TRADING DAY AND EASTER HOLIDAY IN SEASONAL ADJUSTMENT METHODS

Raj K. Jain, Bureau of Labor Statistics, 2 Massachusetts Avenue, NE, Room 3105

Washington, DC 20212-0001

Key Words: Kalman Filter, X-12 ARIMA, State Space

1. Introduction

Several retail sales and whole sales series published by the Bureau of Census are not only seasonal but are also affected by other periodic movements such as those generated by the number of trading days in a month and the Easter holidays. In this paper we present the empirical results of a study of the comparative performance of three seasonal adjustment methods (i) State Space Model-Based (SSMB) method of Jain (1992), (ii) Census X-12 ARIMA and (iii) ARIMA Model-Based method of Burman (1980) as applied to two Census seasonal series which are also affected by trading days and Easter holidays. The two series analyzed in this paper are (i) Whole Sales of Hardware Plumbing and Heating Equipment etc. (HDW) (ii) Retail Sales of Men's & Boy's Clothing Stores (MBC).

2. SSMB Method and the Structural Models

In the SSMB method, a structural model of a time series consists of a decomposition equation with unobserved components such as trend, seasonal etc., and regression components such as trading day and Easter holiday, and the models for the various such components. The structural model is cast in a state-space form and Kalman filtering and smoothing technique is used to estimate all the components. The Kalman filter is initialized by a diffuse prior. The hyper-parameters of the model are estimated by EM algorithm and a quasi-Newton algorithm. The following are the minimum AIC structural models for the HDW and MBC series respectively:

Optimal Structural Model (OSM) for HDW

$$y_t = \mu_t + \gamma_t + TD_t + \varepsilon_t \quad (1.1)$$

$$\mu_t = 2\mu_{t-1} - \mu_{t-2} + \eta_t \quad (1.2)$$

$$\gamma_t = \gamma_{t-12} + \omega_{1t} \quad (1.3)$$

$$\gamma_t = -\sum_{j=1}^{11} \gamma_{t-j} + \omega_{2t} \quad (1.4)$$

$$TD_t = \sum_{j=1}^{6} \alpha_{jt}(d_{jt} - d_{7t}) \quad (1.5)$$

$$\alpha_{jt} = \alpha_{jt-1} + \delta_{jt} \quad (1.6)$$

All errors are assumed to be serially and contemporaneously uncorrelated with means zeros and finite variances. d_{jt} j=1,2,...6,7. are the number of jth days in a month. See Kitagawa & Gersch(1984).

Optimal Structural Model (OSM) for MBC

$$y_t = \mu_t + \gamma_t + TD_t + ED_t + \varepsilon_t \quad (2.1)$$

$$\mu_t = 2\mu_{t-1} - \mu_{t-2} + \eta_t \quad (2.2)$$

TRIGNOMETRIC MODEL OF

SEASONALITY (Harvey 1990) (2.3)

$$TD_t = \sum_{j=1}^{6} \alpha_{jt}(d_{jt} - d_{7t}) \quad (2.4)$$

$$\alpha_{jt} = \alpha_{jt-1} + \delta_{jt} \quad (2.5)$$

$$ED_t = \beta_t(h_t - 1/12) \quad (2.6)$$

$$\beta_t = \beta_{t-1} + \delta_t \quad (2.7)$$

where h_t is the proportion of days before Easter that falls in month t. It is zero for all months except March and April.

3. Census X-12 ARIMA Method (X12)

X12-ARIMA method is an enhanced version of X-11, in which those series affected by trading day, Easter holiday, outliers etc. are first adjusted for these effects by REGARIMA models of this method and then seasonally adjusted by X-11. REGARIMA models are regression models with ARIMA errors. Six trading day regression variables are defined as: $T_{jt} = (D_{jt} - D_{7t})$ j=1,2,....6 where D_{jt} is the number of jth days of the week in month t. Easter holiday regression variable is defined as: $E(\omega, t) = (1/\omega)$* (number of ω days before Easter in month t).This variable is zero in all months except March and April so long as $\omega \leq 21$

4. ARIMA Model-Based Method (BRM)

As in Census X-12A method, the series under consideration are first adjusted for trading day and Easter holiday effects. Trading day variables are defined as: $T_{jt} = D_{jt} / N_t \quad j = 1,2,\ldots 7.$ where D_{jt} is the number of j days in month t and N_t is the length of month t. Easter holiday variable is defined as in X-12A method. This method first finds the optimal (minimum AIC) ARIMA model for the trading day and Easter holiday adjusted series under consideration. Then it estimates the optimal ARIMA model by maximum likelihood method. The parameter estimates and the forecasts and backcasts of the chosen model are used to partition the model spectrum, generate the trend and seasonal filters which are then applied to the series together with its forecasts and backcasts. For this paper, Burman's computer program called PROPHET was used

