THE SEASONAL ADJUSTMENT OF THE CONSUMER PRICE INDEXES OF WOMEN'S APPAREL: AN APPLICATION OF STATE SPACE MODEL BASED APPROACH TO INTERVENTION ANALYSIS

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I. Introduction

In most department stores and women's apparel stores, women's summerwear is offered for sale sometimes in the month of February of each year. However, these apparel goods are sold at regular prices and no markdown is taken at that time. Before 1990, BLS did not collect the prices of this summerwear until later in April or May of that year; but in 1990, BLS decided to collect the prices for these goods as soon as these apparel came to the market. The effect of this shift in the timing of price collection was an increase in the February and later monthly price indexes of some women's apparel categories. Moreover, in the switchover to summerwear from the winterwear in the year's first price collection, the comparison of new season's regular prices of summerwear with the end of last season's low sale prices of winterwear result in steep price increases applying upward pressure on women's apparel indexes. The policy shift combined with the effects of switchover may be treated as an "intervention" since the effects of different temporal supply and demand forces are introduced into these indexes. Since the interventions affect the quality of seasonal adjustment of a series, their effects must be accounted for by seasonal adjustment methods used. The X-11 ARIMA method which is currently being used to seasonally adjust BLS series and the State Space Model-Based (SSMB) method which has been proposed as an alternative to X-11 ARIMA method, treat the effects of intervention differently. In this paper, the performance of the two methods is compared empirically using a women's apparel series which is effected by the intervention mentioned above.

II. Seasonal Adjustment Methods and the Treatment of Interventions

The Census X-11 method or its modification X-11 ARIMA method is an empirical method of seasonal adjustment of a time series. It uses a sequence of moving- average filters to decompose a time series into it components. Since X-11 methods are "ad-hoc", non-parametric methods, statistical methods cannot be used to test their accuracy for seasonal adjustment. Moreover, there is no internal mechanism in the X-11 methods to take account of the effects of interventions in a time series if there are any. In practice, at the Bureau of Labor Statistics (BLS), when an index series is affected by interventions such as the ones which will be discussed in this paper, these series are first adjusted by another "ad-hoc" procedure called "RAMP" (see Buszuwski and Scott[1988]) to remove the effects of interventions and then the prior intervention adjusted series is seasonally adjusted by the X-11 ARIMA method. (A statistical procedure is said to be "ad hoc" in nature if it lacks a theoretical basis and well defined criteria of performance. Both X-11 methods of seasonal adjustment and the method of intervention/outlier adjustment used by the BLS fall into this category of procedures. The problem with this approach is that, if the intervention component is not orthogonal to the other components, trend and seasonal, then the results of seasonal adjustment would be unsatisfactory from a statistical point of view. In addition, the X-11 ARIMA procedure does not provide an estimate of the standard error of the seasonal component so that one cannot be sure of the precision of the resulting seasonally adjusted series.

On the other hand, SSMB is a model-based method in which every unobserved component of a time series such as trend and seasonal is specified by a structural model. In addition, an intervention variable is introduced as a separate component and its coefficient is given a dynamic specification. The effect of intervention is thus simultaneously estimated as the other unobserved components of the time series.

In this paper CPI of Women's Dresses 1984-01 to 1991-12 is analyzed. Several structural models were estimated for this series and the following model was found to be optimal (OSM) for it.

$$y_t = \mu_t + \gamma_t + \delta_t I_t + \varepsilon_t \quad (1)$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t + \theta_1 \eta_{t-1} + \theta_2 \eta_{t-2} \quad (2)$$

$$\beta_t = \beta_{t-1} + \zeta_t \quad (3)$$

$$\delta_t = \delta_{t-1} + \phi_t \quad (4)$$

$$I_t = 0 \qquad t \leq \text{JANUARY 1990}$$

$$= 1 \qquad t > \text{JANUARY 1990} \quad (5)$$

γ_t follows a sinusoidal function with white noise error. See

Harvey [1990] (6)

In the models above, y_t is the observed series, μ_t is the trend, γ_t is the seasonal component, I_t is the intervention variable, β_t is the slope of the trend. $\beta_t^{(12)}$ is the slope of the seasonal, δ_t is the coefficient of the intervention variable. The random errors η_t in the trend model (2) follow a second order moving average process. The random errors in all equations are assumed to be mutually and serially uncorrelated having zero mean and constant variance.

