Moral Hazard, Asymmetric Decisions, and the Shadow
Price for Quality Adjustment in Medical Services

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Abstract

The Consumer Price Index (CPI) does not adjust for many of the quality changes in medical care. This has often been used as one factor to justify the conclusion that the CPI has upward bias. When using this factor as evidence of the upward bias in the CPI, one is assuming that the shadow price for quality enhancement is greater than zero. We show that under certain conditions, the shadow price of quality enhancement is not guaranteed to be positive. When there are negative shadow prices for quality increases, the CPI or a Laspeyres index no longer is an upward bound on the true cost of living. The shadow prices for quality enhancement are also affected by the reimbursement system for health care services and by the consumer's inability to choose the quality characteristics of his medical consumption. As health care coverage increases, the quality shadow price under a “fee for service” reimbursement converges to a negative value; however, under a capitated reimbursement the limiting quality shadow price as coverage expands is indeterminate.
I. Introduction

The Consumer Price Index (CPI) that is disseminated by the US Bureau of Labor Statistics is not a true Cost of Living Index (COL), but rather it is a "modified Laspeyres index". Traditional price index theory has established the Laspeyres index as an upper bound on the true COL. This result is based on the assumptions that all preferences are not Leontief, and that in equilibrium, the marginal rates of substitution between any two goods is equal to their price ratio. Since the CPI is a "modified Laspeyres" index, it also does not incorporate the utility gains from substitutions when relative prices change.

Another factor that is often cited for the upper bound nature of the CPI is that it does not incorporate many of the quality enhancements in the medical industry. It is automatically assumed that these quality enhancements have positive shadow prices. Using partial equilibrium analysis, Fisher and Shell[1972] show that exogenous quality enhancements will always have positive shadow prices as long as the marginal utility of the quality characteristic is greater than zero. Rosen[1974] develops the hedonic theory for using regression models to estimate quality shadow prices in a setting of perfect competition and divisible quality characteristics. More recently Berry, Levison, and Pakes[1995] and Goldberg[1995] both critiqued the perfect competition framework of Rosen and developed discrete choice models with differentiated markets. However, all the studies mentioned in this paragraph have the consumer making decisions on the quality characteristics of his consumption bundle, and the consumer pays the full equilibrium market price for all the commodities in his consumption bundle.

In the medical care market, the major conditions of the studies in the previous paragraph are not satisfied. The presence of insurance does not ensure that the consumer directly pays the full equilibrium market price for the health goods in his commodity bundle, nor does he choose (or maybe even observe) the quality characteristics when he decides to purchase medical goods. In the medical market, the provider usually chooses the characteristics for the treatment given to the patient. The only choice left to the patient is to accept or reject the prescribed treatment. I will refer to this consumer constraint as an asymmetric decision (ASD). When these anomalies in the medical market are combined with the property
that medical goods and nonmedical goods are complements rather than substitutes, we can no longer
guarantee that the shadow price for quality enhancements in medical goods is always nonnegative nor can
we always ensure that the Laspeyres price index is an upper bound for the true COL.

Shadow prices for quality enhancements are affected by the mechanism that finances health care
purchases. Medical purchases are often financed by third party reimbursements that incorporate a
copayment or coinsurance feature. These financing mechanisms influence the quantities of medical goods
purchased by the consumer as well as the provider’s allocation of quality characteristics. The moral
hazard effects(MH) are defined as the effects on the equilibrium prices and allocations that come from the
financing mechanism for health care. The MH effects on quality enhancements are not only the result of
the level of insurance coverage, but also by the third party’s reimbursement policy to the provider. In the
traditional “fee for service” (here in after referred to as FS) the provider is reimbursed according the
quality of the service. The newer capitated plans (hereinafter referred to as CS) reimburse at a constant
rate per treatment and the provider must bear the marginal costs of quality enhancements. The functional
form of the ratio of observed price changes to the changes in quality levels is dependent on the type of
reimbursement for providers. This should have important implications when using hedonic regressions to
recover the shadow prices of quality changes.

The major conclusions of this paper is that a Laspeyres index is not guaranteed to be an upper bound on
the true COL when it has a medical component where the medical care industry’s allocation is subject to
both MH and ASD. An important secondary result is that under ASD the shadow price of any quality
enhancement depends on the financing mechanism for health care.

In reality, the medical market is heterogeneous. Some types of providers are more competitive than
others. As a result of this, I start my modeling of the provider under the assumption of monopolistic
behavior and then I model the provider as a perfect competitor. The true behavior of a provider probably
falls between these two extremes.
Section II establishes the consumer model and we show that an exogenous increase in quality does not always have a positive shadow price. However, quality is an endogenous variable and in order treat it properly, we must do an equilibrium analysis. Therefore, in section III, we establish a separate supplier model under FS and CS. Since quality allocations are endogenous in a general equilibrium setting, in section IV, we perturb a variable that is exogenous to the model and compare the observed prices changes to the changes in quality levels for medical services provided under FS and CS.

