergent	.3642
diapers	.3424
tooth brushes	.2739
salty snacks	.2454
paper towels	.3785
yogurt	.3616



Laspeyres-Paasche Indexes

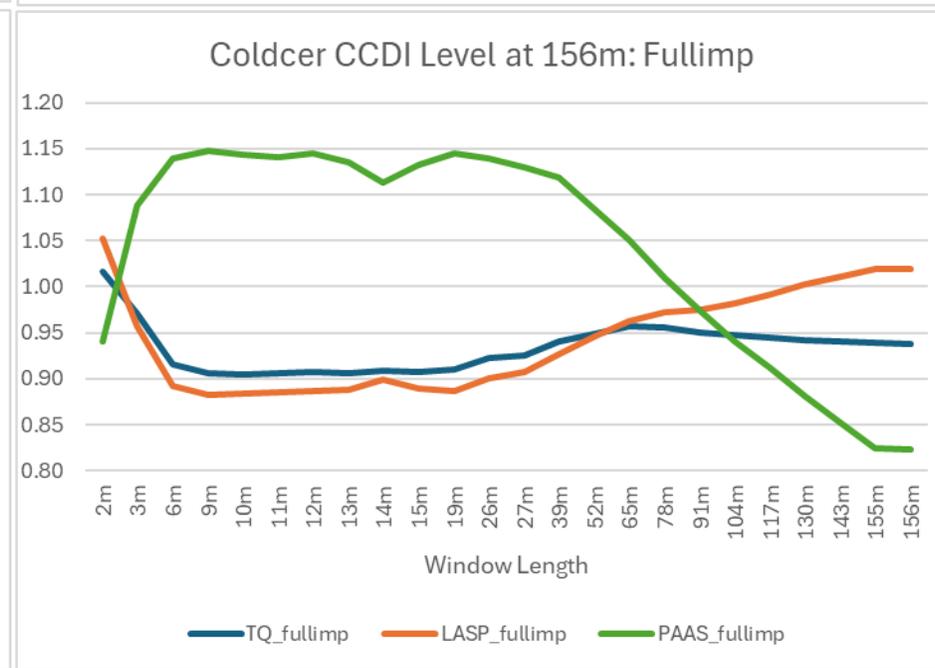
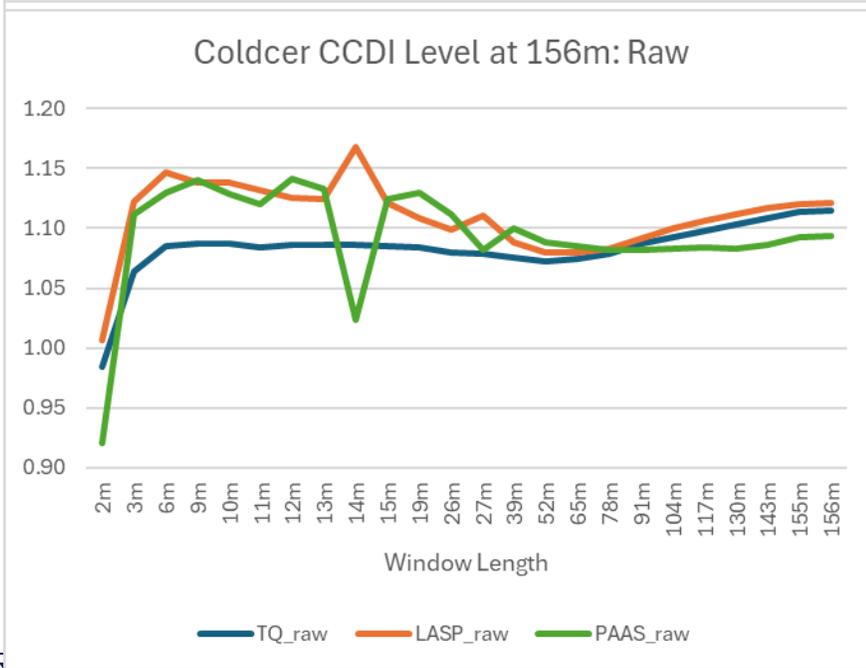
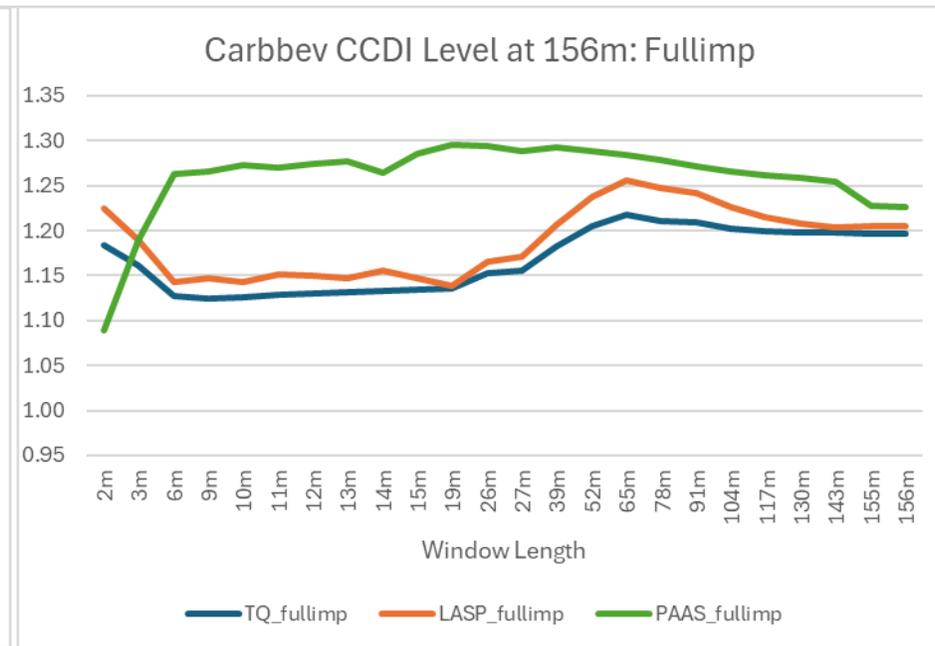