In order to compare the two methods of seasonal adjustment, the structural model approximation of X-11 (XSM) given by Burridge and Wallis [1984] in equation (3.3) is also estimated. XSM was estimated using the prior intervention adjusted series. This prior intervention adjustment was done by the "RAMP" procedure used by the BLS currently. The structural model of seasonal adjustment discussed above is cast into the state-space form and Kalman filtering and smoothing techniques are used to estimate all the unobserved components. The estimation of hyper-parameters done by EM algorithm and by a quasi-Newton numerical optimization technique. The Kalman filter is initialized with a diffuse prior.

Structural models OSM and XSM are compared with respect to(i) various tests of misspecifications including the tests for the adequacy of a model to explain the series and tests for nonlinearity, hetroscedaticity and normality of the residuals; (ii) goodness of fit of the structural models to the data series and (iii) the forecasting performance of the models. The various statistical test and criteria for model comparison used in this paper are described in Harvey [1990]. We also use two nonparametric statistics: BDS and Modified BDS statistics to test the adequacy of the structural models. These statistics are developed in the Chaos literature.(see Mizrack [1991] and Brock, Dechert, and Scheinkman [1987].)

The comparison of the quality of seasonal adjustment includes the comparison of the smoothness of the trend and seasonally adjusted series, the presence of residual seasonality etc. The residuals from the X-11 ARIMA run for the series are also used to estimate various diagnostic tests for comparison.

III. Empirical Results

In this section the OSM and XSM models fitted to each series are evaluated and compared with respect to the specification, goodness of fit and the forecasting performance of the models. Some of the diagnostics are also compared to those obtained directly from the X-11 ARIMA seasonal adjustment of each series. The graphs of various components of the time-series and some other statistics are used to compare the quality of seasonal adjustment by the two methods.

(a) CPI of Women's Dresses (WDR): This is a non-stationary time series with a significant seasonal component. This series is affected by the intervention of February 1990 mentioned earlier. The empirical results in Table 1 show that for this series, OSM residuals are uncorrelated since all tests for the adequacy of model including Ljung-Box Q*, BDS and Modified BDS accept the null hypothesis of no autocorrelation. Other diagnostic tests indicate that the OSM residuals are linear, homoscedastic and normal. On the other hand, the XSM residuals are not uncorrelated since the null hypothesis is accepted by BDS test only. Q* and MBDS tests reject the null hypothesis indicating that XSM is not an adequate model for this series. Moreover, the chi-square tests indicate that XSM residuals are not linear either. Even the diagnostic tests estimated from X-11 ARIMA residuals indicate that those are correlated and not normally distributed since the null hypothesis of no autocorrelation and normality are rejected by the statistical tests. Only the null hypothesis of linearity and homoscedasticity are accepted for the X-11 residuals. However, the goodness of fit statistics such as AIC, BIC, and PEV in Table 2 are lower for XSM than OSM. But the negative RBARSQUARE (SEAS) and the presence of auto correlation is the residuals for XSM suggest that even though the goodness of fit is better for XSM, X-11 is not the appropriate method to use for seasonal adjustment of this series. A similar argument was given by Cleveland and Tiao [1976] in rejecting X-11 as an appropriate method of seasonal adjustment for the telephone data. Moreover the forecasting performance of OSM is superior to that of XSM for this series with lower Mean Prediction Error Sums of Squares (MPESS) for both single step (SS) and multi-step (MS) ahead forecasts. See table 3. The predictive t-test and F-test for both models indicate that the same model is appropriate for the prediction period as well. Also in figure 4a, single step and multistep forecasts for OSM seem to be slightly better than those of XSM.

In figure 1a, the intervention component and the intervention adjusted series obtained by the two methods are shown. As mentioned earlier, BLS uses an ad hoc procedure to adjust a series for outliers and interventions before it is seasonally adjusted by X-11 ARIMA method. On the other hand, SSMB introduces an intervention component in the model and estimates its effects simultaneously with other components. The intervention component estimated by OSM starts out with zero effect until the intervention took place in February 1990; then the effect increases sharply and remains positive till the end of the sample period. Obviously the intervention adjusted series is thus lower that he original series after February 1990. However, the intervention adjusted series used by X-11 ARIMA seems to be affected by the prior adjustment from the beginning of the sample period and continuing after the intervention took place although the prior adjustment is very small in the later half of the series. Since the intervention effect is negative in the first 40 or so months, the effect of this prior adjustment is to raise the level of the series for that period and almost no adjustment is made just before or after the intervention took place. It is, however, difficult to find a reasonable statistical or economic interpretation of this prior adjusted series, before it is seasonally adjusted by X-11 ARIMA procedure.