II. The Consumer Model

We observe that medical goods and services have several important distinguishing characteristics. First, they are complements to other nonmedical goods. The incidence of illness inhibits one from enjoying the consumption of his non-medical goods. The purpose of medical expenditures is to provide healing services so that the consumer can obtain higher enjoyment from the consumption of non-medical goods. The second distinguishing aspect of the medical market is that the consumer does not choose the quality characteristics of the medical good that he consumes. (For instance, a patient does not choose the quantities of the hospital’s radiology, cardiac care, and operating room services or even in most cases whether he can use the inpatient hospital as a source of treatment. In the automobile sector, the consumer can choose characteristics such as anti-lock brakes, gasoline mileage, automobile size, etc.) A third distinguishing characteristic is the financing of medical goods. Third party payments are an important source of financing. These third party payors either pay a fraction of the total price or pay on the basis of a fixed schedule of payments made by the payor and the beneficiary. In either case, the result is that the patient faces a combination of predetermined insurance payments which are not affected by his current consumption of medical goods, and a net price that is lower than the full reimbursement.

We model the consumer sector first without a third party payor, and derive a shadow price for quality changes. There are two goods, one medical and the other is nonmedical. The price of the nonmedical good is the numeraire. The consumer receives an exogenous source of income (denoted as Y) in terms of the nonmedical good. He draws a random sickness variable, x, which has nonnegative support. If x
equals zero then the consumer is in perfect health. Otherwise, the consumer is inhibited from fully enjoying his nonmedical income. He then chooses to exchange a fraction of his nonmedical endowment to purchase a certain quantity of medical goods (denoted as $M$) which is nonnegative. (At this point $M$ can be either a binary variable, the set of nonnegative integers, or the set of nonnegative reals.) In this section, we will assume that $M \in \mathbb{R}$ so that we can use derivatives in deriving our results. In the next section, $M$ will be a binary variable that takes on the values of 0 or 1.

After his draw of $x$, the consumer must choose $M$ to maximize

$$
U = U(Y - MP + h(x, M, I))
$$

$U(\cdot)$ is monotonically positive and strictly concave with respect to its argument. The price of the medical good is $P$, and $I$ is a vector of characteristics of the medical good. The function $h(\ldots)$ is a mapping that quantifies the reduction in the consumer's ability to enjoy his nonmedical endowment after a draw of $x$. $h(\ldots)$ is zero if $x$ is zero, and is monotonically negative with respect to $x$. If $x$ is greater than zero $h(\ldots)$ is monotonically increasing with both $M$ and each element in $I$. The consumer takes $I$ as a given. This represents a departure from the other hedonic models where the consumer does have choice over the quality characteristics.

To keep our analysis simple and to avoid distraction from the main idea of this study, we will let $I$ be a scalar. The function $h(\ldots)$ is then an equivalence mapping in the following sense. If a consumer draws a nonzero value of $x$, then the consumer is indifferent between receiving no medical goods and non medical goods equal to $h(x, M, I) - h(x, 0, I)$ or a quantity of $M$ medical goods with characteristic $I$. $M$ and $I$ are compliments where the following holds:

$$
(2.2) \quad h_{MI}(x, M, I) > 0
$$
In this paper functional subscripts, such as $I$ and $M$ in $h_M$, will refer to derivatives of that function so that $h_M(....)$ is the second derivative of $h(....)$ with respect to $I$ and $M$. In this study, $h(....)$ is strictly concave in each of its three arguments.

In this simple problem the first order conditions are:

$$P = h_M(x, M, I)$$

The second order condition $h_MM(....) < 0$ is immediately satisfied given our restrictions in the previous paragraph. We denote the solution as $M^*$ which is a function of $x, I$, and $P$. For now, we will suppress the arguments on $M^*$.

We are now ready to derive the shadow price for an exogenous increase in $I$. We need to solve for $dP/dI$ such that total utility remains constant. We solve the following

$$U_x \frac{dY}{dP} + U_y \frac{dY}{dI} = 0. \tag{2.4}$$

Using the Envelope theorem, this problem reduces to

$$-M^* dP + h_I dI = 0. \tag{2.5}$$

Therefore, $dP/dI = h_I / M^*$ which is always nonnegative. It is important to note here that all the conditions of price index theory have been satisfied. In particular, the consumer chooses his medical consumption so that his marginal rate of substitution equals the price ratio between the two goods. Secondly, all expenditures are financed out of income.