5. Empirical Results

Wholesale of Hardware, Plumbing and Heating Equipment etc. (HDW): This is a monthly series which is very seasonal and is non-stationary. This is affected by the number of trading days in a month but not the Easter holiday. For analysis, we used 96 observations of this series from January 1985 to December 1992. The empirical results in Table 1 show that for this series, the residuals generated by all three methods are correlated since all Ljung-Box Q* tests for the adequacy of the model reject the null hypothesis. BDS and Modified BDS tests (See Jain (1993) for references), however, accept the null hypothesis of no auto-correlation in the residuals at 5% level of significance for all three methods. Other diagnostic tests indicate that for OSM and BRM models, the residuals are linear, homoscedastic and normally distributed and for X12 only the null hypothesis of linearity with 12 and 24 autocorrelations and null hypothesis of homoschedasticity are accepted. The Chi-square test of normality rejects the null hypothesis of normality for X12 residuals. In Table 2, the two statistics, Akaiki Information Criterion (AIC) and SGMASQ, the mean of the sum of squared model residuals, give mixed results on the goodness of fit of the three models. AIC is lowest for the BRM and highest for X12 whereas SGMASQ is lowest for OSM but highest for X12 indicating that the X12 has the worst fit of all three methods. All three RBARSQ's are highest for BRM and lowest for OSM indicating a superior fit by BRM. The forecasting performance also is best for BRM and worst for OSM since RMPESS, root mean prediction error sum of squares, is lowest for BRM.

In table 3 the F-tests for stable seasonality estimated for all three models reject the null hypothesis of no stable seasonality which obviously confirms our visual belief that the series is seasonal. However the F-tests for all three methods accept the null hypothesis of no moving seasonality in the series. The m7 statistics for all three methods are within acceptable range which indicates that the seasonality is identifiable. In figure 1. first row, the trends from all three methods are smooth although the BRM trend is a bit less smooth than the other two. The TRS statistics in table 3 bear this out being the smallest for BRM. The seasonally adjusted series (not shown) are not very smooth for any method although the SAS statistics as given in Table 3 indicate that BRM gives the least unsmooth series. The reason for the roughness in seasonally adjusted series is the presence of the periodic movements generated by the differing trading days in a month in the monthly series. Thus when we compare the combined seasonally and trading day adjusted series for the three methods in figure 1. row 2, OSM gives the smoothest adjusted series whereas for X12 and BRM, the combined adjusted series are still pretty unsmooth. The reason for this phenomenon can be found in the graphs of the residual errors of the three methods

in row 3. The OSM residuals have the smallest range between -0.03 and +0.03; the X12 residuals have the biggest range and lie between -500.00 and +375.00; and the BRM residuals have the range much smaller than that of X12 but much bigger than that of OSM. This indicates that the X12A and BRM methods do not decompose the series as well as OSM does.

Retail Sales of Men's & Boy's Clothing Stores (MBC): This also is a monthly non-stationary time series with a significant seasonal component. This series is affected by trading days and Easter holiday as described above. For analysis, we used 96 observations of this series from January 1985 to December 1992. The empirical results in Table 1 show that for this series , Ljung-Box Q*, the adequacy tests for both OSM and BRM models accept the null hypothesis of no autocorrelation in the residuals but reject the null for X12 method. On the other hand, BDS and MBDS tests accept the null hypothesis of no autocorrelation in the residual for OSM and X12 models but reject the null for BRM. Other diagnostic tests indicate that the OSM and X12 residuals are linear, homoscedastic and normal whereas BRM residuals are linear and homoscedastic but not normal. The goodness of fit statistics AIC and SGMASQ favor OSM but all three RBARSQ's favor the BRM model. The forecasting performance of BRM is superior to those of OSM and X12 for this series with lower RMPESS (Table 2)for multi-step forecasts. In Table 3 the F-tests indicate the presence of stable but not moving seasonality for all three methods. The m7 statistics for all three methods are within acceptable range which indicates that the seasonality is identifiable. The TRS and SAS statistics in table 3 indicate that BRM gives the smoothest trend and smoothest seasonally adjusted series; however these statistics are not very far apart as the graphs in figure 2 row 1 indicate that the trend for all methods is quite smooth; and the graphs of the seasonally adjusted series (not shown) are quite unsmooth for all three methods because of the presence of trading day and Easter holiday effects in them. The combined seasonally, trading day and Easter holiday adjusted series as shown in figure 2, row 2, on the other hand, are smoother for BRM and OSM and is least smooth for X12 method. It is not clear, however, what has caused the emergence of annual upward blips in the combined adjusted series for OSM and a single big upward blip in the combined adjusted series for BRM. The residual errors as shown in figure 2 row 3 show that OSM residuals lie in the range -0.20 and 0.20 and the X12 residuals lie in the widest range -80.00 and 80.00. The BRM residuals except for February 1990 also lie in a narrow range. The implication of this is that the OSM model decomposes the series better than the other two methods except that BRM also does that quite well but for a single point.