In figure 2a, the unadjusted series, the estimated trend component and the estimated "pseudo-trend" component obtained by OSM, XSM and X-11 ARIMA procedure are shown. The "pseudo-trend" components is the sum of the trend component and the intervention component. The trend components estimated by OSM and X-11 ARIMA are both quite smooth, the later being slightly smoother than the former and both are smoother than the trend components of XSM. The same is true for "pseudo-trend" component. It is interesting to note that the X-11 trend component and the X-11 "pseudo trend" component are almost the same except for the first 40 or so months. On the other hand the OSM trend component is below the series after February 1990 suggesting what the trend would be if there was no intervention. The XSM and the X-11 trend components before the intervention may be interpreted as to what the trend would be if the intervention had taken place at the beginning of the sample period. However, this is not operationally a very useful interpretation of the trend components from the policy maker's point of view

The figure 3a shows the seasonal component and its standard error, the seasonally adjusted series and the unadjusted series for OSM, XSM, and X-11 ARIMA. The standard errors of seasonal components for both OSM and XSM indicate that the seasonality is significant in this series. This is also borne out by the F-statistics for stable seasonality which are computed for both one-way ANOVA and two-way ANOVA of the pseudo-trend adjusted series. The F-statistics for moving seasonality indicates that there is significant moving seasonality as well. The changing amplitude of the estimated seasonal component for both OSM and X-11 ARIMA support that conclusion. The seasonally adjusted series for OSM seems to be slightly smoother than that of X-11 ARIMA and XSM. The slightly lower value of SASR (seasonally adjusted series rough: not shown in any table) for OSM than those of XSM and X-11 AIRMA support that observation.

IV. Summary and Conclusion

The two methods of seasonal adjustment, SSMB and X-11, handle the interventions differently and hence the estimates of various components are found to be different. A comparison of the structural models indicates that the OSM performs better than the XSM. In addition, the OSM estimates of various components of the time series especially the intervention and trend component are more meaningful than those obtained by X-11 ARIMA or XSM. A complete version of the paper can be obtained from the author.

V. References

Brock, W.A., W.D. Dechert, and J. Scheinkman (1987), "A Test for Independence Based on the Correlation Dimension," University of Wisconsin-Madison, Working Paper 8702.

Burridge, P., and Wallis, K.F. (1984), "Unobserved-Components Models for Seasonal Adjustment Filters," *Journal of Business & Economic Statistics*, 2, 350-359.

Buszuwski, J.A. and Scott, S. (1988), "On the Use of Intervention Analysis in Seasonal Adjustment," *Proceedings of the ASA section on Business & Economics, 337-342*

Cleveland, W.P., and Tiao, G.C. (1976), "Decomposition of Seasonal Time Series: A Model for the Census X-11 Program," *Journal of American Statistical Association*, 71, 581-587.

Harvey, A.C. (1990), "Forecasting, Structural Time Series Models and the Kalman Filter," Cambridge,

Mizrack, B. (1991), "A Simple Nonparametric Test for Independence of Order (P)", Unpublished.

Table 1. TESTS OF MISSPECIFICATION OF A MODEL

		ADEQUACY OF A MODEL TESTS					OTHER DIAGNOSTIC TESTS				
		LJUNG-BOX CHI-SQUARE TESTS FOR DIFFERENT NUMBERS OF AUTO-CORRELATIONS IN RESIDUALS (degrees of freedom)			BDS	MBDS (Min/Max)	CHI-SQUARE TEST OF NON-LINEARITY IN RESIDUALS FOR DIFFERENT NUMBERS OF AUTOCORRELATIONS IN RESIDUALS (degrees of freedom)			F-TEST OF HETRO-SCHEDASTI-CITY IN RESIDUALS (degrees of freedom)	CHI-SQUARE TEST OF NORMALITY OF RESIDUALS (degrees of freedom)
Series	Model	7	12	24			7	12	24		
WDR	OSM	-	4.69* (3)	29.04* (15)	0.16*	-1.36/* 1.48	2.63* (7)	4.25* (12)	24.29* (24)	0.62* (27,27)	2.39* (2)
	XSM	43.32 (7)	51.28 (12)	153.57 (24)	1.07*	-0.25/ 2.09	15.43** (7)	28.50 (12)	62.38 (24)	1.50* (19,19)	2.96* (2)
	X11	35.65 (7)	47.11 (12)	92.59 (24)	2.72	0.92/ 2.89	13.91* (7)	17.38* (12)	32.36* (24)	0.14* (32,32)	36.30 (2)

Note 1: One star (*) indicates that null hypthesis accepted at 5% level of significance;
Two stars (**) indicate that null hypothesis accepted at 1% level of significance.