We now introduce a third party payor. The consumer now pays a set fraction, $c$, of his total medical expenditures and the third party payor pays $(1-c)$. Again, to keep the analysis simple, we assume that there is no deductible on the policy. The consumer pays an actuarially fair premium before he draws his
value of \(x\). (If he paid his premium after his draw of \(x\), there would be a certainty of illness rather than just a risk.) Letting \(f(x)\) denote the probability density of \(x\), the actuarially fair premium is

\[
(2.6) \quad r = (1-c) \int_{\mathbf{X}} P M'' f(x)dx
\]

where \(\mathbf{X}\) is the support of \(x\), and \(M''\) is the solution to the consumer's problem when he pays the out-of-pocket price, \(cP\). In the above representation, the arguments in \(M''\) are determining factors on the setting of the premium. We now derive \(M''\). Given any \(r\), the consumers' problem is changed to choose \(M\) to maximize:

\[
(2.7) \quad U = U(Y - r - cMP + h(x, M, I))
\]

The first order condition is

\[
(2.8) \quad cP = h_m(x, M'', I)
\]

Notice that \(r\) does not enter into the first order conditions. Therefore, \(r\) can be easily derived in (2.6) by using the solution (2.8). \(M''\) now has \(c\) as an additional argument. For a given draw \(x\), price \(P\), and quality \(I\), \(c<1\) implies that \(M'' > M^*\). Notice that the net price ratio, \(cP\), equals the marginal rate of substitution, and that \((1-c)\) of the total medical expenditure is financed by the third party payor.

We now wish to solve for the shadow price for an exogenous increase in \(I\). The following must be satisfied:

\[
(2.9) \quad U_y (-r_p - cM''') dP + U_y (-r_i + h_1) dI = 0
\]

Now,

\[
(2.10) \quad r_i = (1-c) \int_{\mathbf{X}} P M'' f(x)dx > 0
\]
and

$$r_P = (1 - c) \int PM^{**} f(x) dx + (1 - c) E_X(M^{**})$$

$r_P$ is greater than zero if the demand price elasticity for medical care has an absolute value that is greater than one. Since the absolute price elasticity for medical care is usually hypothesized to be below one, we will assume that $r_P$ is positive. The shadow price for I is now:

$$\frac{dP}{di} = \frac{-n_1 + h_1}{r_p + cM^{**}}$$

The sign of $dP/di$ depends on the sign of $-n_1 + h_1$. $n_1$ is the MH effect on the shadow price of I. The sign of this expression is ambiguous. The important result here is that the shadow price is no longer guaranteed to be positive. In this economy, one cannot guarantee that the Laspeyres index that is not adjusted for quality changes is an upper bound on the true COL.

III. The Provider Model

In section II, we derived the shadow price of an exogenous increase in the quality characteristic. However, the analysis in section II is still inadequate if we are to understand the relationship between the observed price changes in medical care and the changes in its quality characteristics. Under ASD, we must control for the reimbursement policy in order to recover the shadow price of quality changes from the observed equilibrium prices of the medical good. In the medical care market under ASD, only the provider chooses the quality characteristics of medical goods and services. The consumer's only choice is whether or not to purchase the medical good or service.

In this section we develop and compare the quality allocation decision made by a provider under FS and CS. Under FS, the provider is reimbursed according to the characteristics of the treatment. More resources imply a greater reimbursement. Under CS, the unit of reimbursement is the discharge or the treatment. All treatments are reimbursed at a fixed amount, and the marginal costs of any additional
resource is paid by the provider. Therefore, one might wish to determine whether the provider’s incentives toward the allocation of the quality of medical goods depends on the reimbursement system. When modeling the provider’s decision, we borrow from the results of Hodgkin and McGuire[1994] where they show that under CS, the provider’s allocation of service intensity (or quality) is less than the allocation under FS.

We start our model with N consumers that each have the income and utility that are outlined in section II. Each consumer pays the same actuarially fair insurance premium and each consumer faces the same probability distribution of the sickness variable f(x). Each consumer will take a i.i.d. draw of x. All subscripts of the variable x will index the consumer so that \( x_i \) is the sickness draw of the \( i \)th consumer.

After the consumer draws his sickness variable, he will visit the medical provider if his draw of x is greater than zero. The provider will then observe this x and prescribe a medical treatment with a certain I. The choice of I can vary by consumer. Under FS the provider is reimbursed according the amount of I prescribed. We denote this rate of reimbursement as \( R \). Under CS, there is a fixed reimbursement (denoted as \( \alpha \)) for each treatment. Thus under FS the observed total price for a treatment is \( RI_i \), and is \( \alpha \) under CS. The variable M that was introduced in section II is now a binary variable. If the consumer chooses to consume the prescribed treatment then \( M \) equals one; otherwise \( M \) equals 0. Under FS, the consumer will choose to purchase the prescribed treatment if

\[
(3.1) \quad U(Y - r_{FS} - cRI_i + h(I, x_i, I_i)) > U(Y - r_{FS} + h(0, x_i, I_i)) \quad \text{(FS)}
\]

\( I_i \) is the quality level prescribed for treatment for the \( i \)th consumer \( r_s \) is the actuarially fair insurance premium under FS. Under CS, the consumer will choose to purchase treatment if

\[
(3.2) \quad U(Y - r_{CS} - c\alpha + h(I, x_i, I_i)) > U(Y - r_{CS} + h(0, x_i, I_i)) \quad \text{(CS)}
\]

We have subcribed the insurance premium variable as either \( FS \) or \( CS \) since the actuarial value of the policy depends on the reimbursement system. If we let
\( g(x, I) = \frac{h(I, x, I) - h(0, x, I)}{c} \)

the choice rules for treatment can be further simplified to

\( M_i = 1 \text{ if } RI_i \leq g(x, I) \) \hspace{1cm} (FS)

and

\( M_i = 1 \text{ if } x \leq g(x, I) \) \hspace{1cm} (CS)

It should be clear that the following conditions hold based on \( h(\ldots) \).