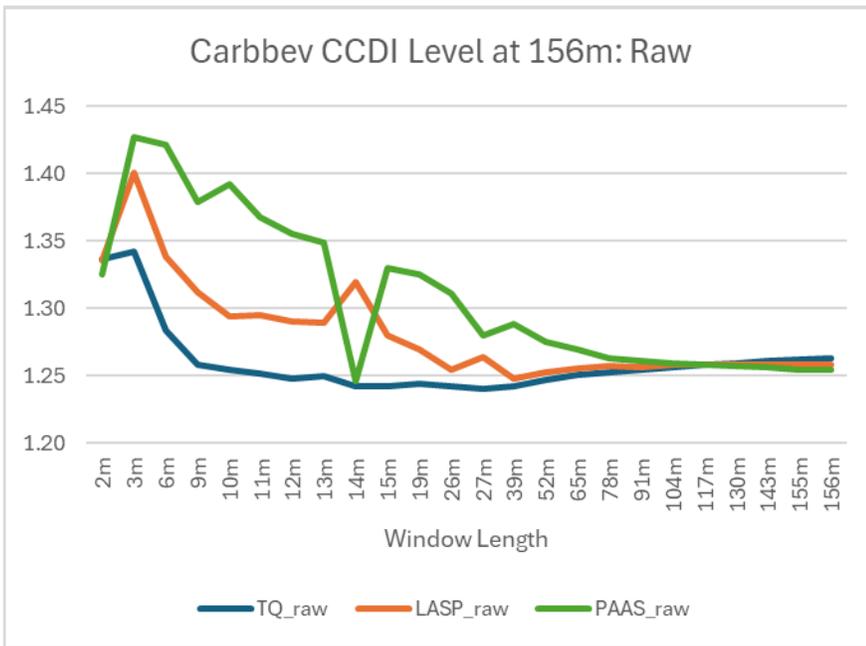
- Odd MPID results may be due to bilateral indexes longer than 2 months are less reliable
- An alternative measure of drift, Hill (2006)
- Simple chained Laspeyres and Paasche Indexes have too much drift to be useful