5. Summary and conclusions

This paper has presented empirical comparison of the three methods of seasonal adjustment in the presence of trading day and Easter holiday effects. The SSMB and Burman's ARIMA model-based methods perform much better than Census's X-12 ARIMA method in case of both series. Although the model statistics for Burman's method are superior to those of SSMB method, the estimated trend and seasonally, trading day and Easter holiday adjusted series are smoother for the SSMB method.

6. References

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Table 1. TESTS OF MIS-SPECIFICATION OF A MODEL

		ADEQUACY OF MODEL TESTS					OTHER DIAGNOSTIC TESTS				
		LJUNG-BOX CHI-SQUARE TESTS FOR DIFFERENT NUMBERS OF AUTO-CORRELATIONS IN RESIDUALS (degrees of freedom)			BDS	MBDS (Min/Max)	CHI-SQUARE TEST OF NON-LINEARITY IN RESIDUALS FOR DIFFERENT NUMBERS OF AUTOCORRELATIONS IN RESIDUALS (degrees of freedom)			F-TEST OF HETRO-SCHE-DASTI-CITY IN RESIDUALS (degrees of freedom)	CHI-SQUARE TEST OF NOR-MALITY OF RESIDUALS (degrees of freedom)
Series	Model	12	24	36			12	24	36		
HDW	OSM	30.31 (3)	42.41 (15)	54.44 (27)	-1.23*	-0.49/1.48*	11.72* (12)	17.65* (24)	30.04* (36)	0.89* (32,32)	1.51* (2)
	X12	68.26 (2)	177.68 (14)	250.87 (26)	-0.24*	-2.74/ -1.23	10.40* (12)	27.03* (24)	73.94 (36)	1.68* (32,32)	20.41 (2)
	BRM	69.86 (8)	143.43 (20)	180.40 (32)	0.36*	-2.07/ -0.45*	14.47* (12)	26.92* (24)	54.63** (36)	1.64* (32,32)	0.25* (2)
MBC	OSM	-	15.44* (10)	22.01* (22)	0.35*	-1.02/ -0.33*	8.72* (12)	21.13* (24)	44.11* (36)	0.39* (32,32)	6.68** (2)
	X12	34.81 (0)	98.02 (12)	165.50 (24)	0.73*	-2.78/ -0.65*	4.16* (12)	14.71* (24)	30.52* (36)	0.52* (32,32)	2.23* (2)
	BRM	18.45* (10)	28.97* (22)	40.53* (34)	2.53	-1.11/ 4.61	14.35* (12)	14.94* (24)	16.06* (36)	0.81* (32,32)	3751.50 (2)

Note One star (*) indicates that null hypthesis accepted at 5% level of significance;
Two stars (**) indicate that null hypothesis accepted at 1% level of significance

Table 2. GOODNESS OF FIT AND FORECASTING PERFORMANCE OF A MODEL

SERIES	MODEL	GOODNESS OF FIT					FORE-CASTING
		AIC	SGMASQ	RBARSQUARE			RMPESS
				Regular	Difference	Seasonal	
HDW	OSM	1046.48	1.02	0.94	0.80	0.58	803.13
	X12	1081.66	16663.33	0.95	0.81	0.61	507.11
	BRM	982.02	5076.57	0.98	0.94	0.88	500.05
MBC	OSM	711.82	1.00	0.99	0.99	0.77	136.68
	X12	786.04	640.90	0.99	0.99	0.74	140.56
	BRM	-	96.92	0.999	0.999	0.96	103.01