C.1) \( g(\ldots) \) is zero for all \( x \) equal to zero.

C.2) \( g(\ldots) \) is monotonically increasing in both arguments and is strictly concave in both arguments.

The provider's marginal cost for each \( I \) is \( C \) which includes opportunity costs. After the provider observes \( x \), he then chooses a treatment with \( I \). It is obvious that \( R \) must be at least as great as \( C \) for services to be rendered under FS. Therefore, under FS, the provider will attempt to maximize \( I \) subject to (3.4). Under CS, the provider will wish to minimize \( I \) subject to (3.5).

We start with the condition that there is a monopolist provider, and then will change the condition so that the provider exhibits behavior under perfect competition. In a monopoly situation for a given \( R \) and \( \alpha \), the provider can extract all of the consumer surplus from the consumption of the medical good. Therefore, for a given \( R \) and \( \alpha \), the decision rule for \( I \) under a monopolistic FS provider is \( I = \frac{g(x, I)}{cR} \), and under a monopolistic CS, the decision rule is \( \alpha = \frac{g(x, I)}{c} \). Notice that each consumer's level of \( I \) will not always be greater under FS than CS. If \( \alpha > RI_i \), then the provider under CS will provide a higher level of \( I \), in order to induce the \( i \)th consumer to purchase the good. If the provider under CS, must provide a level of \( I_i \)
where \( CI > \alpha \) in order to induce the \( i \)th consumer to buy the medical good then the patient will not be treated. Therefore \( \alpha/C \) is an upper bound on \( I_i \).

We now focus on the choice of \( R \) and \( \alpha \) under monopoly. The provider will choose \( R \) and \( \alpha \) under FS and CS respectively in order to maximize profit. The problem under FS is to choose \( R \) to maximize:

\[
(3.6) \quad (R - C) \sum_{i=1}^{N} I_i \chi_i
\]

where

\[
(3.7) \quad \chi_i = \begin{cases} 
1 & \text{if } x_i > 0 \text{ and } \exists I_i \text{ such that } I_i \leq \frac{g(x_i, I_i)}{cR} \\
0 & \text{otherwise}
\end{cases}
\]

The problem under CS is to choose \( \alpha \) to maximize:

\[
(3.8) \quad \sum_{i=1}^{N} \psi_i (\alpha - CI_i)
\]

where:

\[
(3.9) \quad \psi_i = \begin{cases} 
1 & \text{if } x_i > 0 \text{ and } \exists I_i \text{ such that } \alpha \leq g(x_i, I_i) \text{ and } CI_i \leq \alpha \\
0 & \text{otherwise}
\end{cases}
\]

If we substitute \( I_i = \frac{g(x_i, I_i)}{cR} \) into (3.6) we get the first order conditions under FS

\[
(3.10) \quad \sum_{i=1}^{N} \chi_i (I_i - Rg_{x_i}) = 0
\]

Here, \( g_{x_i} \) denotes the partial derivative of \( g(x_i, I_i) \) with respect to \( I_i \).

Under, CS since \( \alpha = g(x_i, I_i) \), the following holds:
\[
\frac{\partial l_i}{\partial \alpha} = \frac{1}{g_{l_i}}.
\]

First order conditions are

\[
\sum_{i=1}^{N} \psi_i (1 - \frac{1}{g_{l_i}}) = 0
\]

In this analysis, when the provider acts as a monopolist, he can maximize his profit through the selection of both the individual characteristics and the charge.

We impose one additional condition on \(g(\cdot)\).

(C.3) \(g(\cdot)\) is linearly homogenous.

We now state the result from this section:

**Theorem 3.1** Given C.1, C.2, and C.3, the expected shadow price of an exogenous increase in \(l_i\) is less under FS than under CS when the provider exhibits monopolistic behavior.

The proof is outlined in the appendix.