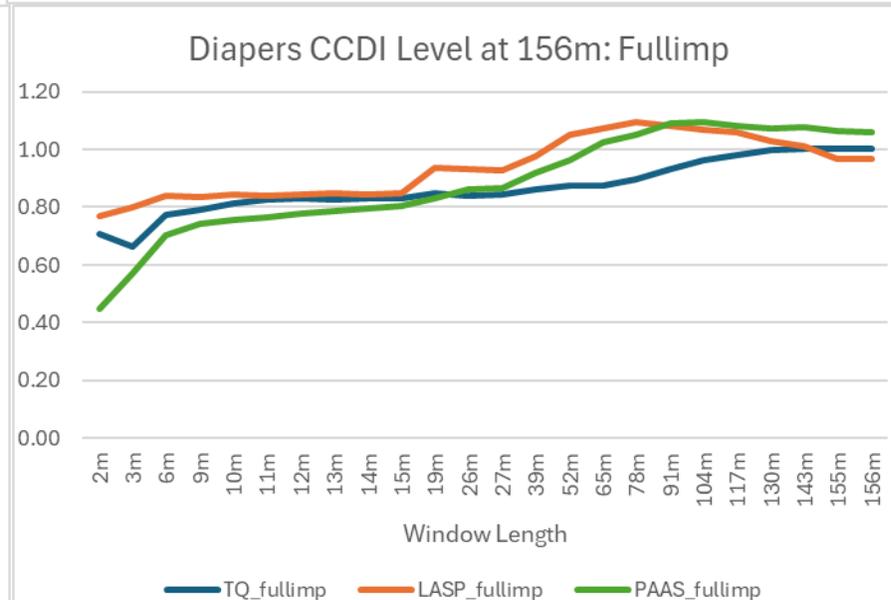
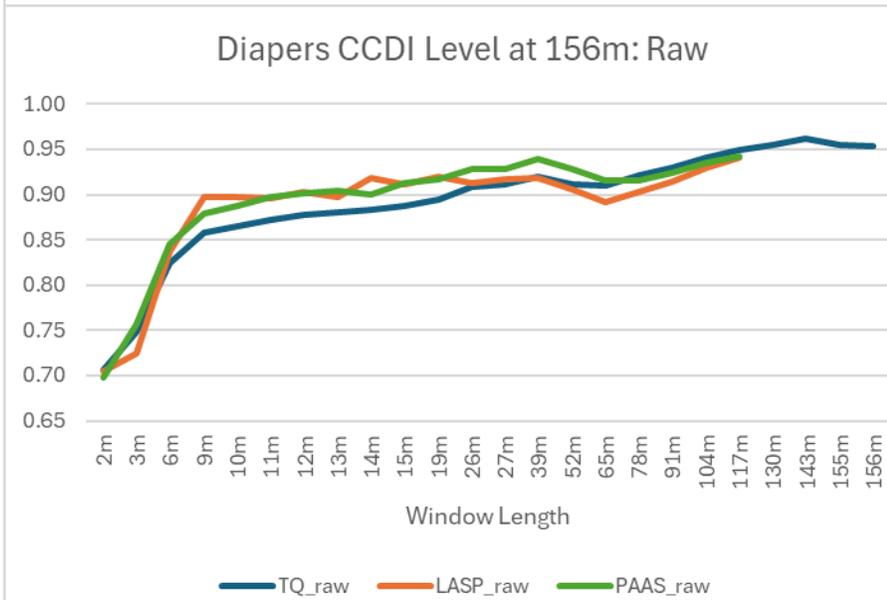
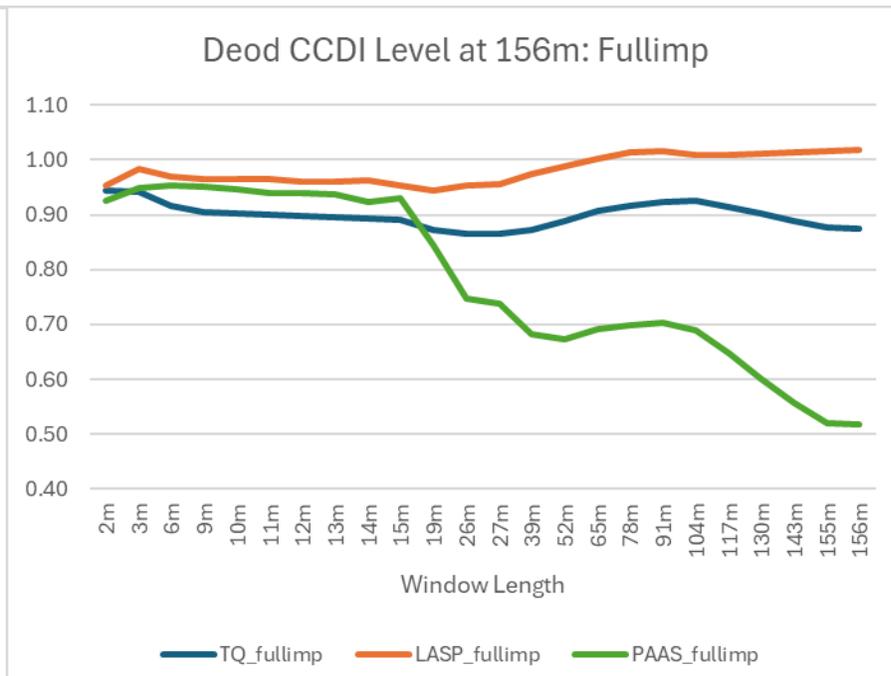
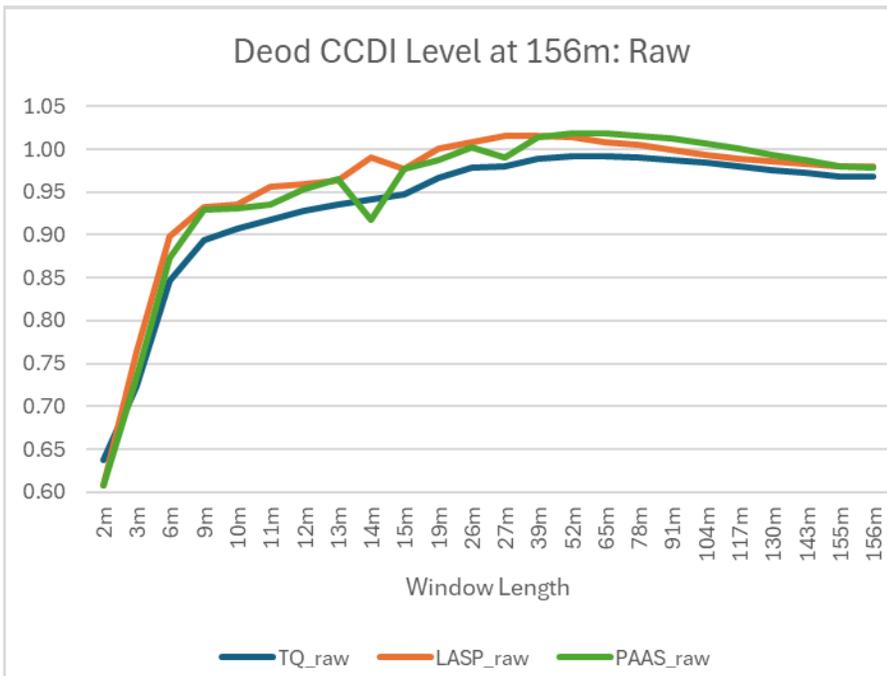