Table 3. QUALITY OF SEASONAL ADJUSTMENT

Series	Model	Presence of Seasonality			Identifiable Seasonality Measure m7	Orthogo-nality		Smoothness of Trend TRS	Smoothness of Seasonally Adjusted Series SAS
		Stable S. F(df)	Moving S. F(df)	Residual S. F(df)		OG1	OG2		
HDW	OSM	27.07 (11,77)	0.16* (7,77)	0.03* (11,84)	0.37	0.04	0.04	0.59	3.67
	X12	10.67 (11,77)	1.50* (7,77)	0.56* (11,84)	0.73	0.01	0.04	0.58	4.04
	BRM	12.82 (11,77)	0.86* (7,77)	0.00* (11,84)	0.61	0.05	0.04	0.73	2.78
MBC	OSM	748.19 (11,77)	0.12* (7,77)	0.30* (11,84)	0.07	0.02	0.04	0.32	2.88
	X12	595.18 (11,77)	1.73* (7,77)	0.24* (11,84)	0.10	-0.00	0.03	0.36	3.41
	BRM	557.95 (11,77)	1.10* (7,77)	0.71* (11,84)	0.10	0.07	0.04	0.30	2.36

Note: One star (*) indicates that null hypthesis accepted at 5% level of significance;
Two stars (**) indicate that null hypothesis accepted at 1% level of significance

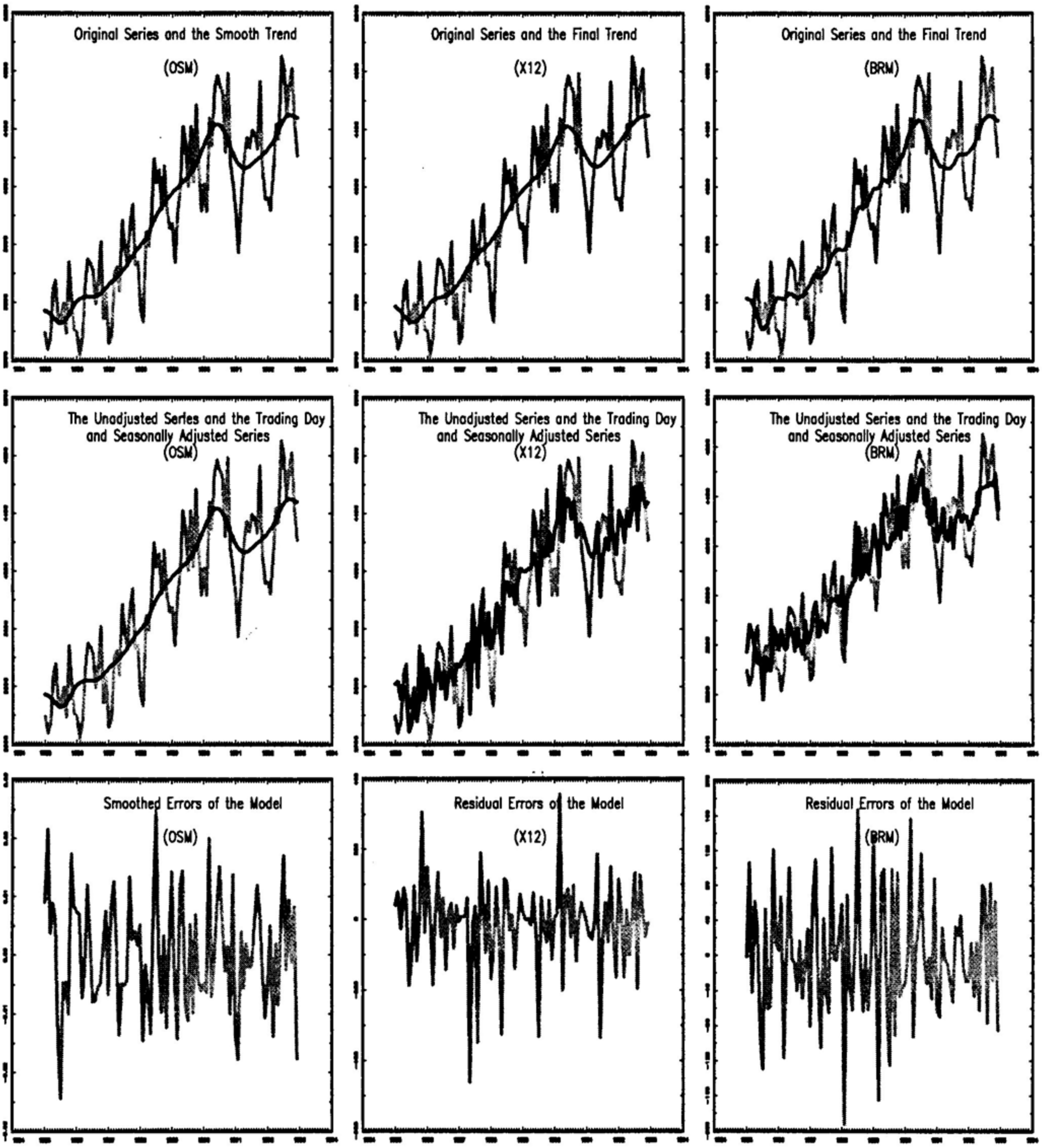

Figure 1. Whole Sales of Hardware Plumbing and Heating Equipment etc.