We give a brief intuition behind Theorem 3.1. Under CS, providers have incentives to economize on treatment quality. Although some consumers with low draws of \(x\) might demand higher levels of \(l_i\) in order to pay \(\alpha\), the provider will not give treatment if the cost of treatment is greater than \(\alpha\). For the consumers with a relatively high draw of \(\alpha\), the provider can provide relatively low levels of \(l_i\) and still induce these relatively ill consumers to purchase treatment. Therefore, under CS there is an upper bound on the level of \(l_i\) that will be allocated to any consumer where

\[
(3.13) \quad l_i \leq \frac{\alpha}{C_i} \quad \forall i = 1, \ldots, N
\]
This result will in turn place a lower bound on the consumers marginal value of I under CS. Under FS there is no upper bound placed on the level of I. Under the assumptions of the model, sicker consumers will have higher values of the first allocations of I and will be willing to accept less I for a payment of α. This will produce an allocation were the consumer’s marginal value of I (h₁) will be inversely related to the sickness variable x. Under, FS the h₁ will be the same across all consumers. Thus, the variance of the shadow price for medical care under CS will be positive, but given conditions C.1-C.3 will be zero under FS. Therefore, h₁ for the sicker consumers under CS is higher than under FS, and the reimbursements are lower for the sicker consumers under CS, there will be greater upper pressures on the shadow price of I under CS.

The results in this section should not imply that welfare is improved if the reimbursement system under monopoly changes from FS to CS. Neither situation is pareto optimal because the consumer does not choose I to maximize his utility, and because the presence of coinsurance induces welfare loss.

The condition of a monopolistic provider is an extreme condition, and perhaps unrealistic. Under monopoly, we would get the result that higher coverage(i.e. a lower c) would not induce a change in I. If c drops to c’ then R and α will increase by c/c’ and, the payment made by the consumer will stay constant and the insurance premium will rise to finance the increase in the prices and the coverage. However, we have observed increases in medical quality over time, and this observation should allow us to conclude that not all providers act in a monopolistic manner.

At the other extreme is the perfectly competitive provider. Under this scenario, the consumer is able to extract all produce surplus from the provider if the cost function is linear with respect to quality. Under perfect competition and FS, R=C. (Remember that C includes an opportunity cost.) Since the patient can choose among many providers, the provider will treat the patient as if the following first order condition is satisfied

\[(3.14) \quad g_i(x_i, I_i) = C\]
This condition would bring the economy close to pareto optimal equilibrium if there was no insurance market, because the patient would have some indirect control over the level of I that is embodied into his medical service. The shadow price of an exogenous quality change under a perfectly competitive FS provider is at least as great as the shadow price under an FS monopolistic provider. \( h_i \) is the same under both scenarios, but \( r_{\text{pol}} \) is smaller.

Under CS, if every provider faces the same unit cost, \( C \), and the same reimbursement per service then all patients would receive the same allocation of I regardless of there sickness level. Since \( g_i \) is always greater than zero and the payments are fixed, consumers would gravitate to the providers that would allocate the highest quality. The producer would only provide treatment if he could cover his costs. As result, the provider would allocate \( \alpha/C \) units of I to each patient. The reimbursement rate, \( \alpha \), would be chosen before the draw of the random sickness variable. Since the provider now operates as a perfect competitor, the insurance company as a perfect agent of the consumers would negotiate a rate, \( \alpha \) that would maximize the expected utility of the household. The negotiated rate would be the solution to the following problem.

\[
\begin{align*}
\max_{\alpha} & \int_A U(Y - r_{\text{cs}} - c\alpha + h(1, x, \alpha/C))f(x)dx \\
& + \int_{A^c} U(Y - r_{\text{cs}} - h(0, x, 0))f(x)dx
\end{align*}
\]

where

\[
\begin{align*}
A &= \{x: x > 0 \text{ and } c\alpha \leq g(x, \alpha/C)\} \\
A^c &= \{x: x = 0 \text{ or } c\alpha > g(x, \alpha/C)\}
\end{align*}
\]

We let the scalar \( \bar{x} \) be defined as:

\[
\bar{x} = \text{closure}(A) \cap \text{closure}(A^c)
\]

Then we can characterize the first order conditions to (3.15) as:
\[
(3.17) \quad \int_A U_Y(\frac{h_i}{C} - c)f(x)dx - \int_{A + A'} U_Y \frac{\partial r_{es}}{\partial \alpha} f(x)dx \\
+ U(Y - r_{es} + h(0, x, 0)f(\bar{x})\bar{\alpha}) / \partial \alpha - U(Y - r_{es} - c\alpha + h(1, x, \alpha / C)f(\bar{x})\bar{\alpha}) / \partial \alpha = 0
\]

The last two terms offset each other so that we can simplify the first order conditions to:

\[
(3.18) \quad \int_A U_Y(\frac{h_i}{C} - c)f(x)dx - \int_{A + A'} U_Y \frac{\partial r_{es}}{\partial \alpha} f(x)dx
\]

We end this chapter by comparing the shadow prices of an exogenous increase in I between CS and FS under perfect competition. We can look the first order conditions (3.14) and (3.18). Similar to the monopolistic FS, \(g_i\) will be constant across all consumers due to condition C3, whereas it will vary across consumers under CS. Under CS, \(g_i\) increases with \(x\). However (3.18) no longer ensures that \(E(g_i)\) under CS is greater than it is under FS. We can rewrite (3.18) as

\[
(3.19) \quad E((g_i - C)U_Y|A) = \frac{(1-c)}{c}CE(U_Y)(1 + \alpha f(\bar{x})\frac{\partial \bar{x}}{\partial \alpha}) > 0
\]