Table 2. GOODNESS OF FIT OF A MODEL

		AIC	BIC	PEV	SGMASQ	RBARSQUARE		
SERIES	Model					Regular	Difference	Seasonal
WDR	OSM	322.36	350.56	14.03	1.01	0.94	0.73	0.23
	XSM	304.65	304.65	9.47	1.11	0.91	0.72	-0.26

Table 3. FORECASTING PERFORMANCE OF A MODEL

		MPESS		Post-Sample PEV	Predictive t-test (degrees of freedom)	Predictive F-test (degrees of freedom)	
SERIES	MODEL	SS	MS			SS	MS
WDR	OSM	20.49	25.78	282.52	0.88* (85)	1.45* (15,96)	0.33* (15,96)
	XSM	33.43	939.19	37318.92	1.67* (96)	3.52 (15,96)	1.70* (15,96)

Note 1: One star (*) indicates that null hypthesis accepted at 5% level of significance;
Two stars (**) indicate that null hypothesis accepted at 1% level of significance.

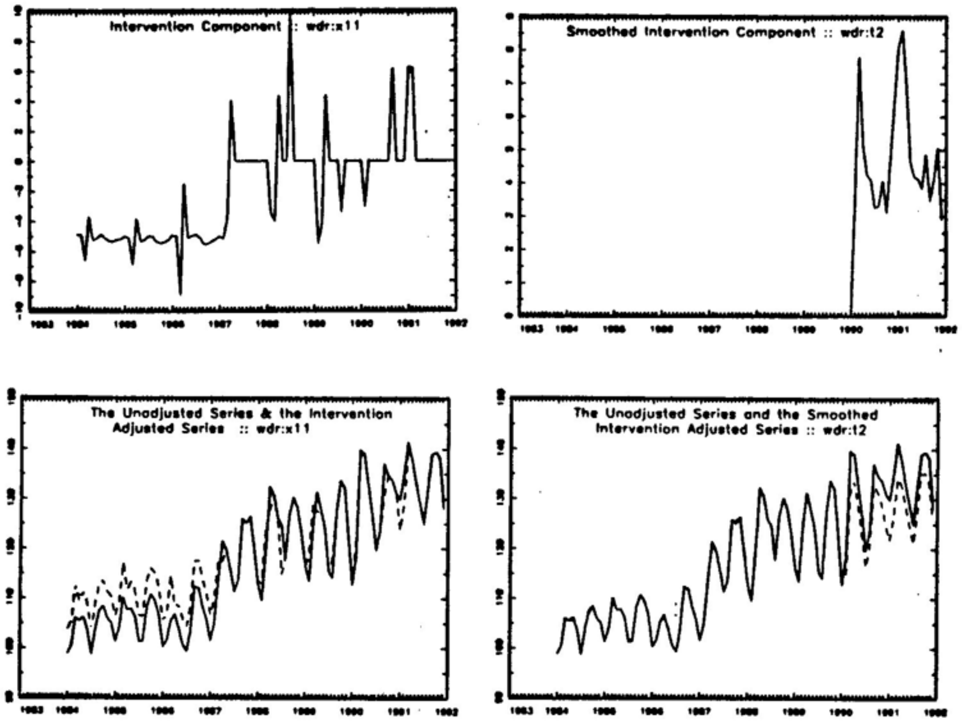

Figure 1a. The Intervention Component & Intervention Adjusted CPI of Women's Dresses