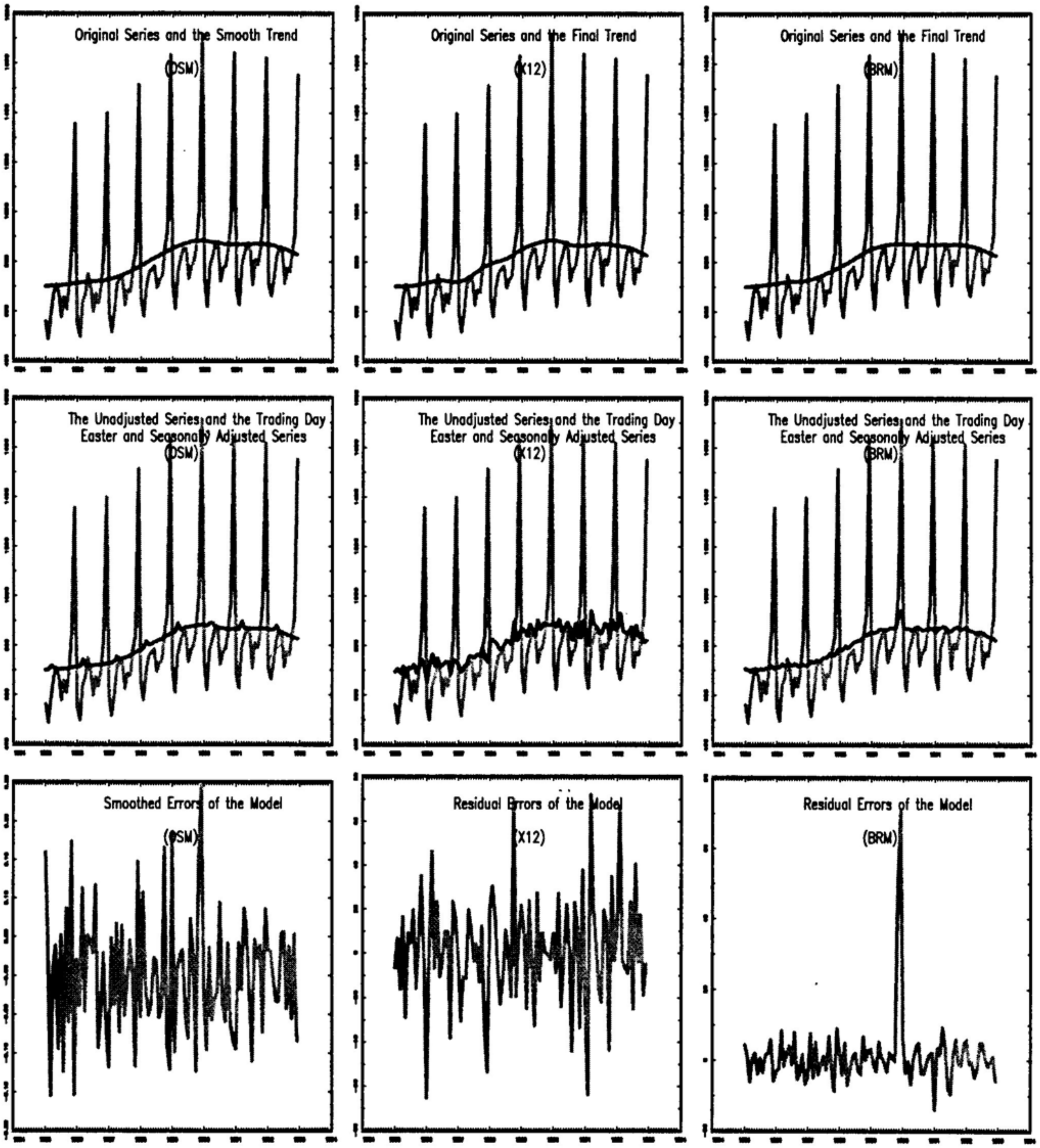

Figure 2 Retail Sales of Men's and Boys' Clothing Stores