The result (3.19) tells us that the expected value of \(g_i\) that is weighted by the marginal utility for income is greater than \(C\), but in fact \(E(g_i)\) could be less than \(C\) and (3.18) and (3.19) would still be satisfied. Since the treatment price for the consumer is constant under CS. The insurance premium effect under FS is

\[
r_{FS,1} = (1-c)C \times Pr(x; g_i \geq C \text{ for some } I)
\]

and under CS

\[
r_{CS,1} = (1-c)\alpha f(\bar{x}) \left( \frac{g_i}{\bar{g}_i} \right)_{1, \alpha / C}
\]

The difference between \(r_{FS,1}\) and \(r_{CS,1}\) is indeterminate.
IV The Relationship Between Observed Prices and Quality Changes

In section II, we established the result that not all shadow prices for quality increases are positive, and in section III we established that the expected shadow price for medical care depends on the reimbursement mechanism. In this section, we perturb the exogenous variable $c$ by lowering its value. This perturbation represents an exogenous increase in insurance coverage. Looking at the Health Expenditure Accounts from the United States Health Care Financing Administration, we observe that the fraction of total payments that are financed out of pocket has declined over time. We derive the the observed price change, the increase in quality, and the shadow price for this increase under both FS and CS respectively. The results in this section are an important first step if one wishes to use hedonic regressions to recover a consistent estimator for the shadow price of medical care quality.

Under the monopolistic scenario, any perturbation in $c$ will not affect the final equilibrium allocation of the quality allocated to each patient. If $c$ drops then the provider adjusts either $\alpha$ or $R$ so that the net payment made by the patient is exactly the same amount as before the drop.

Under perfect competition, an exogenous drop in $c$ should increase the final allocation of quality and the price observed by the consumer should also increase. We first look at the results under FS. Since the cost structure $C$ has not changed, we can implicitly differentiate (3.14) with respect to $c$, and we get

\[
\frac{\partial l_i}{\partial c} = \frac{g_{l_i}}{c} = \frac{h_{l_i}}{c} = \frac{h_{l_{l_i}}}{h_{l_{l_l}}}
\]

(4.1)

Therefore the average observed change in the observed treatment price is

\[
\int C \frac{\partial l}{\partial c} f(x) dx = \int C \frac{h_{l_i}(l, x, I(x))}{h_{l_{l_i}}(l, x, I(x))} f(x) dx
\]

(4.2)

$I(x)$ denotes the allocation of $I$ given sickness level $x$ when the first order conditions, (3.14), are satisfied. (4.1) and (4.2) show that both the observed price and the allocation of $I$ go to infinity as $c$ goes to zero.
For the ith consumer the observed price is \( P_i = CI_i \) and the shadow price is the solution to:

\[
(4.3) \quad \int \{ U_Y(-\frac{\partial q}{\partial P} - c)xP / \partial l + U_Y(-\frac{\partial q}{\partial I} + h_1)f(x)\}dx
\]

\[
(4.4) \quad \frac{\partial P / \partial l}{\partial I} = \frac{\int U_Y(-\frac{\partial q}{\partial I} + h_1)f(x)dx}{\int (U_Y(\frac{\partial q}{\partial P} + c)f(x)dx}
\]

The denominator to (4.4) is always positive, and as \( c \to 0 \), the numerator converges to a negative value.

As \( I \) goes to infinity and \( c \) goes to zero, \( h_1 \) will converge to zero. Therefore under a perfectly competitive FS provider, a continued increase in coverage will eventually cause the shadow price for quality to be negative. Thus at some point the ratio of the observed price change to the shadow price of the quality change will become automatically negative when the coinsurance rate goes below a critical level.

The results for a perfectly competitive CS provider are different from the perfectly competitive FS provider. Recalling from section III, the CS provider will provide a constant amount of quality per patient and the amount is equal to \( \alpha/C \). Therefore if \( \alpha \) goes to infinity as \( c \) goes to zero, then the allocation of \( I \) will go infinity as \( c \) goes to zero. However, we can easily show that \( \alpha \) is bounded and will not go to infinity as \( c \) goes to zero. We state the theorem here and the proof is in the Appendix.

**Theorem 4.1** Under perfect competition, and CS if conditions C.1 through C.4 hold, then there is an upper bound on \( \alpha \).

The intuition behind the bound on \( \alpha \) is that before the draw of the random variable \( x \), the insurer sets \( \alpha \) accord to the first order conditions in (3.18). Even if under full coverage where \( c=0 \), the consumer is better off with a finite \( \alpha \) and a finite allocation of \( I \), than an infinite \( \alpha \) and \( I \).
Finally, under CS we cannot ensure that the shadow price of quality increase converge below zero. Since there is an upper bound on I, there is an upper bound on r_t and h_t. This in turn places a lower bound on the shadow price for quality increases.