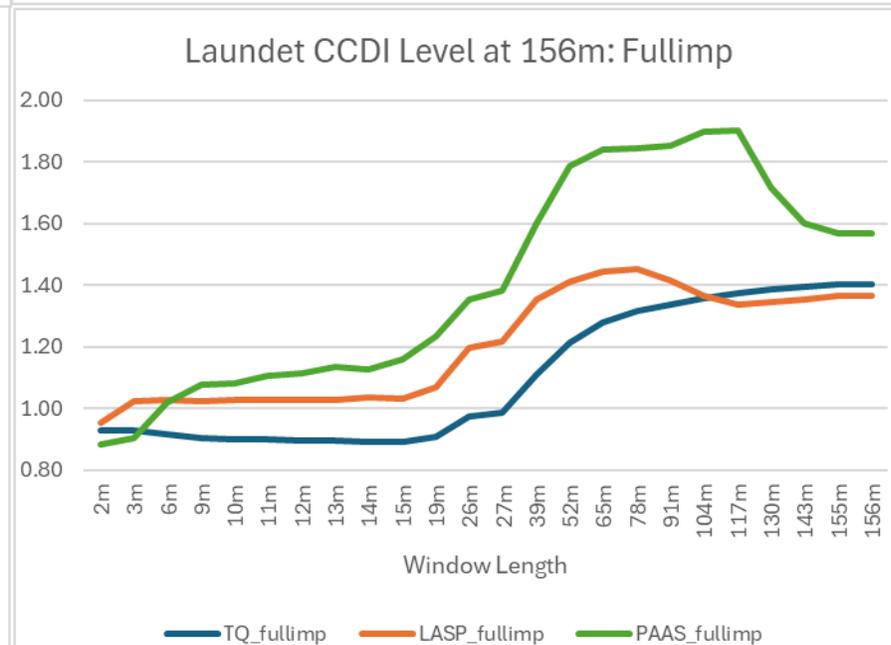
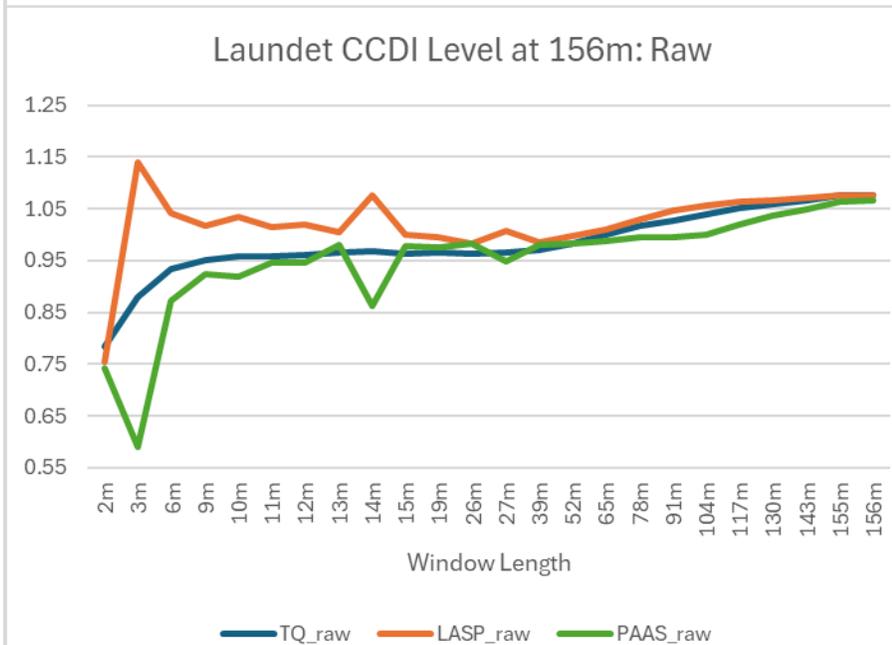
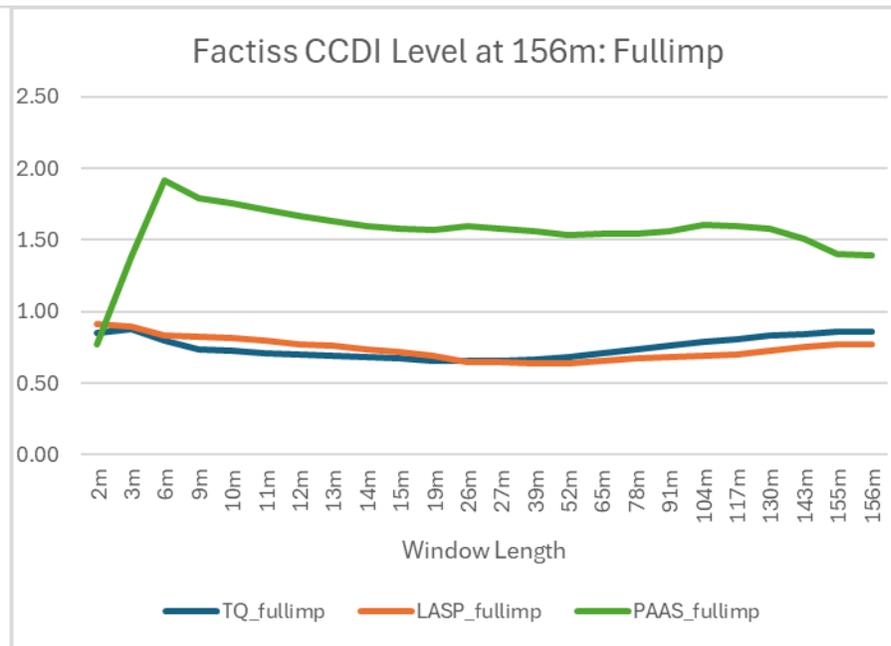
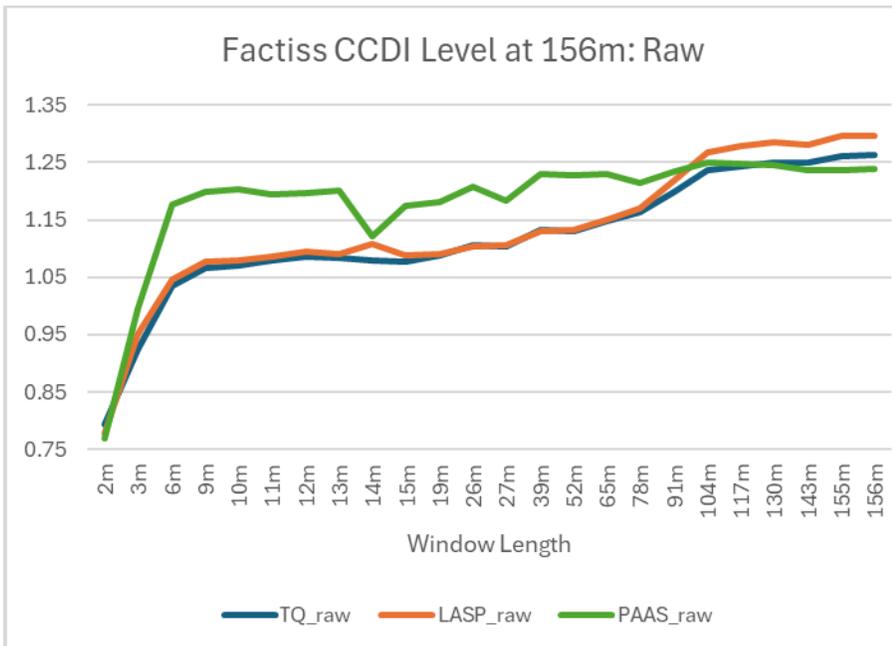