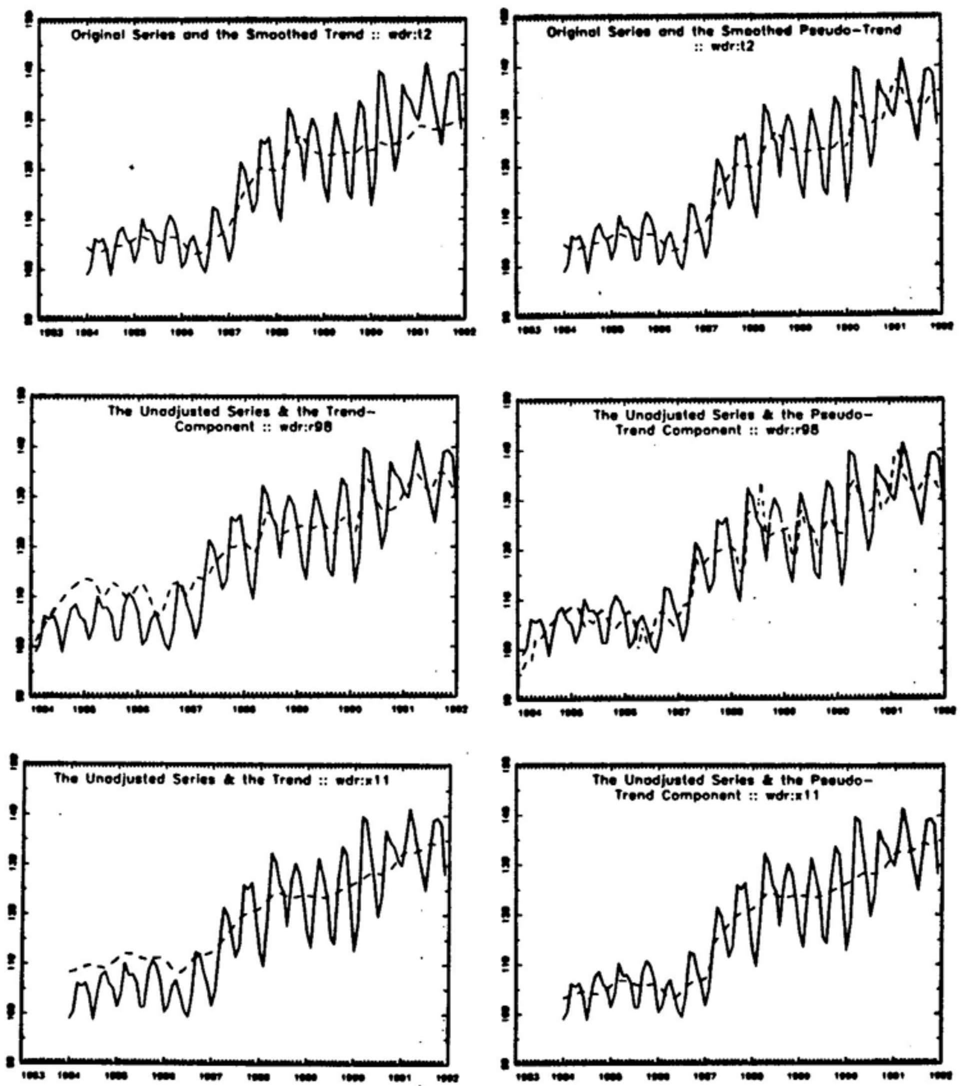

Figure 2a. The Trend & Pseudo-Trend component and the Unadjusted CPI of Women's Dresses