Table 1 summarizes the effects of continued insurance coverage on the expected allocation of quality, the expected observed price, and the expected shadow price. Historically, out-of-pocket expenditures have fallen from 55% in 1960 to less than 20% in 1993. Up until the late 1980’s, most services fell under FS reimbursement. This would seem to indicate that the higher level of national coverage induced higher quality allocations, and continued growth in observed relative medical prices. It remains uncertain whether these quality enhancements actually had positive shadow prices. It is clear however that one cannot automatically conclude that the quality enhancements of the 1970’s and early 1980’s had positive shadow prices. The results of this study also imply that one cannot use the increases in the allocation of medical quality as an automatic
V Conclusions

We have demonstrated in this study that the shadow price for the increase in the quality of medical services is not always positive. Because of this result we can no long ensure that the Laspeyres Index which is not adjusted for quality is an upper bound on the true COL. We further show that under ASD the shadow price of medical quality depends on the reimbursement mechanism for medical care. Under either perfect competition or monopolistic behavior the expected shadow price of an exogenous increase in quality is higher under CS than under FS.

However, the allocation quality characteristics across consumers is not an exogenous event. Therefore, we perturbed the coinsurance rate and derived the observed price changes and the shadow prices of the increased quality allocation. Under FS and perfect competition we found that as coverage increased, observed prices would grow in an unbounded manner as shadow prices fell below zero. Whereas under CS, continuous increases in coverage would not induce observed prices to continuously grow, and there would be a lower bound place on the shadow price of medical care. These results are crucial when the econometrician wishes to use hedonic regression to recover the shadow price of medical quality from the observed price and quality increases.

<table>
<thead>
<tr>
<th>Table 1</th>
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<td>Effects of a Continued Expansion in Coverage</td>
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<td>(c → 0)</td>
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<table>
<thead>
<tr>
<th>Variable</th>
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<th>CS</th>
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<tr>
<td>Expected Quality Allocation - E(T)</td>
<td>$\rightarrow \infty$</td>
<td>Converges to a finite value</td>
</tr>
<tr>
<td>Expected Observed Price - E(P)</td>
<td>$\rightarrow \infty$</td>
<td>Converges to a finite value</td>
</tr>
<tr>
<td>Expected Shadow Price - E(∂P/∂q_{obs})</td>
<td>Converges to a negative value</td>
<td>?</td>
</tr>
</tbody>
</table>
Appendix

Proofs

Before proving Theorem 3.1, we need to prove Lemma A.1 which is later used in the proof of Theorem 3.1

Lemma A.1: Under conditions C.1 through C.3, and given a monopolistic provider, total social medical expenditures under CS will be no greater than the revenues under FS.

Proof: First let $X = \{ x_i : \chi_i = 1 \}$ and $\Psi = \{ x_i : \psi_i = 1 \}$ then $\Psi \subset X$, because $X$ contains all consumers whose marginal value $g_i$ of medical care is greater than $C$. However, $g_i > C$ is a necessary but not sufficient condition to be in $\Psi$. Therefore the volume of treatment under CS will be no greater than the volume under FS. The average revenue per patient will also be less under CS. To see this, we suppose that given the optimal $\alpha^\ast$ equals $E(R, I, \Psi)$.

Given the linear homogeneity of $g(\ldots)$, $\alpha^\ast = g(E(x_i | \Psi), tE(x_i | \Psi))$ for some $t$, and $g_i(E(x_i | \Psi), tE(x_i | \Psi)) = C$.

Letting $I_i^\ast$ be the solution that satisfies $\alpha^\ast = \frac{g(x_i, I_i^\ast)}{c}$, we show through Jensen's inequality that

$$E \left[ \frac{1}{\bar{E}_i(x_i, I_i^\ast(x_i)) \Psi} \right] > \frac{1}{g_i(E(x_i | \Psi), tE(x_i | \Psi))} = \frac{1}{C}$$ and this implies

$$E \frac{C}{g_i(x_i, I_i^\ast(x_i))} > 1$$ and the first order conditions for CS are violated. Since $d\alpha/dI > 0$, $\alpha$ must be lower than $E(R, I, \Psi)$.

QED

Proof of Theorem 3.1

We need to show that $-r_1 + E(h_0)$ is less under FS than under CS since the denominator of (2.12) is the same for both FS and CS. We will first show that under FS $E(h_0) = E(g_i)$ is less than it is under CS. Then we will show that $r_1$ under CS is less than $r_1$ under FS.