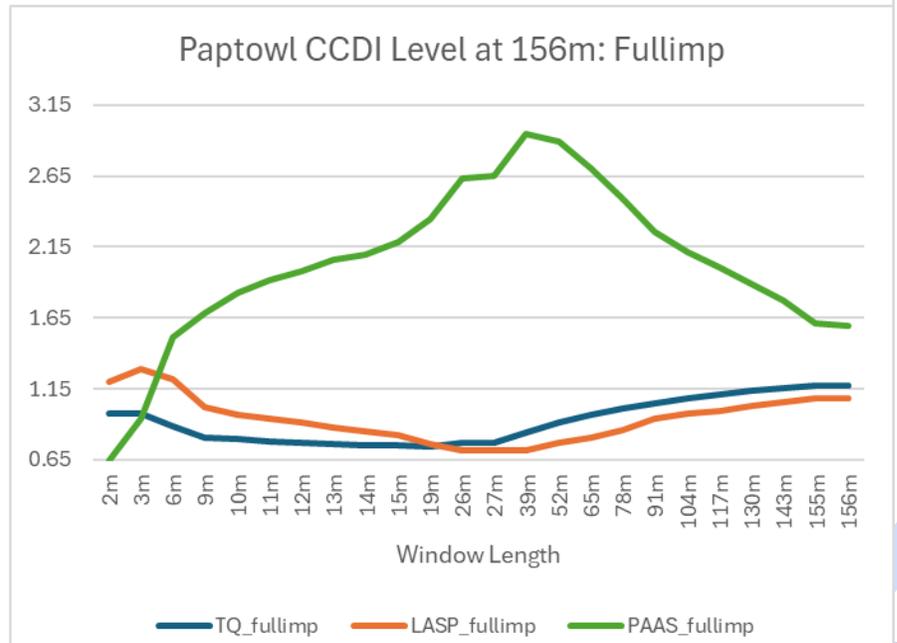
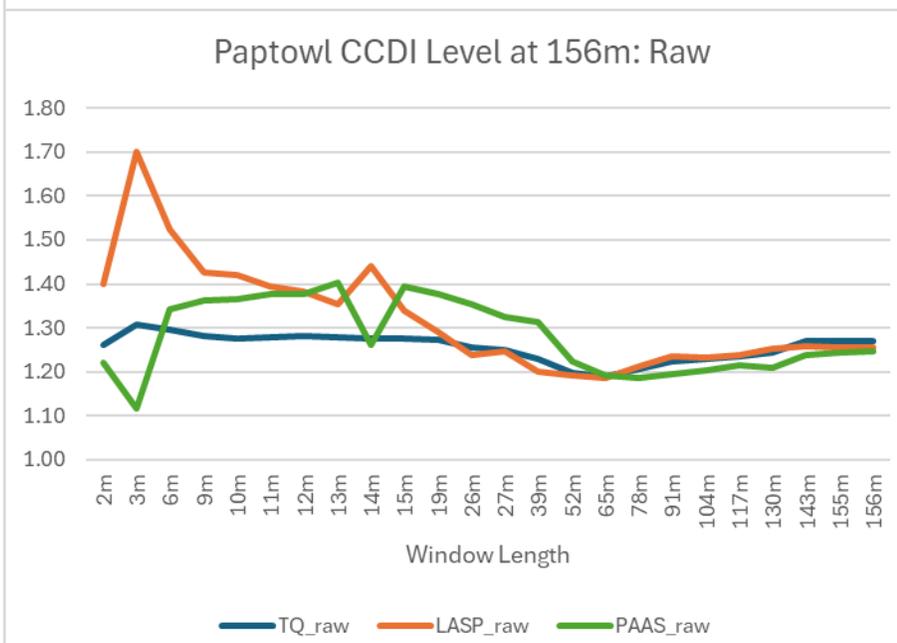
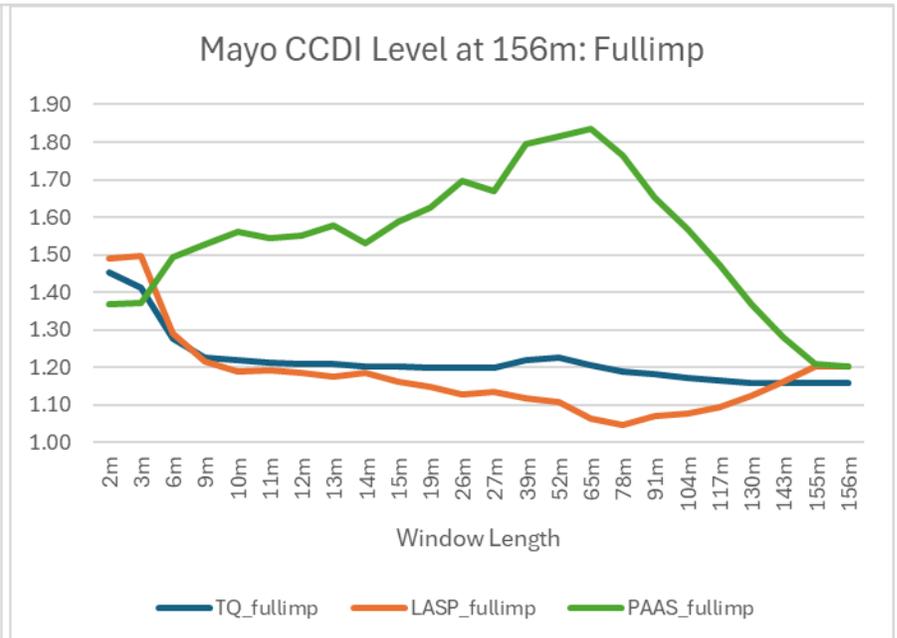
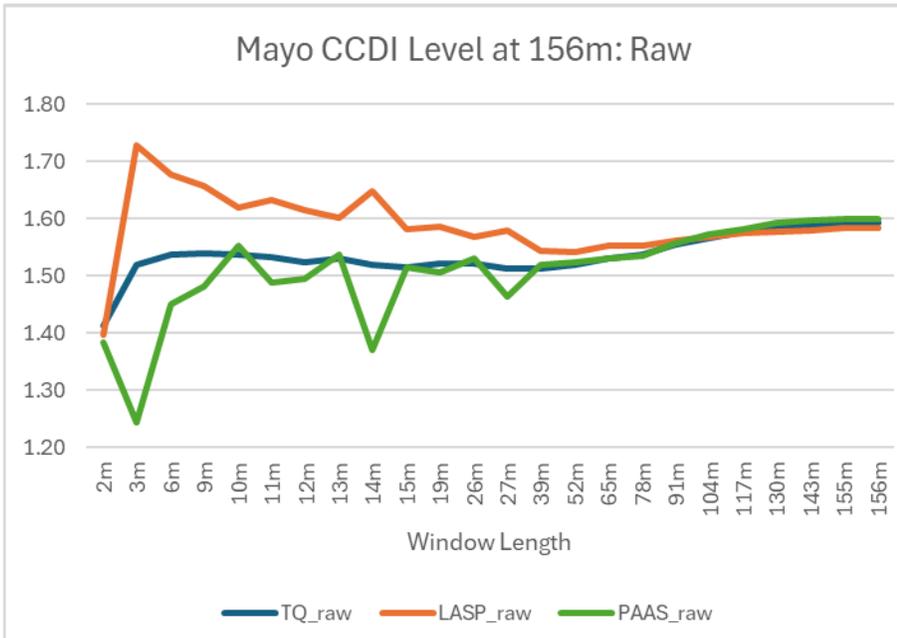
- GEKS Laspeyres and Paasche indexes constructed to approximate bounds
 - ▶ Not fully circular even for full window
 - ▶ 2 month not same as chained but still averages of bounds