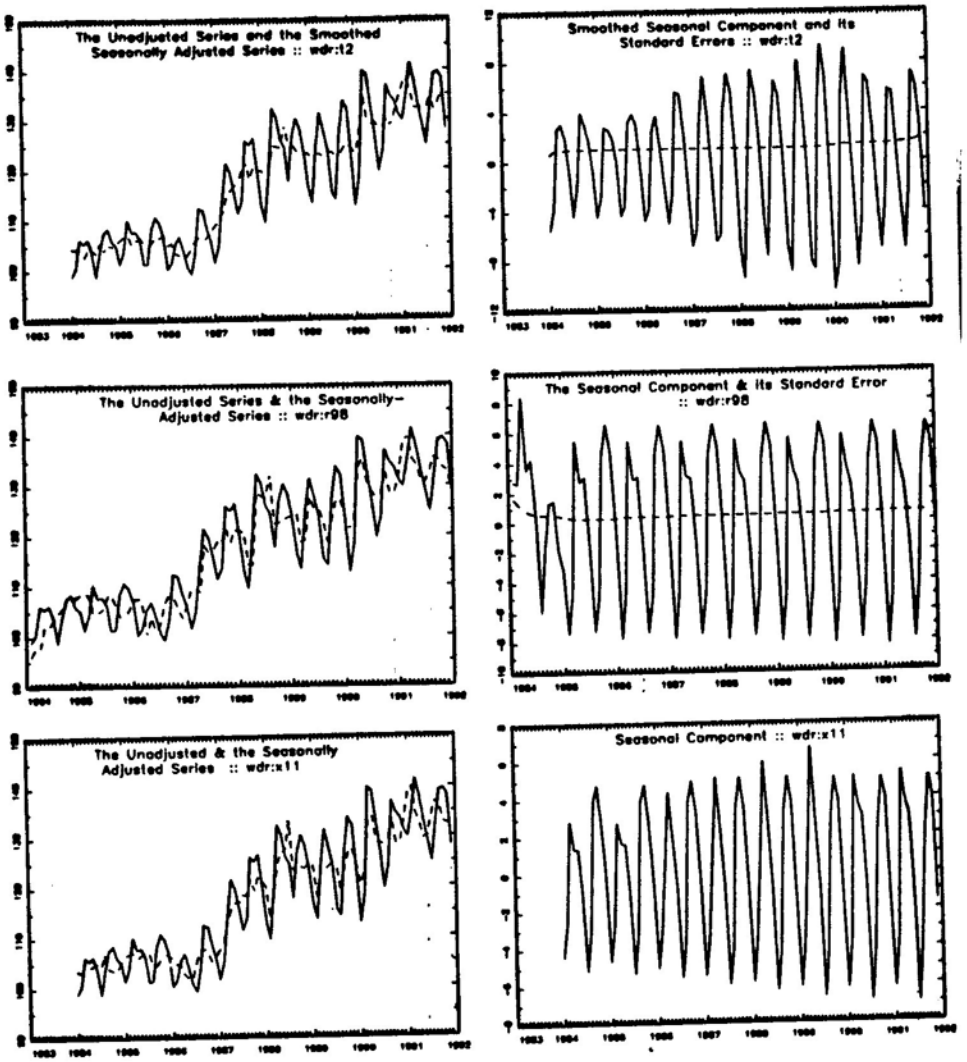

Figure 3a. The Seasonal Component and the Seasonally Adjusted CPI of Women's Dresses

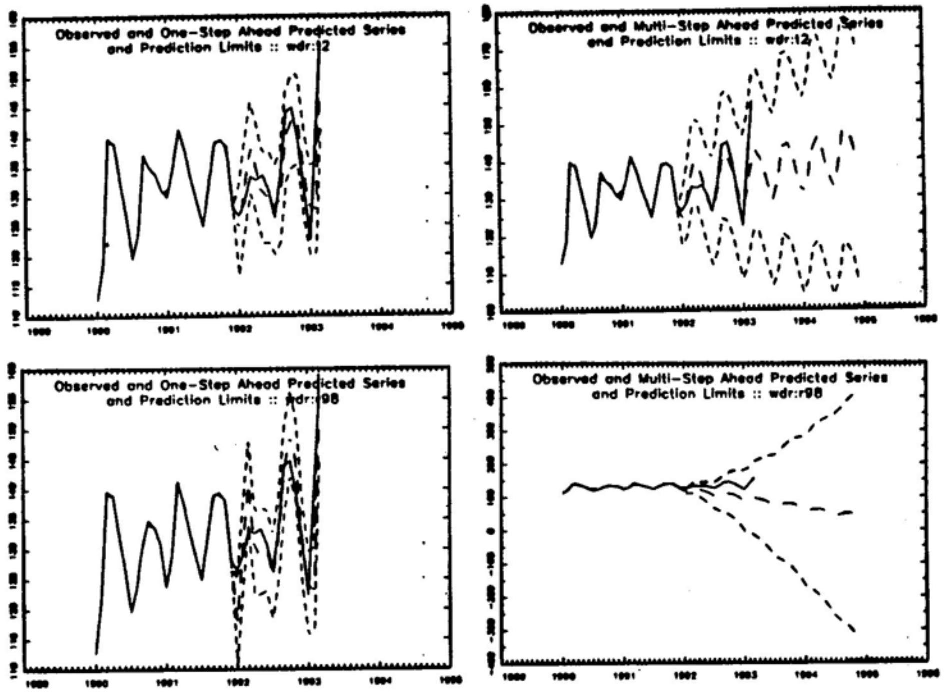

Figure 4a. The Single-Step & Multi-Step Forecast of the CPI of Women's Dresses