Under FS, the first order conditions for choosing $R$ are

$$\sum_{i=1}^{N} I_i^\star \chi_i - (R - C) \sum_{i=1}^{N} \frac{dI_i^\star}{dR} \chi_i = 0$$

$$\frac{dI_i^\star}{dR} = \left( 1 - \frac{\bar{E}_i}{cR} \right)^{-1} I_i^\star = \left( \frac{cI_i^\star}{cR - \bar{E}_i} \right)$$

where $I_i^\ast$ is the resulting allocation of $I$ given the optimal choice of $R$.

Given (3.4), and the linear homogeneity of $g(\ldots)$, $g_i$ is constant for all $x$. Therefore, the first order conditions can be written as

$$1 = -\frac{R - C}{\sum_{i=1}^{N} \left( \frac{cI_i^\star}{R - g_i} \right) \chi_i} = -\frac{R - C}{(R - \bar{E}_i)}$$

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This implies that at equilibrium $g_i = C$ for all consumers that purchase the medical good.

Under CS, we first create an order statistic from $x_i$ and denote it as $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(N)}$. Next, we let

$$
\psi_{(j)} = \begin{cases} 
1 & \text{if } x_{(j)} > 0 \text{ and } \exists 1_{(j)} \text{ such that } \alpha \leq g(x_{(j)}, 1_{(j)}) \\
0 & \text{otherwise}
\end{cases}
$$

create the series of compact sets:

$$A_j = (\alpha: \psi_{(j)} = 0, \psi_{(2)} = 0, \ldots, \psi_{(j-1)} = 0, \psi_{(j)} = 1, \psi_{(j+1)} = 1, \ldots, \psi_{(N)} = 1)$$

Let

$$\hat{\pi}_j = \max_{\alpha \in A_j} \sum_{i=1}^{N} \psi_i (\alpha - C l_i^{**})$$

where $l_i^{**}$ is the resulting allocation of $I_i$ given the optimal allocation of $\alpha$, and let $\alpha_i^*$ be the solution.

Then the first order condition for the above problem is:

$$(N - j - C) \sum_{i=1}^{N} \frac{1}{g_i(x_i, 1_{(i)})}$$

There will be $N$ $\alpha_j^*$'s and $\alpha_1^* \geq \alpha_2^* \geq \ldots \geq \alpha_N^*$. The final solution is $\alpha^* = \alpha_j^*$ if $\hat{\pi}_j \geq \hat{\pi}_h \forall h$.

The final first order solution is

$$l = \frac{C}{N - j} \sum_{i=1}^{N} \frac{1}{g_i(x_i, 1_{(i)})}$$

By Jensen's inequality under CS,

$$E(R) > C,$$

where as under FS $E(g_i) = C$.

Under CS $\alpha$ does not vary with $I$ and therefore,

$$r_{CS} = (1 - c) \alpha \partial E(\psi) / \partial I$$

where $r_{CS}$ is the derivative of the insurance premium under CS.

Under FS $r_{FS} = (1 - c) \partial E(\chi) + (1 - c) \partial E(R) / \partial I$. In Lemma A.1, we showed that $E(R) > \alpha$, and $E \partial \psi / \partial I < 1$. Therefore $r_{FS} \geq r_{CS}$.

**QED**

**Proof of Theorem 4.1:**

We establish the second order conditions for the maximization problem in (3.15).

$$\text{SOC} = \int \left[ \frac{h_1}{C} U_Y + U_Y \left( \frac{h_1}{C} \frac{\partial \bar{r}_i}{\partial \alpha} - c \right) \right] f(x) dx - \int \left[ U_Y \frac{\partial^2 \bar{r}_i}{\partial \alpha^2} - U_{YY} \left( \frac{\partial \bar{r}_i}{\partial \alpha} \right)^2 \right] f(x) dx < 0$$

We now differentiate the first order conditions in (3.17) with respect to $c$. We get
\[
\frac{\partial \text{FOC}}{\partial c} = \int -U_Y + U_{YY}\left(\frac{h_i}{C} - \frac{\partial \alpha}{\partial c} - \alpha\right)\left(-\frac{\partial \alpha}{\partial c} - \alpha\right)f(x)dx + (1 - F(\bar{x})) + (1 - c)f(\bar{x})\frac{\partial \bar{x}}{\partial c} + f(\bar{x}) - (1 - c)f'(\bar{x})\frac{\partial \bar{x}}{\partial c}
\]

We can solve for \(\frac{\partial \alpha}{\partial c}\), and

\[
\frac{\partial \alpha}{\partial c} = \frac{\partial \text{FOC}}{\partial c}
\]

SOC

Since \(-\alpha < \frac{\partial r_{cs}}{\partial c} = -\alpha \Pr(x > \bar{x}) + (1 - c)\alpha f(\bar{x})\bar{x} / \partial c < \alpha\), when \(c=0\) and

\(- (1 - c) - \alpha < \frac{\partial r_{cs}}{\partial \alpha} = (1 - c)\Pr(x > \bar{x}) + (1 - c)\alpha f(\bar{x}) < \alpha + (1 - c)\), \(\frac{\partial \alpha}{\partial c}\) is bounded.

QED
References