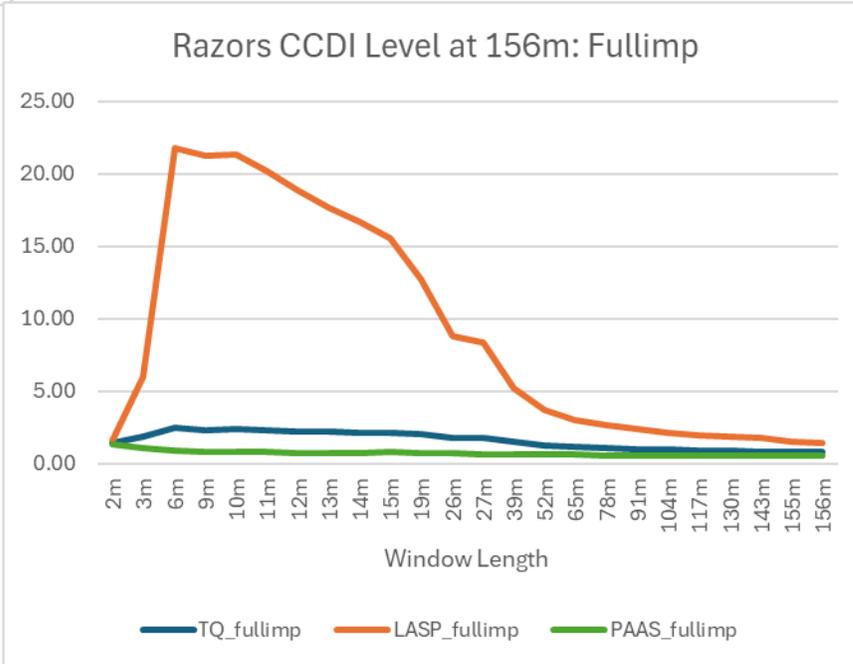
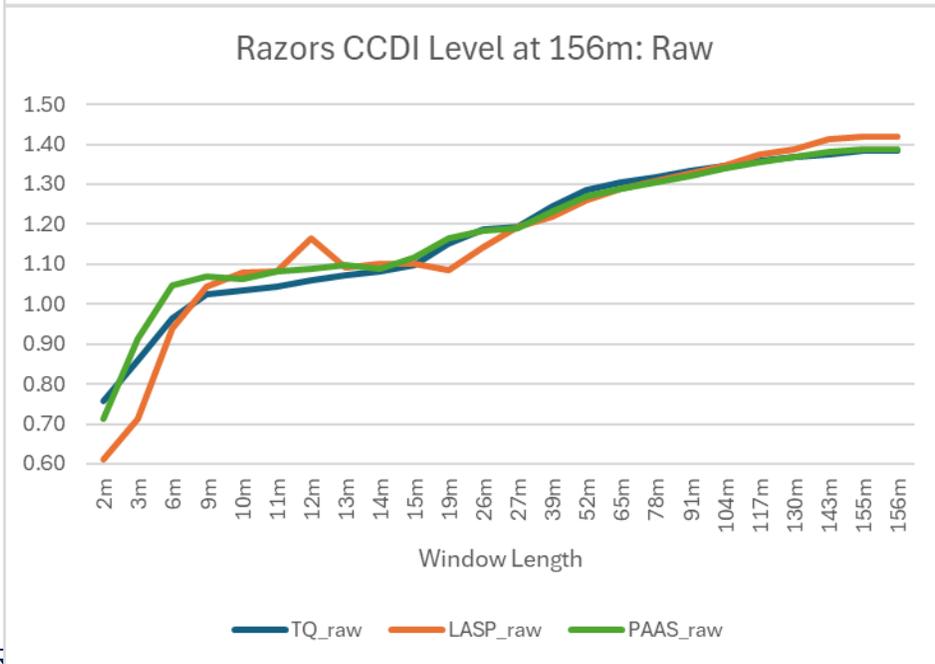
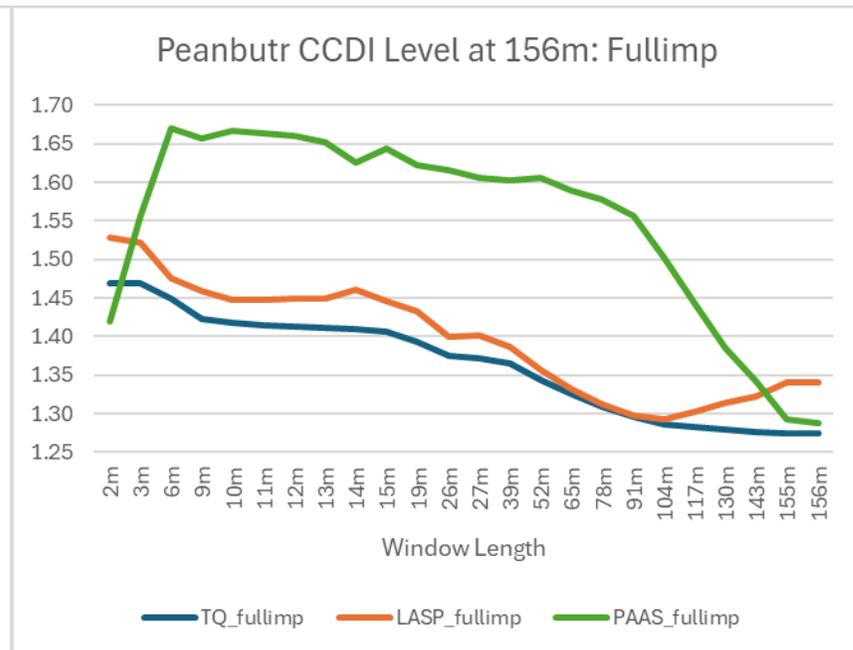
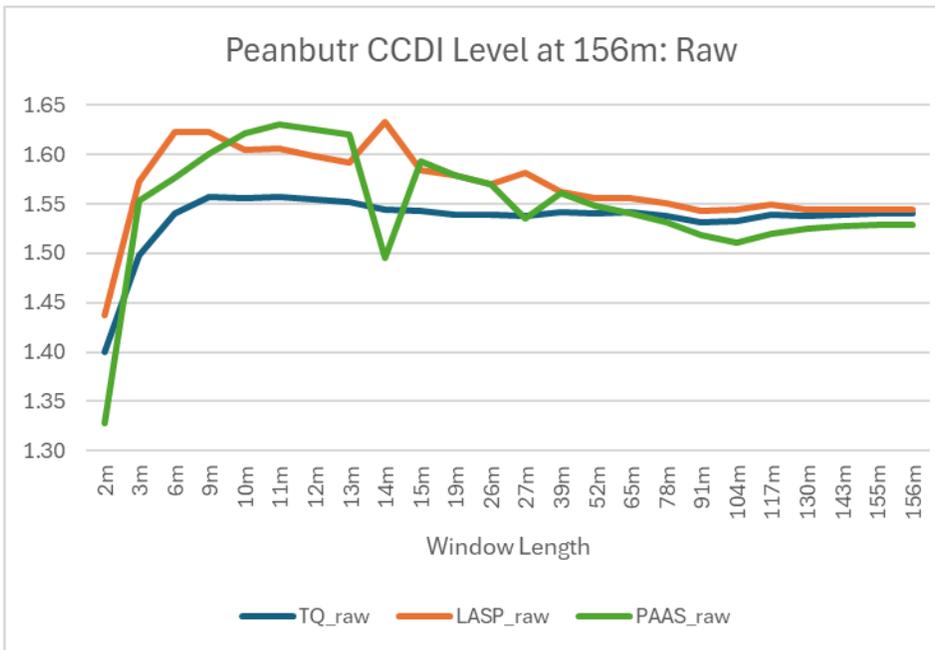


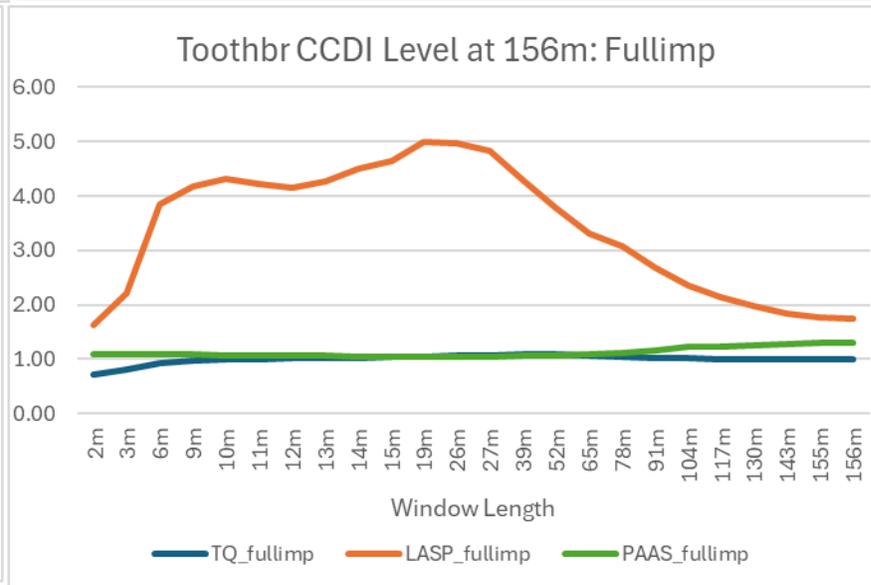
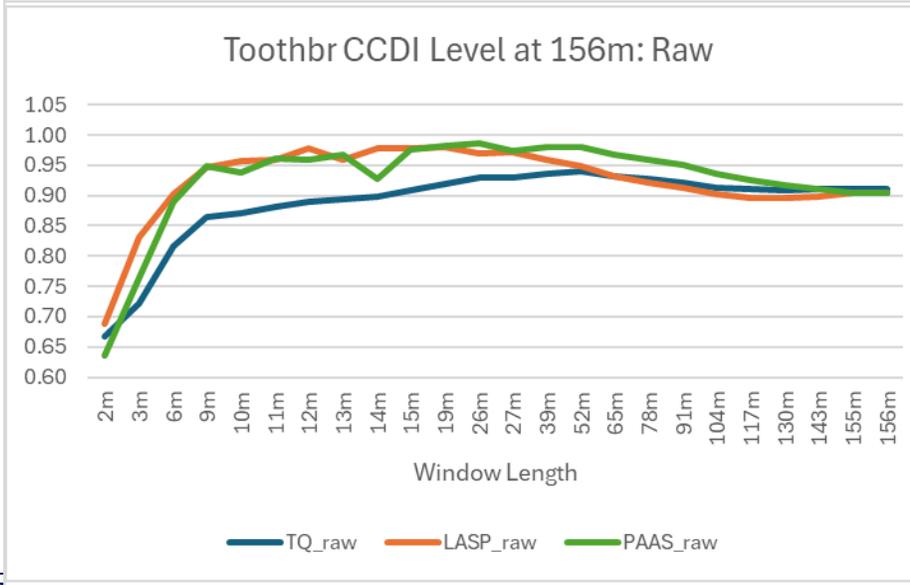
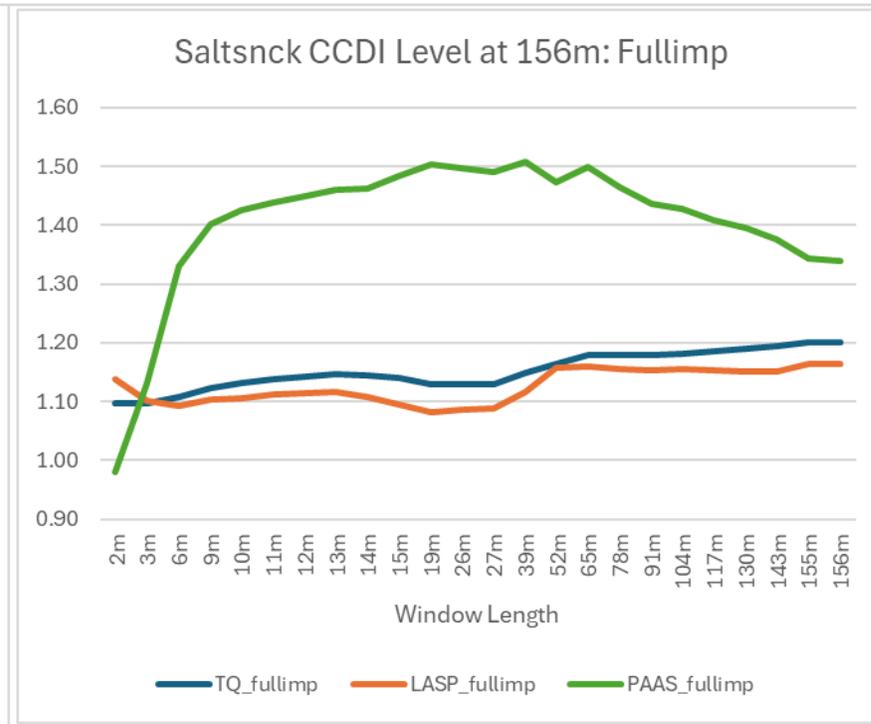
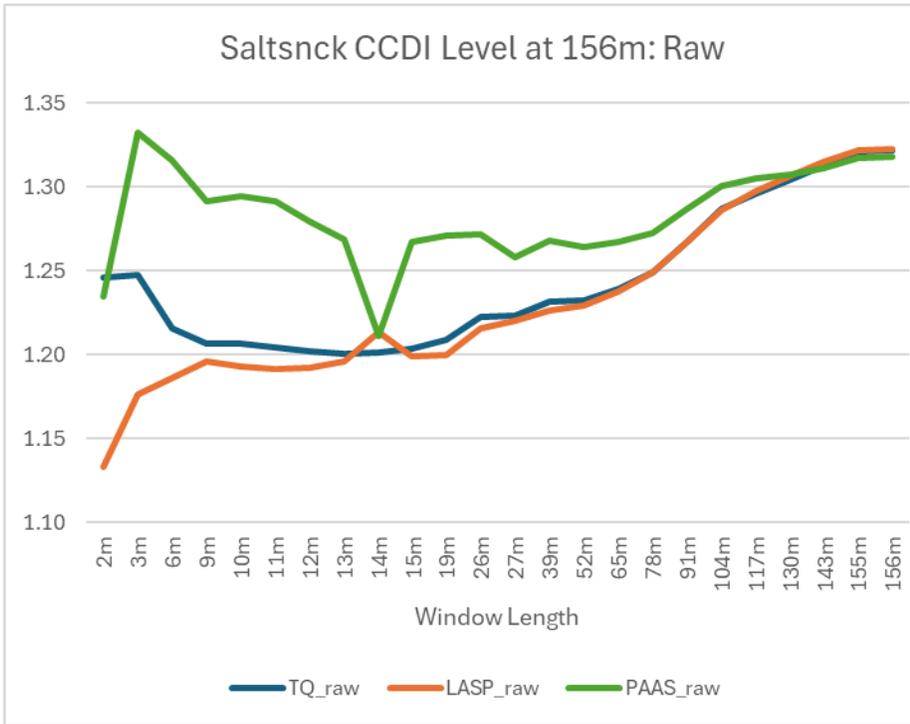


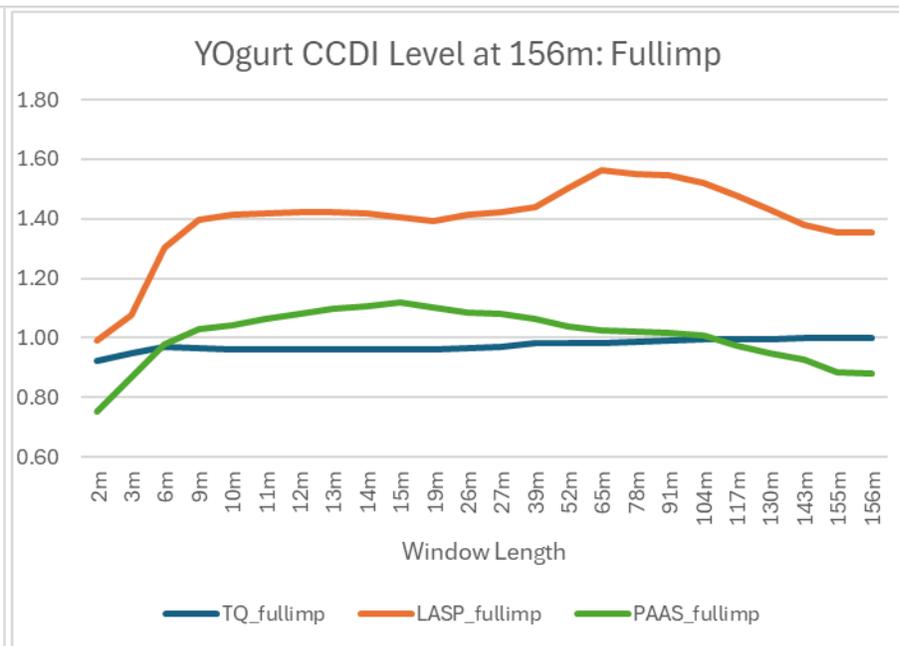
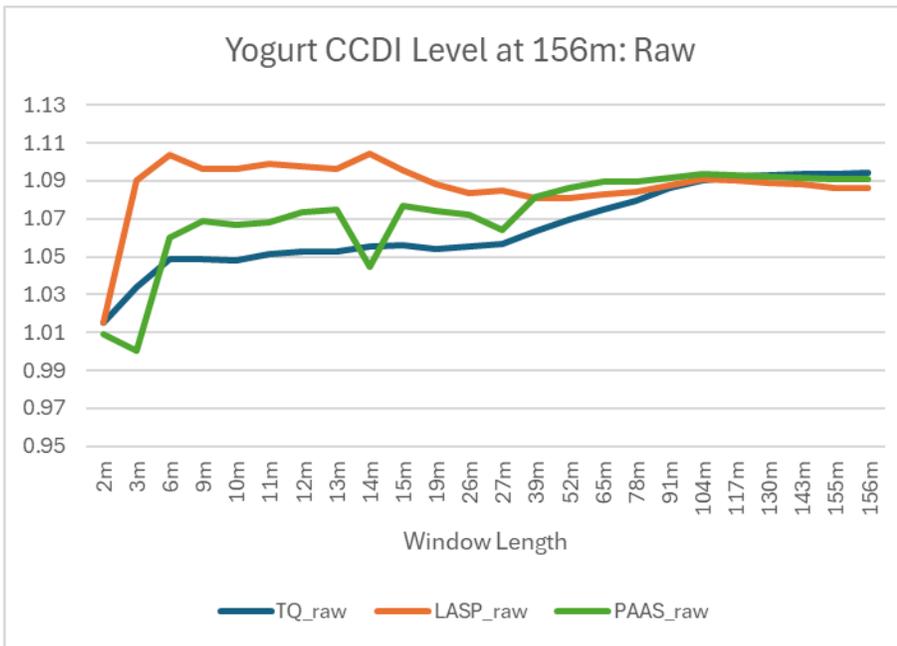












Laspeyres-Paasche Spread Results

- The raw data indexes converge at longer lengths
 - ▶ What would be expected of Divisia approximations with short interval and no chain drift
- Paasche often above Laspeyres
- Full imputation formulas don't converge nearly as much

Conclusions

- Full imputation hedonic regression, except for a small number of months, has manageable variance
- Hedonics can reduce chain drift by itself
- A short or medium length rolling window hedonic multilateral method could make drift worse instead of better
 - ▶ Not necessarily reliable
- Index formulas converge for raw indexes

Contact Information

Gregory Kurtzon

Research Economist

Division of Price and Index Number
Research/Office of Prices and Living
Conditions

www.bls.gov/pir

kurtzon.gregory@bls.gov

